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"Teoria Financeira Aplicada à Seleção de Portfólios"

"Financial Theory Applied to Portfolios Selection"

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TEORIA FINANCEIRA APLICADA À SELEÇÃO DE PORTFÓLIOS

Resumo

O objetivo desta tese é explorar e analisar os modelos multicritérios, em especial a aplicação da metodologia ELECTRE III, com o intuito de construir carteiras defensivas, numa estratégia *buy and hold*, onde todos os critérios e alternativas têm pesos iguais, utilizando rácios financeiros (ROA, ROE, FA, GL e RL). Assim sendo, primeiramente selecionamos ações transacionadas no PSI-Geral, de 1999 a 2011, e seguidamente definimos um período inicial / histórico, onde oito carteiras foram construídas uma para cada período, e um período de acompanhamento, considerando um, dois e três anos de monitorização. Ao realizar uma análise descritiva a cada carteira individualmente, considerando todos os períodos, e uma análise estatística apenas para os períodos de acompanhamento, comparamos as rentabilidades e índice de Sharpe calculados para o ELECTRE III, CAPM, e PSI-20 TR. No caso específico de análise estatística realizamos testes paramétricos (*t-student* e ANOVA *one-way*) e testes não paramétricos (Kruskal-Wallis e Mann-Whitney), subdividido em duas amostras: uma única amostra de 2005-2011; e duas sub amostras, uma de 2005 a 2007, e outra de 2008 a 2011.

A análise descritiva indica-nos que, para algumas carteiras, o ELECTRE III e o CAPM comportaram-se melhor que o mercado (PSI), sendo também mais eficiente (índice de Sharpe). Entre o ELECTRE III e o CAPM, num período de três anos (a longo prazo), o ELECTRE apresenta uma rentabilidade média superior ao mercado e ao CAPM, obtendo também uma maior rentabilidade por unidade de risco. Apesar disso, estatisticamente encontramos resultados diferentes, provavelmente devido a limitações do tamanho da amostra. Assim, considerando os testes estatísticos para uma única amostra e para as duas sub amostras, os resultados obtidos levam-nos a concluir, em geral, que não podemos inferir que uma forma de cálculo da rentabilidade média da carteira e índice de Sharpe é melhor que a outra, estatisticamente, embora o teste Mann-Whitney tenha conseguido diferenciar certas médias. Ao comparar os dois sub períodos, e como esperado, os resultados paramétricos e não paramétricos mostram que as médias podem ser diferenciadas estatisticamente.

Palavras-chave: Rácios financeiros, mercado de capitais português, gestão de portfolios, CAPM, e ELECTRE III.

FINANCIAL THEORY APPLIED TO PORTFOLIOS SELECTION

Abstract

The scope of this thesis is to explore and analyze the multicriteria models, in particular the application of ELECTRE III methodology, in order to construct defensive portfolios, in a buy and hold strategy, where all criteria and alternatives are equally weighed, using financial ratios (ROA, ROE, FA, GL and RL). So being, we firstly selected shares trading in PSI-Geral, from 1999 to 2011, and then an initial/historical period were defined, where eight portfolios were established one for each period, and a follow-up period considering one, two and three years holding. By conducting a descriptive analysis to each portfolio individually considering all periods, and a statistical analysis only to follow-up periods, we compare profitabilities and Sharpe's index means calculated to ELECTRE III, CAPM, and PSI-20 TR. In the specific case of statistical analysis, we conducted parametric (*t*-student and ANOVA one-way tests) and nonparametric tests (Kruskal-Wallis and Mann-Whitney tests), subdivided into two analysis: a unit sample analysis from 2005 to 2011; and a two sample analysis, one from 2005 to 2007, and another one from 2008 to 2011.

Descriptive analysis tell us that, for certain portfolios, ELECTRE III and CAPM behaved both better than market (PSI), also being more efficient (Sharpe's index). Between ELECTRE III and CAPM, in a three years holding (long term), ELECTRE had greater average profitabilities than market and CAPM, with also greater profitability by unit of risk. Despite this, statistically we found different results, probably due to sample size limitation. So, considering statistical tests for a unit sample and for two samples, results obtained lead us to conclude generally that in every follow-up we cannot infer that one way of calculating portfolio's average profitability and portfolio's Sharpe's index is better than the other statistically, although Mann-Whitney test differentiated certain means. When comparing both sub periods, and as expected, parametric and nonparametric results show us that means can be differentiated statistically.

Key Words: Financial ratios, Portuguese index, portfolio management, CAPM, and ELECTRE III.

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LIST OF ABBREVIATIONS AND ACRONYMS

ADELAIS – A Multiobjetive Mathematical Programming;

AHP – Analytic Hierarchy Process;

APT - Arbitrage Pricing Theory;

CAPM – Capital Asset Pricing Model;

CML – Capital Market Line;

DM – Decision Maker;

ELECTRE – Elimination and Choice Translating Reality;

FA – Financial Autonomy;

GL – General Liquidity;

ICAPM - Intertemporal Capital Asset Pricing Model;

IGCP - Institute for the Management of Treasury and Public Credit;

MCDM – Multicriteria Decision Model;

MINORA – The ordinal Regression Methodology;

MPT - Modern Portfolio Theory;

p – Preference threshold;

PER – Price Earning Ratio;

PROMETHEE – Preference Ranking Organization Method and Enrichment Evaluations;

PSI – Portuguese Stock Index;

q – Indifference threshold;

RL – Reduced Liquidity;

ROA – Return on Assets;

ROE – Return on Equity;

TOPSIS - Technique for Order Performance by Similarity to Ideal Solution;

β – Coefficient Beta.

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“All truth passes through three stages:

First, it is ridiculed.

Second, it is violently opposed.

Third, it is accepted as being self-evident.”

Arthur Schopenhauer (1851)

INTRODUCTION

Over the last decades the financial market has been changing tremendously specially due to globalization. So, when comes the time to define which investment(s) to undertake, or in what asset(s) to invest, the decision is often not clear given a set of alternatives. Furthermore, the investment decision may be influenced by all kinds of constraints, both explicit and implicit, generating a multicriteria decision problem. In this sense, and as we all know, for the decision maker expected returns on various asset classes are key inputs in portfolio decisions. Unfortunately, determine the best method to estimate expected returns, in a more rigorous and assertive way, is a difficult task.

Therefore, this thesis has a double objective. On one hand, by transforming portfolio management into a multicriteria problem, and using ELECTRE III method, we explore the application of financial theory (financial ratios) to select which assets choose to form a certain portfolio, and test if outperform the market (PSI-20TR). On the other hand, also form a certain portfolio, but this time using Capital Asset Pricing Model (CAPM) to select the shares, and also test if outperforms the market. Finally, take some conclusions about witch method is the best to determine which assets should be selected to form a portfolio, ELECTRE III or CAPM.

Taking this circumstances and the scientific lack, as far as we know, by investigating the traditional models strength considering financial factors, this thesis is well timed and relevant. Therefore, we study the process to establish a portfolio selection by proposing a new methodology, a multicriteria approach, based on financial ratios.

In traditional models, portfolios are defined based on the “average” investor, instead of the particular (typically no average) investor at hand. However, the assumptions made to describe this average investor are often inadequate and may even be misleading. For instance, CAPM, a contribution from Sharpe (1964), Lintner (1965) and Mossin (1966), have a very important assumption: it formerly defined the notion of risk in terms of mean-variance, in a portfolio context. However, this assumption may be too restrictive, in practice, because it is based on the preferences of the investor and/or the representation of investment alternatives. Variance, as a risk measure, may miss its link with an investor's preference structure or with the distribution of assets and with portfolio returns. Besides that, information concerning mean and variance is not always sufficient to adequately discriminate between investment alternatives.

This model was criticized and augmented by Markowitz (1987), which showed that if one assumes a given set of constraints, then the following CAPM conclusions will not verify: the market portfolio is efficient, and the expected returns are linearly related to the betas (as risk measure). Therefore, Markowitz (1987) analysis, or the classical analysis, assumes the existence of two important assumptions: the investor is never satisfied, and the investor is risk averse.

In 1993, Fama and French developed a three-factor model. This new approach augmented the CAPM model with certain factors (CAPM market factor plus a size factor and a value/growth factor) to explain what the CAPM cannot. This famous model defined the basis for others studies: Jegadeesh and Titman (1993), Campbell, Hilscher and Szilagyi (2008), Fama and French (2008), and Cooper, Gulen and Schill (2008). However, over the past two decades, it has become increasingly clear that even the highly influential Fama and French model cannot explain many cross-sectional patterns.

Chen and Zhang (2009) developed a new thee-factor model, based on q-theory (on the marginal productivity of a firm's investments). They proposed a market factor, an investment factor, and a return-on-assets factor combine to summarize the cross-sectional variation of expected stock returns.

In the late '80s and early '90s, the development of new operational research techniques, as well as the computer power, enabled new approaches in the optimal portfolio modelling selection: the "Expert Systems” and the Multicriteria modelling approach (Lee, Kim & Chu, 1989; Shane, Fry & Toro, 1987).

As stated by Soares (2000), the multicriteria approach takes into account, besides the two basic factors, risk and return (the classic mean-variance model), one number of important additional factors such as market liquidity, the PER, the dividends growth rate, among others.

As already mentioned, conducting a different approach, a multicriteria approach based on financial theory, the decision to invest should be supported in the desires (preferences) of the investor and the characteristics of the investment alternatives, adequately understood and related to each other.

As referred by Yap, Yong and Poon (2010), financial theory has been very successful in distinguish the weak companies from the healthy ones, assuming financial ratios, a particularly importance: they have long been regarded as barometers of corporate health, being used for reporting liquidity, leverage, activity and profitability, and an investor may use them to appraise a company performance and its future prospect of success (Green, 1978, Gibson, 1982; Chen & Shimerda, 1981; Gardiner, 1995; Yap et al., 2010).

In this sense, supported in the company's financial statements, financial theory provides information about its solvency position and its borrowing power, and whether if it is a suitable investment to consider, throws financial ratios and studies of trends. This idea is enhanced by several studies, for instance, Beaver (1966), Altman (1968), Ohlson (1990), Taffler (1983), Wood and Piesse (1987), Inman (1991), Ganelasingam and Kumar (2001), Cybinski (2001), Sori and Jalil (2009) among many others.

Inspired in Spronk and Hallerbach's (1997) work, our framework for individual decisions is given by: (1) Decision alternatives. (2) Express each of these alternatives in terms of certain criteria, as financial ratios. (3) Define criteria and thresholds in order to reflect investor preferences. (4) Choose the best alternative according to a distillation process. This process helps finding a (set of) suitable stream(s) of decision alternatives, by eliminating all inferior ones. (5) Monitoring over time the performance of the alternatives selected: initially to a certain period (historical behavior), and then to a follow-up period (holding period of one, two and three years). (6) Calculate Beta, Treynor´s index (1965), Sharpe's index (1966), and Jensen's performance index (1968), and finally (7) conduct parametric and nonparametric tests to confirm conclusions.

Thus, this work is organized as follows. Firstly, multicriteria decision making problem and ELECTRE III method main features are presented. Secondly, linkage between financial ratios and portfolio management is explored. Then, empirical test is performed, followed by the results analysis. Finally, the main conclusions are drawn.

PART 1 – THEORETICAL BACKGROUND

CHAPTER 1: PORTFOLIO MANAGEMENT: KEY ISSUES

Portfolio selection problem is probably one of the most subjects studied in finance. And why? Because, the selection of assets or equities is not just a problem of finding attractive investments. Defining the right portfolio of assets cannot be done only by human intuition.

Any investor would like to have the highest return possible from an investment. However, we must take into account the amount of risk the investor is able or desires to undertake. The expected return and the risk measured by the variance (or the standard deviation) are the two main characteristics of a portfolio beyond classical approach. Besides that, it's important to note that the behavior of a portfolio can be quite different from the behavior of individual components of the portfolio.

Harry Markowitz (1952), Tobin (1958), William Sharpe (1964), John Lintner (1965), and Merton Miller (1991) gave the most representative contributions to modern financial theory. They consolidated a new scientific area, within the economy, in which the key concept is the economic study of the capital market.

Markowitz (1952) developed the mean-variance model which was a pioneering attempted to focus the demand for risky assets. In Markowitz's model, an investor selects a portfolio at time *t-1* that produces a stochastic return at *t*. The model assumes that investors are risk averse and, when choosing among portfolios, they care only about the mean and variance of their one-period investment return. According to the mean-variance model, investors maximize the expected return for a given risk level, where risk is measured by the variance. The portfolio model provides an algebraic condition on asset weights in mean-variance-efficient portfolios. As a result, investors choose "mean-variance-efficient" portfolios, in the sense those portfolios (Fama & French, 2004; Fernando, 2002):

1- minimize risk for a specified expected return;

2- maximize the expected return for a specified risk;

3- minimize the risk and maximize the expected return using a specified risk aversion factor;

4- minimize the risk regardless the expected return;

5- maximize the expected return regardless the risk;

6- minimize the expected return regardless the risk.

Fernando (2002) adds that the first three conditions are related with the Efficient Frontier (the efficient points in the Return-Risk). The fourth condition gives minimum variance solutions which are for cautious investors, and it is used to compare and benchmark with other portfolios. The fifth condition gives the upper bound of the expected return which can be obtained, being useful to establish comparing. The last condition indicates a worst case scenario.

Another important question is the Markowitz's diversification[1]. This can be defined as the process of combining assets, whose returns are not positively correlated, in order to reduce portfolio risk without sacrificing is own expected return. The lower the correlation between assets returns, greater the benefit of diversification.

Ultimately the goal is, through diversification and the Principle of Dominance [2], select the most efficient portfolios: those with the highest expected return for a given risk. The portfolios that respect this relationship, for different risk levels, are the so-called efficient frontier.

The classic efficient frontier corresponds to the Minimum Variance Set: the number of portfolios with the lowest levels of variance in relation to expected returns allowed. The Minimum Variance Set incorporates two important properties[3]:

— If one combines two or more portfolios, which are in the Minimum Variance Set, this will result in a new portfolio in the Minimum Variance Set.

— Given a set of a, there is a linear relationship between the factors and the beta of its expected return if, and only if, the betas are calculated using a market index, as a portfolio, with minimum variance.

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[1] Markowitz, H. (1952). Portfolio Selection.

[2] The Principle of Dominance states: "Among investments with the same rate of return, the one with the least risk is most desirable. In addition, given a group of investments with the same level of risk, the one with the highest return is most desirable" (Porter, 1972).

[3]In this regard see: Haugen, R. (1990). *Modern Investment Theory*. Prentice-Hall International eds, second edition,pp.116-117.

In the sub-chapters that follow, we make a brief summary of the most important points about portfolio classical optimization theory; in particular, asset allocation, diversification, modern portfolio theory, CAPM, capital market line, efficient frontier and portfolio monitoring and rebalance.

1.1. The asset allocation

A portfolio manager is faced with the need to select, within a universe of hundreds or thousands of assets classes such as equities, fixed income securities, properties, mutual and hedge funds, insurance policies and cash, some titles to analyze and, maybe, to invest. So, before defining asset allocation, it is important to select witch assets to invest. An important tool in this selection is the already mentioned Principle of Dominance. This principle states that:

1) Of all the entire assets invested, which given a certain expected return, the asset with the lowest risk is the most desirable.

2) Of all the entire assets invested, which hold a particular risk, the asset with the highest expected return is the most desirable.

Concerning asset allocation, Kuan (2008) defined it as being the process of determining the weight given to certain asset, in portfolio. The asset allocation model works in two phases. First, we must determine which asset classes will be considered in the portfolio. Subsequently, set up each asset weighting.

In order to determine the exact weigh, we must consider a number of factors: the asset classes expected returns, standard deviation, the investor's risk tolerance level, the need for capital preservation, and economic expectations.

The Markowitz's model is the typically model to be employed in terms of selecting assets classes. Despite the number of covariance estimates needed when dealing with a large number of assets, investors decides which asset classes to own and in what proportions. So, setting an appropriate asset allocation is crucial to maximizing investment returns.

Since market returns are unpredictable in the short term, when choosing the classes of assets to invest, it is important to have as reference a strategic benchmark portfolio (based on long-term, equilibrium assumptions of the return and risk profile of the investment universe and the investor's attitude toward risk).

In order to study how investors may react to these fluctuations, Perold and Sharpe (1988) studied four strategies: buy-and-hold, constant-mix, option-based portfolio, and constant-proportion portfolio strategies. They conclude that there is no reason to believe that any particular type of strategy is best for everyone (and, in fact, only buy-and-hold strategies could be followed by everyone). In general, when we talk about investing in capital market, about 80% of the total investment return comes from strategic asset allocation (Perold & Sharpe, 1988; Kuan, 2008).

As mentioned by Kuan (2008), after the asset allocation decisions, investors should adjust the portfolio mix through the use of short-term market information. The stress remains on short-term performance, which leaves investors' long-term risk tolerances and preferences unaffected. These strategies, also called by tactical strategy, "are strategies which attempt to deliver a positive information ratio by systematic asset allocation shifts." (Lee, 2001)

When confronted with market fluctuations, investor can have one of two attitudes. On one hand, have an active strategy. For example, select a potential company based on expectations of strong growth in earnings (the "Growth Investing"), switch between two asset classes such as stocks and cash, based on market predictions (the "Market Timing")[4], to shift sector weighs in the portfolio in order to take advantage of those sectors that are expected to behave relatively better, and avoid those sectors that are expected to behave relatively worse (the "Sector Rotation"), or even measure a portfolio value using fundamental analysis, such as price earnings ratio and price to book ratio (the "Value Investing"[5]).

On the other hand, he can have a passive attitude. In this case, investor believes that market is efficient. So, he does not seek to outperform the market but simply do as well as the market: he tries to minimize transaction costs and time spent in managing the portfolio,

[4] Damordan (2003) indicates that there are four ways to do it: to adjust the mix of assets, to switch investment styles and strategies to reflect expected market performance, to shift the funds within the equity market from sector to sector, and finally, to speculate on market direction.

[5] Some of the common value measures are: the book value multiple, to buy low price-to-book value companies, the market value to replacement cost or Tobin's Q, the earnings multiples, the revenue multiples and the dividends yields.

because any expected benefits from active trading or analysis are likely to be less than the costs. So, he can buy-and-hold or benchmark. Based on the study of Perold and Sharpe (1988), and Kuan (2008), buy-and-hold strategy is the only strategy that can be followed by everyone: the investor simply buys a portfolio of assets, based on some criteria, and keeps it over some investment horizon.

Considering benchmark strategy, any tactical strategy is always measured against a benchmark portfolio. For this purpose, if a portfolio manager outperforms better than the benchmark portfolio (have higher return), then the portfolio manager is said to have a positive “alpha”. Besides this, it is also important to analyze the ability to consistently out-perform. To do so, and according to Fabozzi’s (1999) study, portfolio manager can use several methods, so being, capitalization method, stratified method and Markowitz’s mean-variance.

As Kuan (2008), also Levišauskait (2010) considers that a portfolio manager can be: an active portfolio manager or a passive portfolio manager. So, in an active portfolio management, investor acts as if assets markets are inefficient, constantly rebalancing portfolio. Different is the passive portfolio management, where investor holds assets in the portfolio for a relatively long period with small and infrequent changes, believing markets are relatively efficient. Passive investor only rebalance his portfolio when his preferences changes, when the risk free rate changes or when forecast about the risk and return of the benchmark portfolio changes. Table below, summarizes active versus passive investment management main features.

Table nº 1 - Active versus passive investment management

Area of comparisons	Active investment management	Passive investment management
Aim	To achieve better results than average in the market	To achieve the average market results
Strategies used and decision making	Short term positions, the quick and more risky decisions; keeping the “hot” strategy	Long term positions, slow decisions

Investor/manager	Tense	Laid-back
Taxes and turnover of investment portfolio	High taxes, relatively high turnover of portfolio	Low taxes, small turnover of portfolio
Performance results before costs and taxes	In average equal to the passively managed portfolios	In average equal to the actively managed portfolios
Performance results after costs and taxes	In average lower than market index after taxes	In average higher than the results of actively managed portfolio returns after taxes
Individual investors	Over 85 % from total individual investors	Over 15 % from total individual investors
Institutional investors	Over 56% from total institutional investors	Over 44% from total institutional investors
Supporters	All brokerage firms, investment funds, hedging fund, specialized investment companies	Passively managed pension funds, index funds
Analytical methods	Qualitative: avoiding risk, forecasts, emotions, intuition, success, speculation, gambling	Quantitative: risk management, long term statistical analysis, precise fundamental analysis

Source: Levišauskait, 2010.

Levišauskait (2010) points out that is common to find investors that invest passively in the markets they consider to be efficient and, actively in the markets they consider to be inefficient, combining both strategies.

The question raised earlier leads us directly to the problematic of portfolio revision. According to Levišauskait (2010), “is the process of selling certain issues in portfolio and purchasing new ones to replace them”. Generally, this author defend that changes in financial market conditions, changes in investor’s preferences and portfolio diversification are the main reasons that supports the necessity of portfolio valuation/revision. In particular,

- As the economy evolves, certain industries and companies become either less or more attractive as investments. Seeing investment decisions are made in a dynamic investment environment, changes occur naturally. So, macroeconomic indicators, such as gross domestic product (GDP) growth, inflation rate, interest rates, as well as the new information about industries and companies, should be observed by investor in a regular basis, because these changes can influence portfolio risk and return;
- The investor over the time may change investment preferences, becoming the portfolio no-optimal;
- The constant need for diversification. Individual assets in the portfolio often change in risk-return characteristics and their diversification effect may be lessened.

Taking these circumstances, investors may have to rebalancing a portfolio, making periodic adjustments to maintain certain original conditions. Levišauskait (2010) defends that, in general, rebalancing reduces the risks of losses, because a rebalanced portfolio is less volatile than one that is not rebalanced. But, investor must be aware that revise a portfolio have costs. On one hand, investor support costs as trading fees and commissions; on the other hand, this revising may also have tax implications which differ from country to country. In order to do so, Levišauskait (2010) points several methods to rebalance portfolios:

a) *Constant proportion portfolio:* All adjustments are made in order to maintain the relative assets weighting in the portfolio as their price changes. For many investors this is a strange way to rebalance portfolio, because requires the purchase of assets that have performed poorly and the sale of those that have performed best. This method of rebalancing must be considered one choice, but not necessarily the best one.

b) *Constant Beta portfolio:* Across time, the values of the portfolio assets and their Betas[6] will change and this may cause the target portfolio Beta to shift. For instance, if the target portfolio Beta is 1.10 and it had risen over the valuation period of time to 1.25, the portfolio Beta could be brought back to the target (1.10) in the following ways:

[6] About this subject see subchapter 1.6.

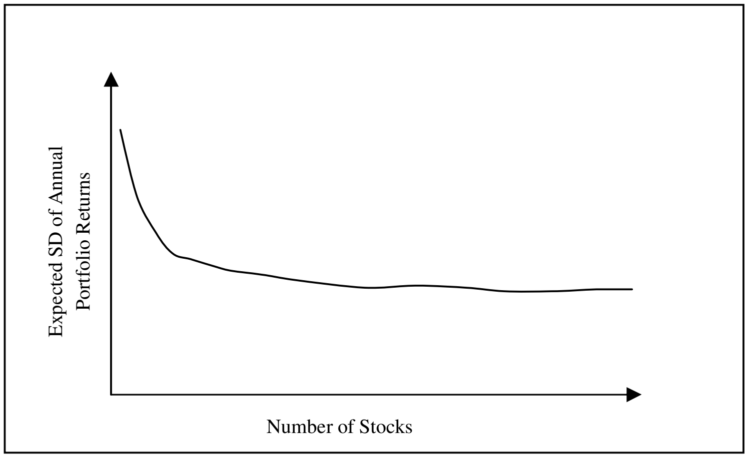
- *Put cash into the portfolio as a component and hold it:* Cash will reduce portfolio Beta (Beta of cash is 0), by diluting the assets in portfolio. Holding cash should be a temporary component of portfolio;
- *Put additional cash into the portfolio and buy assets with a Beta lower than the target Beta.* The problem rises when investor may be not able to invest additional cash.
- *Sell high Beta assets in portfolio and hold cash.* Just like the first alternative, this way reduces the Betas holdings in the investor's portfolio which may be not appropriate.
- *Sell high Beta assets and buy low Beta assets.* The assets bought could be new additions to the portfolio, or the investor could add to existing positions.

c) *Indexing:* These alternatives for rebalancing the portfolio are more frequently used by institutional investors. Allow to eliminate concern about outperforming the market, because by definition, it seeks to behave just like the market averages. Investor attempts to maintain some predetermined characteristics of the portfolio, such as Beta of 1.0.

1.2. The diversification

Kuan (2008) believes the risk of a portfolio depends on assets weights individually consider (for example stocks), their variance, and their covariance. A change in any of these variables will change the portfolio risk. So, considering that assets are randomly selected and combined in equal weights into a portfolio, is commonly accepted that the risk of the portfolio decreases as the number of assets increases. To corroborate this assumption, Evans and Archer (1968) show how natural diversification reduces the dispersion of returns in a portfolio; in their study, 75% of the total portfolio risk considered was eliminated through diversification. In addition, Strong (2003) drew similar conclusions, and also concluded that in a portfolio containing fifteen or twenty securities there is little benefit to add more components. Also Elton, Gruber and Padberg (1976, 1977), investigated the relationship between risk and the number of assets in the portfolio. Their conclusions can be expressed by the following chart.

Chart nº 1 – The impact of portfolio diversification by increasing number of stocks



Source: Kuan, 2008.

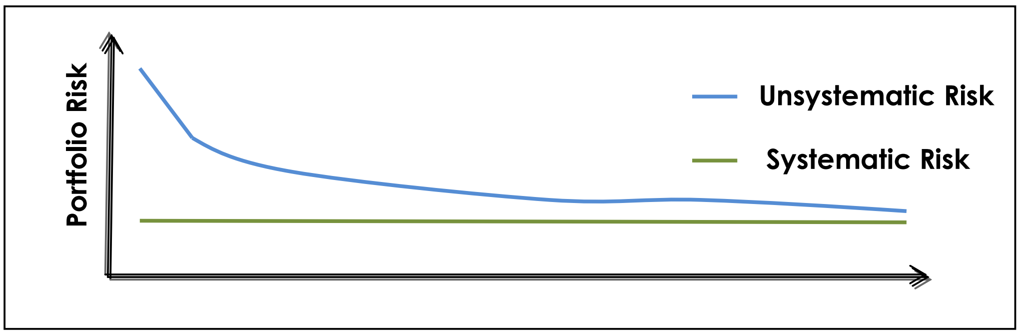
Analyzing chart nº 1 we can understand why they conclude that 51% of a portfolio standard deviation is eliminated when diversification increases from 1 to 10 securities. If we add 10 more securities, standard deviation suffered a decline of around 5%. If we continue increasing the number of securities, for example to 30, we will only have an additional decreasing of 2% in the standard deviation. Beyond 30 securities, almost all of the diversifiable risk is eliminated, and there are very little benefits in adding more securities (Gup, 1983; Constantinides, 1986; Campbell, Lettau, Malkiel, & Xu, 2001; Kuan, 2008). Considering the work of these authors, it is reasonable to say that approximately 50 securities are needed to ensure adequate diversification.

So, to better understand portfolio risk, we must know its components. In Quintero's (2002) works risk measurement is divided into two: in *systematic*, of market, no diversifiable or residual, and in *unsystematic* risk, unique, diversifiable, specific or idiosyncratic.

The *systematic* risk is the one that cannot be eliminated through diversification. This happens because of risks in the economy that threaten all business, and because stocks and shares tend to move in the same direction. By definition, is measured with a Beta equal to one ($\beta = 1$).

The *unsystematic* risk is the one that can be eliminated through diversification. By increasing the number of assets in the portfolio, the standard deviation decreases but with a decreasing rate, seeing additional risk reductions will be relatively smaller after include more than 10 or 15 assets[7]. Evans and Archer (1968), and subsequent critics of Statman (1987), conclude that investors should perform marginal cost-benefit analysis to determine the appropriate number of assets to be included in the portfolio. In practice, there isn't a method to achieve the optimal number, but Statman (1987) showed that many of the strategies used by investors appear to be sub-optimal: the correlations between portfolios return and market performance index increase. In this way, the investor with a portfolio is highly correlated with the market and with the systematic risk, being the last one the main concern for investors.

Chart nº 2 – Diversification Effects



Source: Quintero, 2002.

The distinction between systematic and unsystematic risk is very useful. In a diversified portfolio, unsystematic risk can be eliminated, but investors cannot eliminate systematic risk through diversification. A fully diversified portfolio has a beta equal to one ($\beta = 1$).

We can generally say that one of the key points of diversification is to invest in different markets, domestic and foreign markets. The variety in market performance contributes to the good performance of the portfolio thought diversification, being proved by Solnik (1974) as cited in Kuan's (2008) works. This author compared the benefits of international diversification against United States domestic markets. The results obtained are expressed in the following chart,

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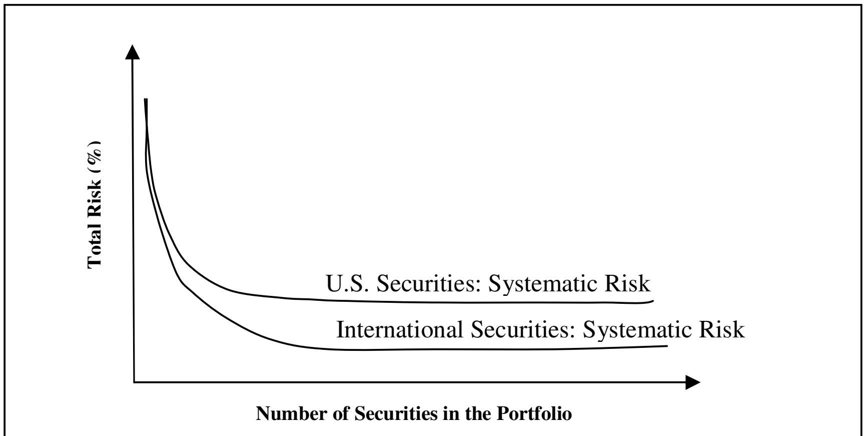
[7] Classical work of Evans and Archer (1968) consider that all diversifiable risk is eliminated in a portfolio with 10 shares. Here, the authors discuss the mathematical relationship between portfolio size and risk; they conclude that each additional share included in the portfolio results in a lower decrease at risk.

Chart nº 3 – Total Portfolio Risk with International Investment



Source: Kuan, 2008.

Solnik (1974) shows that the systematic risk, without international portfolio was 27%, while with a diversified international portfolio were 11.7%. From Solnik's (1974) works, Kuan (2008) stats that there are two motivations for international diversification: on one hand, *ceteris paribus*, a low international correlation allows reduction of the volatility, or total risk, of a global portfolio. On the other hand, a low international correlation also provides profit opportunities for an active investor seeing markets do not move up or down together. The investor can adjust the international asset allocation of global portfolio toward markets with superior expected returns.

1.3. The modern portfolio theory and the capital asset pricing model (CAPM)

Modern portfolio theory (MPT)[8] is the creation of economists, who try to understand the market as a whole, rather than business analysts, who look for what makes each investment opportunity unique. As stated in Kuan's (2008) works, in MPT investments are described statistically, in terms of their expected long-term return rate and their expected short-term volatility. The volatility is equated with "risk", measuring how much worse in average an investment bad years are likely to be. The goal is to identify acceptable level of risk tolerance, and then to find a portfolio with the maximum expected return for that level of risk.

[8] Henceforth called MPT; also called "portfolio theory" or "portfolio management theory".

This method was developed, in first hand, by Professor Harry Markowitz in 1952 (as already mentioned at the beginning of chapter). Later, in 1990, he shared a Nobel Prize with Merton Miller and William Sharpe which has become a reference in investment portfolios.

In his famous article *"Portfolio Selection"*, Markowitz (1952) described how to combine assets in order to build a diversified portfolio and, at the same time, an efficient portfolio. In short, he considers that a portfolio risk could be reduced and the expected rate of return could increase, when assets are not correlated. Thus, keeping assets that tend to move in harmony with each other does not lower our risk. Diversifying is, once again, the watchword. In a diversified portfolio, composed by uncorrelated assets, the portfolio volatility is less than the average of the volatilities of its individual parts.

The MPT has changed the way investors think about their strategies. The theory assumes that financial markets are efficient, meaning the price of any asset incorporates all the information existent[9]. The main task of theory is to determine the asset's rate of return. It is based on CAPM[10] assumptions: perfect capital market, ability to lend and borrow in unlimited amounts to a common risk-free rate, and homogeneity in public expectations. The CAPM provides two basic conclusions. The first concerns the degree of optimal portfolio diversification in market equilibrium; the second is about the appropriate measure of risk assets and the relationship with its expected return rate. In short, the portfolio risk is what matters to investors.

But how does one asset influence another one with different characteristics in terms of risk and return in the same portfolio? And what could be the influence of this relationship to the investor's portfolio?

Dowd (2000) pointed out that answers to these questions are of great importance for the investor when comes the time to form a diversified portfolio. So being, covariance should

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[9] There are three levels of efficiency defined by Roberts (1967): Weak Form Efficiency (prices reflect all information contained in past price movements); Semi-strong form (besides the weak, it also reflects all the other information published); Strong Form (besides the above, reflects all the information that can be gained through analysis of the company and the economy).

[10] The main Sharpe's merit was to extend the Markowitz and Tobin optimal portfolio analysis selection to a model of capital market equilibrium. What we now call CAPM is actually a synthesis of contributions from various authors. Almost simultaneous, Sharpe (1964), Treynor (1961) unpublished, Lintner (1965) and subsequent Mossin (1966), Fama (1968) and Black (1972) gave an important contribution, contributions that take as a starting point Markowitz's (1952,1959) and Tobin's (1958) works. Sharpe (1964) and Lintner (1965) add two key assumptions to the Markowitz model to identify a portfolio that must be mean-variance-efficient. The first assumption is *complete agreement*: given market clearing asset prices at t-1, investors agree on the joint distribution of asset returns from t-1 to t. And this distribution is the true one, that is, the distribution from which the returns we use to test the model are drawn. The second assumption is that there is *borrowing and lending at a riskfree rate*, which is the same for all investors and does not depend on the amount borrowed or lent.

be calculated, because it measures the relationship between two variables, and tells us in what direction this relationship goes. A covariance with a positive number shows that when return on asset A is above its mean of return (positive), the other asset B also tend to be, and when the rate of return of asset A is negative or bellow its mean of return, the returns of other asset tend to be negative too. A covariance with negative number shows that rates of return of two assets are moving in the opposed directions: when return on asset A is above its mean of return (positive), the returns of the other asset B tend to be negative, being true the inverse situation. Thus, in analyzing relationship between assets in the same portfolio using covariance for portfolio formation, it is important to identify which of the three possible outcomes exists:

- positive covariance ("+"): If there is a positive covariance between two assets, investor should not put both assets in the same portfolio, because their returns move in the same direction and the risk in portfolio will be not diversified;

- negative covariance ("-"): If there is a negative covariance between two pair of assets, investor should include both assets in the portfolio, because their returns move in opposed directions and the risk in portfolio could be diversified or decreased;

- zero covariance ("0"): If there is zero covariance between two assets, we conclude there is no relationship between the rates of return of two assets, despite this being a rare case in practice.

Based on the example presented by Kuan (2008), let's consider we are an investor interested in investing on asset *A*, and we want to measure the portfolio risk of this asset (measure the contribution to the risk of overall portfolio from holding asset *A*). To do so, we have to analyze the estimates of *expected returns*, *variance*, and *covariance* of asset *A*.

Firstly, to calculate the *market portfolio variance*, we use the covariance matrix frontier with the market portfolio weights. We highlight asset *A* in this description within the *n* assets in the market portfolio.

Table nº 2 - Covariance matrix frontier of asset *A* with the market portfolio weights

Portfolio Weights	W_1	W_2	…	W_A	…	W_n
W_1	$Cov(r_1, r_1)$	$Cov(r_1, r_2)$	…	$Cov(r_1, r_A)$	…	$Cov(r_1, r_n)$
W_2	$Cov(r_2, r_1)$	$Cov(r_2, r_2)$	…	$Cov(r_2, r_A)$	…	$Cov(r_2, r_n)$
…	…	…	…	…	…	…
W_A	$Cov(r_A, r_1)$	$Cov(r_A, r_2)$	…	$Cov(r_A, r_A)$	…	$Cov(r_A, r_n)$
…	…	…	…	…	…	…
W_n	$Cov(r_n, r_1)$	$Cov(r_n, r_2)$	…	$Cov(r_n, r_A)$	…	$Cov(r_n, r_n)$

Source: Kuan, 2008.

After getting the covariance matrix, it's now time to calculate the *variance*. The variance is calculated by summing all the elements of the covariance matrix, but preceded by the multiplication of each element by the portfolio weights from the row and the column. The contribution of one asset to portfolio variance therefore can be expressed as the sum of all the covariance terms in the row corresponding to the asset, where each covariance is first multiplied by both assets weight. In Kuan's (2008) example, the contribution of asset *A* to the variance of the market portfolio is given by the following expression:

$$C_A = W_A[W_1Cov(r_1, r_A) + W_2Cov(r_2, r_A) + \ldots + W_nCov(r_n, r_A)] \quad [01]$$

where,

$\mathbf{C_A}$, represents contribution of asset *A* to the variance of market portfolio;

$\mathbf{W_A}$, is asset *A* weight in portfolio;

$\mathbf{W_nCov(r_n, r_A)}$, represent the covariance of asset *A* with market portfolio (we can measure asset contribution to the risk of the market portfolio by its covariance with that portfolio).

To note that, when there are many assets outstanding in the capital market, the covariance of a particular asset will be dominated by the others in the total risk of portfolio.

Thus, if on one hand, the covariance between asset *A* and the rest of the market is negative, then asset *A* makes a "negative contribution" to portfolio risk. This happens because asset *A* provides returns that move inversely with the rest of the market. On the other hand, if the covariance is positive, asset *A* makes a “positive contribution” to portfolio risk, because its returns have a “positive contribution” in the rest of the portfolio.

Relatively to the *rate of return on the market portfolio* (r_M), mathematically may be written by the expression,

$$r_M = \sum_{k=1}^{n} W_k r_k \qquad [02]$$

where,

$\mathbf{W_k r_k}$ represents asset positive or negative contribution (measured by respective weight in portfolio) to portfolio return.

and the *reward-to-risk ratio* for investments in asset *A* (or Sharpe’s index[11]) can be expressed as,

$$S = W_A \frac{E(r_A) - r_f}{W_A \mathrm{Cov}(r_A, r_M)} = \frac{E(r_A) - r_f}{Cov(r_A, r_M)} = \frac{E(r_M) - r_f}{\delta_M^2} \qquad [03]$$

where,

$\mathbf{W_A}$, is the weight of asset *A* in portfolio;

$\mathbf{r_f}$ is the risk free;

$\mathbf{W_A[E(r_A) - r_f]}$ is the contribution of asset *A* to the risk premium of market portfolio;

$\mathbf{W_A Cov(r_A, r_M)}$ is the contribution of asset *A* to variance of market portfolio.

[11] About this subject see subchapter 1.6.

In short, this ratio quantifies the premium risk (extra return) that investors demand to support portfolio risk. A basic principle of equilibrium in capital markets is that all investments should offer the same reward-to-risk ratio. If the ratio is better for one investment than another, investors would rebalance their portfolios to alternatives with better trade-off and away from the other alternatives. This behavior would impress pressure on assets prices until the ratios were equalized.

If we continue to analyse formula [03], we can also say that,

$$E(r_A) - r_f = \frac{Cov(r_A, r_M)}{\delta_M^2} * [E(r_M) - r_f] \qquad [04]$$

being the ratio $\frac{Cov(r_A, r_M)}{\delta_M^2}$ also denoted as Coefficient Beta or β[12]. This ratio measures the contribution of asset *A* to the variance of market portfolio as a fraction of total variance of market portfolio. With all this CAPM can be written as,

$$\begin{array}{llcl} & W_1E(r_1) & = & W_1r_f + W_1\beta_1[E(r_M) - r_f] \\ + & W_2E(r_2) & = & W_2r_f + W_2\beta_2[E(r_M) - r_f] \\ + & \dots & = & \dots \\ + & W_nE(r_n) & = & W_nr_f + W_n\beta_n[E(r_M) - r_f] \\ \hline + & E(r_p) & & r_f + \beta_p[E(r_M) - r_f] \end{array} \qquad [05]$$

or generally as,

$$E(r_p) = r_f + \beta_p[E(r_M) - r_f] \qquad [06]$$

[12] About this subject see subchapter 1.6.

So far we have seen portfolios with only one asset. But, what happens if the portfolio is comprised of multiple risky assets and no risk-free assets? The *expected return of the portfolio* (R_p) is simply the weighted average of the expected return of the individual assets defined as follows:

$$R_p = w_1R_1 + w_2R_2 + \ldots + w_nR_n = \sum w_tR_t \qquad [07]$$

where,

$\mathbf{w_nR_n}$ is individual weigth of asset n in expected return.

Obviously, as the number of assets grows, calculating the standard deviation becomes more complicated. So, if we consider a portfolio with only two risky assets, the standard deviation of the returns of a portfolio (δp) is determined as follows:

$$\delta p = (W_1^2\delta_1^2 + W_2^2\delta_2^2 + 2W_1W_2\delta_1\delta_2\rho)^{\wedge 1/2}\ ,\ \ w_1 + w_2 = 1 \qquad [08]$$

where,
$\mathbf{W_1^2\delta_1^2}$ is the weigth of asset in standard deviation of that same asset;
$\boldsymbol{\rho}$ is the correlation coefficient of assets 1 and 2 in the portfolio.

The expression above tell us that the investor is able to reduce the risk of the portfolio through diversification, this is, the efficiency of the diversification depends on the correlation coefficient (ρ) of the two assets in the portfolio.

To understand this concept, it’s important to understand how an investor determines the correlation coefficient between two assets. There are two techniques to doing it: to determine the probability distribution of the future returns of the two assets, also denoted as the “true” correlation coefficient, or to analyze historical information of realized returns of the two assets.

Considering the first technique, based on the probability distribution of future returns ($\boldsymbol{\rho}$), the expression is as follows:

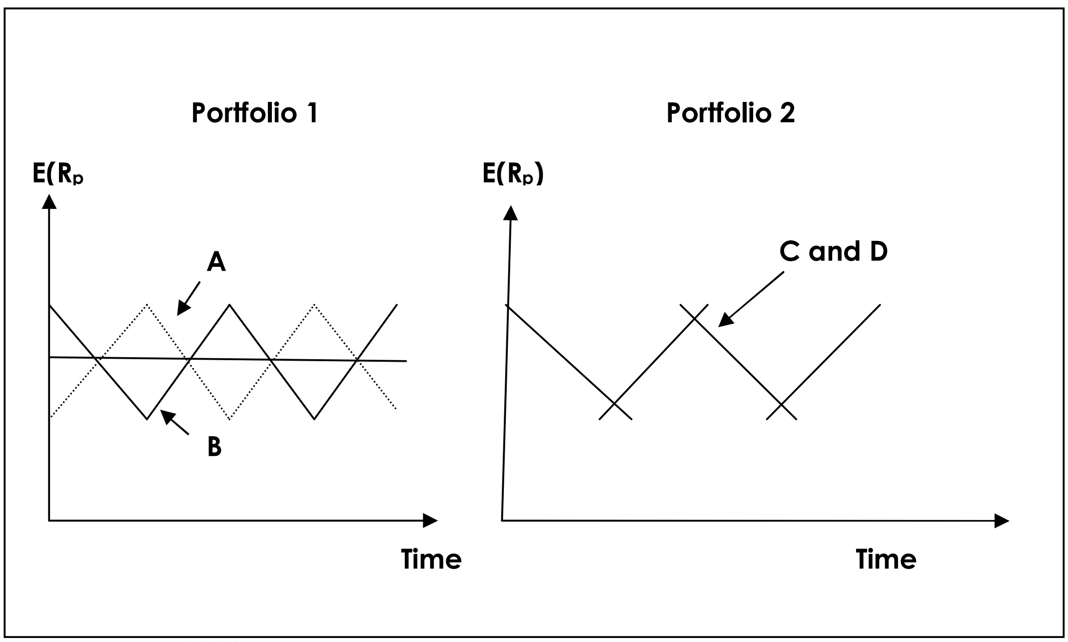
$$\rho = \frac{\sum (x_t - E(x)(y_t - E(y))P_t}{\delta_x \delta_y} \qquad [09]$$

Considering the second technique, based on historical realized returns (***r***), the formula is the following:

$$r = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{\sqrt{(n\sum x_i^2 - (\sum w_i)^2)(n\sum y_i^2 - (\sum y_i)^2)}} \qquad [10]$$

Criticizing both models, Kuan (2008) considers the second technique more effective. Let's look to chart below, is order to illustrated assumptions presented by the mentioned author.

Chart nº 4 - The relationship between correlation coefficient and the effectiveness of diversification (two different pairs of assets in two different portfolios)



Source: Kuan, 2008.

In portfolio 1, consisting of two assets - *A* and *B* - is easily seen that the correlation coefficient between those assets is inversely proportional, because they move in the exact opposite direction (when the return of one asset goes up, the return of the other asset will go down by the exact same amount, being true the inverse situation). Considering the investor invested 50% of the money in asset *A*, and the other 50% in asset *B*, the volatility of the two assets cancels out one another completely, leading to a situation where portfolio 1 has no volatility at all. In this particular case, diversification has been very effective.

Concerning portfolio 2, which contains two assets - *C* and *D* – they are directly proportional, because they move in the same direction (in a positive correlation, the returns of the two assets move in perfect direction). Supposing the investor invested 50% of the money in asset *C*, and the other 50% in asset *D*, by the second chart we can see portfolio 2 contains the same amount of risk as those of assets *C* and *D*, meaning that diversification has not been effective at all.

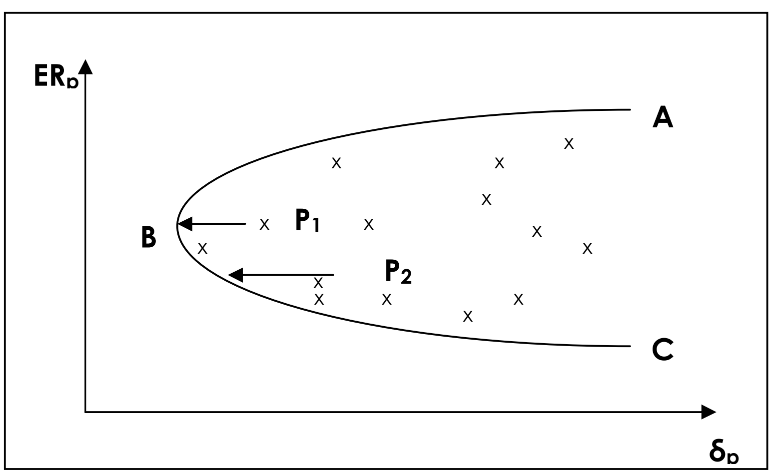
So, it is commonly accepted that diversification allows an investor to lower the risk level of a portfolio, beyond the individual risk levels of the asets it contains. But, we cannot forget that the risk level of the portfolio changes as its composition also changes.

The chart above tells us that is through the minimum variance portfolio that the maximum effect of diversification can be achieved, and the right combination of w_1 and w_2, which will produce the minimum variance of the portfolio, can be achieved throw the following formulas,

$$W_1 = (\delta^2_2 - \delta_1\delta_2\rho) / (\delta^2_1 + \delta^2_2 - 2\delta_1\delta_2\rho) \quad [11]$$

$$W_2 = 1 - W_1$$

Chart nº 5 - Mean-Variance Optimization and Efficient Frontier



Source: Pires, 2010.

When analyzing chart nº 5 what stands outs is the curve *ABC*. This curve, also known as the efficient frontier, represents the relationship between risk and return, this is to say, represents the different portfolios that are reachable to an investor. Portfolios that lie outside the efficient frontier are unreachable to an investor. The upper end of the efficient frontier represents the riskier asset *A*, while asset *C* represents the safer investment. As the composition of the portfolio switches from asset *A* to asset *C*, we will notice that the risk level of the portfolio falls. However, there is an equilibrium point between asset *A* and asset *C*. This particular combination, that causes the lowest risk level of the portfolio with the highest expected return, is known as the minimum variance portfolio represented in *point B*. It is not possible to lower the risk of the portfolio any further. This means that an investor can create a portfolio that has a risk level that is lower than the individual risk levels of the two assets.

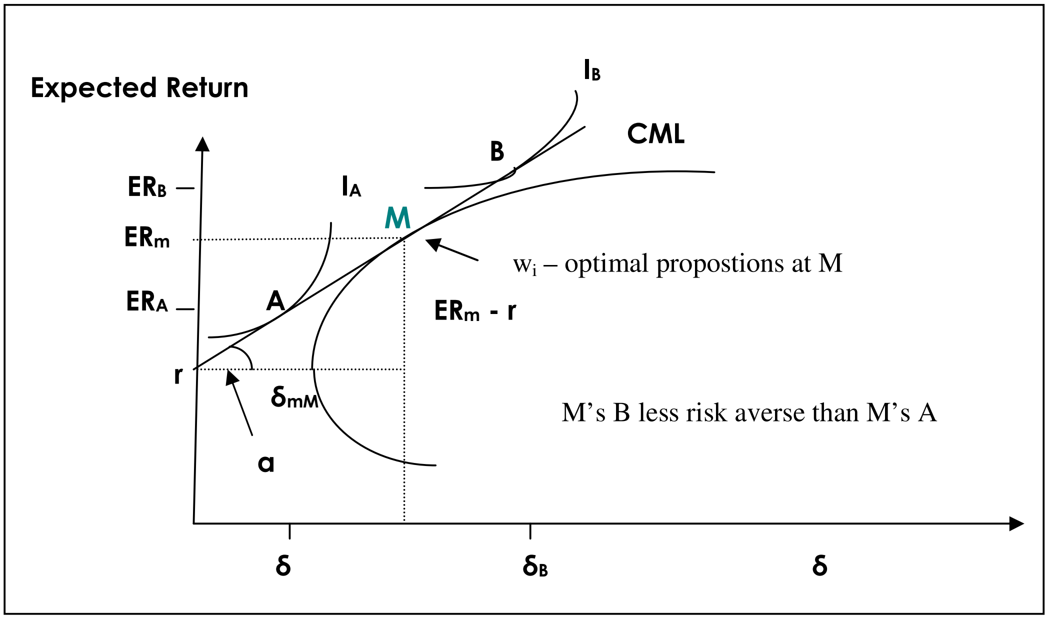
1.4. Capital market line (CML) and separation theorem

If until now we only have considered the risk involved in our portfolio, it's appropriate to look at the risk of all assets operating in the market. Therefore, it is important to point out the differences between portfolio minimum variance and portfolio optimal risky (both located in the efficient frontier). As we all know, the investor's interest in the minimum variance portfolio is to minimize the risk of the portfolio, obtaining the best risk and return available.

As done to mean-variance optimization and efficient frontier, Kuan (2008) mapped (chart nº 6) the efficient frontier for this particular scenario. Looking at chart downwards, the best scenario for the investor is to be located in the line tangent to efficient frontier, or Capital Market Line (CML). The tangency point between efficient frontier and CML represents the optimal risky portfolio, represented in our chart by the portfolio *M* (point *M*). This portfolio is nothing less noting more than a portfolio with all the risky assets in proportion to their market value. Although CML represents a potential portfolio for the investor, there is no obligation to pick up that market portfolio. The investor can choose any of the portfolios along the CML. Everything depends on investor's risk preference, as showed by chart nº 7. So, CML represents the relation between risk and return of a portfolio that contains multiple risky assets and the risk-free asset, and can be defined as,

$$E(R_p) = R_{rf} + \left[\frac{E(R_M) - R_{rf}}{\delta_M} \right] \delta_p \qquad [12]$$

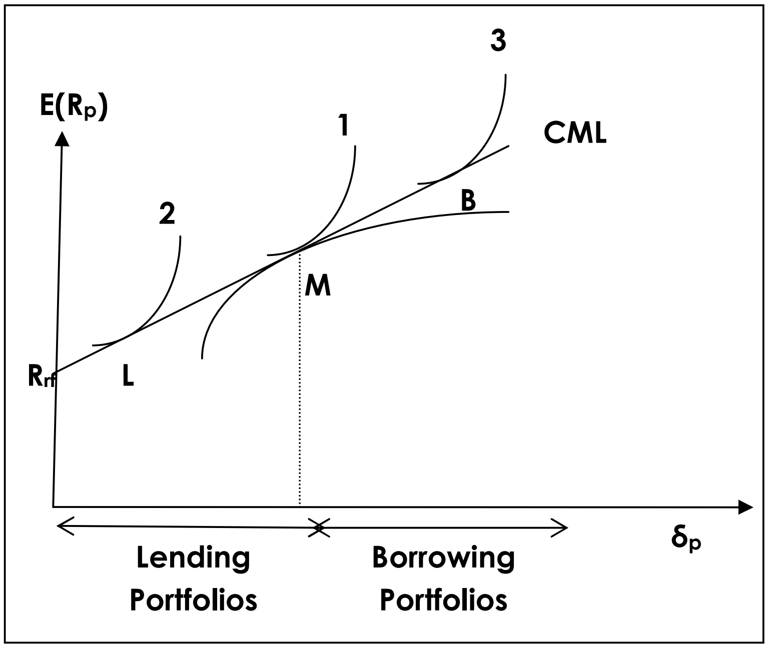
Chart nº 6 - Capital Market Line (CML)



Source: Pires, 2010.

From chart nº 7 Kuan (2008) conclude that, if the investor's risk preference is represented by *indifference curve 1*, so he will pick up portfolio *M*, which is the market portfolio. But, if the investor's risk preference is represented by *indifference curve 2*, he will prefer portfolio *L*, which has a lower level of risk than the market portfolio. In this case, the only way to reduce the risk of the portfolio is to include the risk-free asset in it (seeing investor is "lending" part of his money at the risk-free rate, portfolio *L* is also known as a "lending portfolio"). Finally, if the investor's risk preference is represented by *indifference curve 3*, he will pick up portfolio *B*, which has a higher level of risk than the market portfolio. Now in this scenario, the only way to increase the risk of the portfolio is to borrow at the risk-free rate to increase the investment based (portfolio *B* is also known as a "borrowing portfolio").

Chart nº 7 - Risk preference versus investor investment decisions



Source: Pires, 2010.

According to Tobin's Separation Theorem (1958b), an investor's decision making process is made of two separate decisions: to be on the CML, were the investor initially decides to invest in the market portfolio (in our graph portfolio *M*), or according to their risk preference, investor makes a separate financing decision on whether to lend or borrow at the risk-free rate in order to get the desired portfolio.

As Buiter (2003) pointed out, the assumption behind Separation Theorem is that in a world with one safe asset and a large number of risky assets investor, which is risk-averse, have to choose between the safe asset and the same portfolio of risky assets. The degree of risk aversion only determines the weight in total portfolio accounted by the safe asset and by the portfolio of risky assets. Tobin (1958b) viewed his model of portfolio choice under uncertainty and the separation theorem describes the behavior of an individual portfolio manager, criticizing the use made by Sharpe (1964), Lintner (1965) and Mossin (1966) of the mean-variance model. The separation theorem plus the assumption that all financial market investors expected the same rate of return, means that the portfolio of risky assets held by an investor and every portfolio holder has to be the market portfolio of risky assets.

Although CAPM fails empirically, it is considered to yield important assumptions (Mossin, 1968; Samuelson, 1969; Merton, 1969; Cox, Ingersoll & Ross, 1985). The main reason for Tobin's critics (1958b) on CAPM was the common belief in a perfect market. This idea became popular in the 70s with Lucas (1981, 1990), Kydland and Prescott (1982), Prescott (1986b), Stokey, Lucas and Prescott (1989) and Sargent (1987). All these authors sustained that the massive daily transactions, the government intervention in the market through public debt, among others, are distorted by broker's expectations and speculations

In short, when an investor do not have a risk-free asset, he can only reduce the risk of the portfolio by investing the money in "safer" risky assets, which will move the investor downward along the efficient frontier. However, when the investor has access to the risk-free asset, he can reduce the risk of the portfolio by investing or lending at the risk-free rate, which will move him downward along the CML. The portfolios on the CML provide a higher return than the portfolios on the efficient frontier for the same level of risk. This means that, having risk-free asset helps an investor to reduce the risk of his portfolio, but also to preserve most of the return of the portfolio.

In the same way, in the absence of the risk-free asset, the investor can improve the return of the portfolio by investing his money towards "riskier" risky assets, which will move the investor upward along the efficient frontier. However, with the presence of the risk-free asset, the investor can improve the return of the portfolio by borrowing at the risk-free rate, which will move the investor along the CML. Portfolios on the CML experienced a smaller increase in risk level than the portfolios on the efficient frontier for the same level of increase in return. This means that, the availability of the risk-free asset helps an investor to improve the return of his portfolio without taking much risk.

1.5. The CAPM assumptions and theoretical failings, and the three-factor model

Kuan's (2008) study lists the ten key concepts on which CAPM was constructed:

- There are no transaction costs in buying and selling assets; there are no intermediation costs (broker); there are no spread between bidding and asking prices; there is no place for the payment of any king of fee. Only assets risk determines which asset an investor will buy.

• An investor can take any position of any size in any asset he wishes; market it's perfect, with an infinite liquidity; all assets positions and sizes are available.

• When deciding which assets invest, the investor does not consider taxes, being indifferent to receiving dividends or capital return.

• The investor is rational and risk adverse, being aware of all risk underlying an investment, and making decisions based on that risk. Because of this, he demands a higher return as the volatility increases.

• Investors have the same motivations, whether short-term or long-term, concerning time horizon and return. The only concern remains in volatility (risk).

• An investor measures the risk in exactly the same way as the others, because they have the same information and will buy or sell based on identical assumptions. All investor expect the same from his investment: as a seller, because another asset has a level of volatility corresponding to their desired return; as a buyer, because the asset has a level of risk corresponding to the return that he wants.

• Investors seek to control risk only by the diversification of their portfolios.

• All assets, including human capital, can be bought and sold on the market.

• Investors can lend or borrow at the risk free rate, and can also do short selling without restriction.

• The market is neutral, not being influenced by politics or speculative behaviors.

These theoretical assumptions were criticized by many authors, because of its theoretical failings, for instance, Fama (1973), Haugen and Heins (1975), Murphy (1977), and Fama and French (1992).

Markowitz (1959) and Sharpe (1994), defined risk as volatility: the greater the volatility (measured either in terms of standard deviation or beta), the greater the risk. There are other factors that influence assets movements, as inflation or speculative assets which are extremely volatile.

Haugen and Heins (1975) do no corroborate Markowitz (1959) and Sharpe's (1994) definition of risk. To these authors, there is no correlation between volatility and return: high volatility does not give better results, as lower volatility does not give lesser results. Besides this, and more important, is the failure of the assumption that volatility remains constant over time. Volatility simply does not stay the same for any period of time, and varies drastically from one time period to another. So, assets do not have a fixed volatility, therefore it is absolutely impossible to use that factor to manage a portfolio, unless one knows what volatility is going to be.

Another important paper, from Murphy (1977), reviewed the definition of risk. This author concludes that there isn't any stable relation between the risk taken and the return achieved, and that a high volatility was not necessarily compensated by greater returns.

Also Fama and French (1992, 2004) criticized CAPM's theoretical assumptions. Firstly they consider expected returns on all assets are linearly related to their betas, being the only variable with explanatory power. Second, if risk premium (Beta) is positive, so the expected return on the market portfolio exceeds the expected return on assets whose returns are uncorrelated with the market return. Lastly, in the Sharpe (1964) – Lintner (1965) version of the model, assets uncorrelated with the market have expected returns equal to the risk free interest rate, and the beta is the expected market return minus the risk free rate.

The results conducted them to conclude that, on one hand, the relation between β and average return, when variation in market is allowed, is weak (from 1941-1990); on the other hand, β is not enough to explain average return (other variables, such as earnings/price, cash flow/price, BE/ME, and past sales growth, explained more significantly average return then β). Summing, they conclude that estimates of beta for individual assets are imprecise, creating a measurement error problem when they are used to explain average returns. They also conclude that regression residuals have common sources of variation, such as industry effects in average returns. These results were confirmed by Basu (1983), Chan, Hamao, and Lakonishok (1991), Fama and French (1992, 1993, 1996), and Lakonishok, Shleifer, and Vishny (1994), but contested by Kothari, Shanken, and Sloan (1995).

Merton (1973) extended CAPM considering the investor has different objectives. In the CAPM investors care only about the return their portfolio produces at the end of the current period. But, this new approach considers that investors, besides being concerned with their end-of-period payoff (capital invested plus dividends), they are also concerned with the opportunities they will have to consume or invest the payoff. His model was known as the Intertemporal Capital Asset Pricing Model (ICAPM). Thus, when choosing a portfolio at time *t-1*, ICAPM investors consider how their return at time *t* might vary with future macroeconomic variables, for instance labor income, the prices of consumption goods, and the nature of portfolio opportunities at time *t*. But ICAPM investors also consider expectations about those macroeconomic variables and investment opportunities to be available after time *t*. As a result, optimal portfolios are multicriteria/multifactor efficient, meaning they have the largest possible expected returns, given their return variances and the covariances of their returns with the relevant macroeconomics variables.

Latter, Fama (1996) shows that ICAPM generalizes CAPM, arguing that, if there is risk free borrowing and lending or if short-sales of risky assets are allowed, so market portfolio is multicriteria/multifactor efficient. Besides that, multifactor efficiency implies a relation between expected return and Beta, but it requires additional betas, along with a market beta, to explain expected returns.

Based on this evidence, Fama and French (1993, 1996) propose a three-factor model for expected returns as follows,

$$E(R_{it}) - R_{ft} = \beta_{iM} [E(R_{Mt}) - R_{ft}] + \beta_{is}E(SMB_t) + \beta_{ih}E(HML_t) \qquad [13]$$

where,

SMB_t (small minus big) is the difference between the returns on diversified portfolios of small and big assets;

HML_t (high minus low) is the difference between the returns on diversified portfolios of high and low B/M assets, and

β (the betas) are slopes in the multiple regression of R_{it} - R_{ft} on R_{Mt} - R_{ft}, SMB_t, and HML_t.

One implication of the expected return equation of the three-factor model is that the intercept α_i in the time series regression,

$$R_{it} - R_{ft} = \alpha_i + \beta_{iM}(R_{Mt} - R_{ft}) + \beta_{iS}SMB_t + \beta_{ih}HML_t + \varepsilon_{it} \qquad [14]$$

should be statistically indistinguishable from zero, for all assets *i*. Using this criteria, Fama and French (1993, 1996) find that the model captures much of the variation in average return for portfolios formed on size, book-to-market equity, and other price ratios that cause problems for the CAPM. Fama and French (1998) show that an international version of the model performs better than an international CAPM in describing average returns on portfolios formed on scaled price variables for assets in 13 major markets.

The three- factor model is now widely used in empirical research that requires a model of expected returns. Estimates of α_i from the time-series regression above are used to calibrate how fastly stock prices respond to new information (Loughran & Ritter, 1995; Mitchell & Stafford, 2000). They are also used to measure the special information of portfolio managers, for example, in Carhart's (1997) study of mutual fund performance.

Among practitioners, the model is offered as an alternative to the CAPM for estimating the cost of equity capital (for example, Ibbotson Associates).

From a theoretical perspective, the main shortcoming of the three-factor model is its empirical motivation. The small-minus-big (SMB) and high-minus- low (HML) explanatory returns are not motivated by predictions about macroeconomics variables that concerns to investor. Instead they are constructed to capture the patterns uncovered by previous work on how average stock returns vary with size and the book-to-market equity ratio.

But this concern is not fatal. The ICAPM does not require that the additional portfolios used along with the market portfolio to explain expected returns "mimic" the relevant variables. Both ICAPM and arbitrage pricing theory (APT) suffices that the additional portfolios are well diversified (in the terminology of Fama, 1996, they are multifactor minimum variance) and they are sufficiently different from the market portfolio to capture covariation in returns and variation in expected returns missed by the market portfolio. Thus, adding diversified portfolios that capture covariation in returns and variation in average returns left unexplained by the market is in the spirit of both the ICAPM and the APT.

The behavior analysis of three-factor model typically concede that he captures covariation in returns missed by the market return and that it picks up much of the size and value effects in average returns left unexplained by the CAPM. But, truly the average return premium associated with the model's book-to-market factor – which does the heavy lifting in the improvements to the CAPM – is itself the result of investor overreaction that happens to be correlated across firms in a way that just looks like a risk story. In short, in the behavioral view, the market tries to set CAPM prices, and violations of the CAPM are due to mispricing.

The conflict between the behavioral irrational pricing story and the rational risk story for the empirical failures of the CAPM leaves us at a timeworn impasse. Fama (1970) emphasizes that the hypothesis that prices properly reflect available information must be tested in the context of a model of expected returns, like the CAPM. Intuitively, to test whether prices are rational, one must take a stand on what the market is trying to do in setting prices, that is, what is risk and what is the relation between expected return and risk. Thus, when tests reject the CAPM, one cannot say whether the problem is its assumption that prices are rational (the behavioral view) or violations of other assumptions that are also necessary to produce the CAPM.

Fortunately, for some applications, the way one uses the three-factor model does not depend on one's view about whether its average return premiums are the rational result of underlying state variable risks, the result of irrational investor behavior, or sample specific results of chance. For example, when measuring the response of asset prices to new information or when evaluating the performance of managed portfolios, one wants to account for known patterns in returns and average returns for the period examined, whatever their source. Similarly, when estimating the cost of equity capital, one might be unconcerned with whether expected return premiums are rational or irrational since they are in either case part of the opportunity cost of equity capital (Stein, 1996). But the cost of capital is forward-looking, so if the premiums are sampled specific they are irrelevant.

The three-factor model is hardly a panacea. Its most serious problem is the momentum effect of Jegadeesh and Titman (1993). Assets that do well relative to the market over the last three to twelve months tend to continue to do well for the next few months, and stocks that do poorly continue to do poorly. This momentum effect is distinct from the value effect captured by book-to-market equity and other price ratios. Moreover, the momentum effect is left unexplained by the three-factor model, as well as by the CAPM. Following Carhart (1997), one response is to add a momentum factor (the difference between the returns on diversified portfolios of short-term winners and losers) to the three- factor model. This is again legitimate in applications where the goal is to abstract from known patterns in average returns to uncover information-specific or manager-specific effects. But since the momentum effect is short- lived, it is largely irrelevant for estimates of the cost of equity capital.

Frankel and Lee (1998), Dechow, Hutton and Sloan (1999), Piotroski (2000), among others show that in portfolios formed on price ratios like book-to-market equity, assets with higher expected cash flows have higher average returns that are not captured by the three-factor model or the CAPM. The authors interpret their results as evidence that assets prices are irrational; they do not reflect available information about expected profitability.

In truth, however, one cannot tell whether the problem is bad pricing or a bad asset pricing model. A assets price can always be expressed as the present value of expected future cash flows discounted at the expected return on the asset (Campbell & Shiller, 1989; Vuolteenaho, 2002). It follows that if two assets have the same price, the one with higher expected cash flows must have a higher expected return. And this is true whether pricing is rational or irrational. Thus, when one observes a positive relation between expected cash flows and expected returns that is left unexplained by the CAPM or the three-factor model, one cannot tell whether it is the result of irrational pricing or a miss-specified asset pricing model.

As we all know, CAPM is based entirely on beta. The beta's concept was initially defined by Sharpe (1964), Lintner (1965) and Mossin (1966), in the 60, but the literature showed to have no predictive power (Fama, 1973; Fama & French, 1992). The beta for one period differs dramatically from the beta of the next period, invalidating the chance to use it as a volatility predictor. Fama and French (1992) examined 9,500 assets between 1963 and 1990, concluding that a asset's risk, measured by beta, was not a reliable predictor of performance.

In investor's perspective, Kuan's (2008) study summarizes de main critics. The *transaction costs* have a major effect on whether the investor wants to have a long-term or short-term position. In the same way, *taxes* have a major impact on what kinds of investments are interesting. *Liquidity*, a very important indicator, is crucial for the investor to remains on the market, or out of it, and the *difference between dividends and return* affects the type of asset an investor will buy. The *investors are not rational*. They just want "hot" sectors and markets boom. Because of this, many investors will buy assets based only on rumor or hunches. Besides this, only the government can *borrow at the risk free rate.* No other investor in the world can borrow money at these rates. And, *short selling* is not possible in many countries.

Also Soares and Fernandes' work (2002) summarizes the most important problems, when applying the model, such as:

— Do not allow to determine an optimal schedule for model revision. The model was studied and implemented for a certain period. However, when market conditions change, changes should be introduced in the model, to update the efficient frontier. Nevertheless, the model does not provide this information: when to be revised. In practice, as discussed throughout this thesis, several authors propose the use of a sensitivity analysis, concerning different estimation periods, in order to select the best fit to evidence.

— As time passes and the portfolio is not revised, the frontier points are scrolled for directions, eventually, of lower/highest income.

— Correlation analysis requires a high time to have consistency. However there is always the chance that, in the long term, the structural conditions have changed, so the return will not be stationary in the long term. The model does not answer how to overcome this disadvantage.

— There are errors in the inputs measurement, for instance estimation errors that have impact in subsequent outcome.

1.6. Portfolio monitoring and rebalance / performance analysis

The main idea of monitoring *Portfolio performance* involves determining periodically how the portfolio performed in terms of return earned, but also the risk experienced by the investor. In general, the market value of a portfolio is determined by adding the markets value of all the assets held on the portfolio. So, the Return on the portfolio ($\mathbf{r_p}$) can be given by,

$$\mathbf{r_p} = (V_e - V_b) / V_b \qquad [15]$$

where,

V_e - beginning value of the portfolio;

V_b - ending value of the portfolio.

The essential idea behind performance evaluation is to compare the returns which were obtained on portfolio with the results that could be obtained if more appropriate alternative portfolios had been chosen for the investment. Such comparison portfolios are often referred to as benchmark portfolios. In selecting them, investor should be certain that they are relevant, feasible and known in advance. The benchmark should reflect the objectives of the investor.

Dowd (2000) pointed out that, in portfolio management, the general problem remains in its risk. Generally risk can be defined as the probability that the actual outcome from an investment will differ from the expected outcome. So, the more variable the possible outcomes can occur, the greater the risk.

In a more analytic way, Dowd (2000) considers that risk adjustment has two main aspects. On one hand, the adjustment before the selection of assets to invest: given a set of alternatives, how portfolio managers choose between asset *A*, which has a high expected return and a relatively high risk, and asset *B*, which has a low expected return but is relatively safe? On the other hand, the evaluation of *actual* portfolio performance *after* the decisions already made and the results of those decisions are materialized. For instance, a comparison of asset *A*, who have a high expected return but took a lot of risks, with asset *B*, who have a low expected return but took few risks, must be done.

To distinguish between these two aspects of risk adjustment, Dowd (2000) use the term *risk adjustment* to refer to the first aspect, *ex ante*; and to refer to the second aspect, *ex post*, use *performance evaluation.* So, *risk adjustment* enables us to compare the returns associated with different levels of risk, either those returns expected *ex ante* or those actually made *ex post* (enables us to choose between investment opportunities with differing (expected) returns and risks). *Performance evaluation* enables us to compare the performance of units or portfolios that made different returns but also took different risks. This measure also guide in allocating internal capital and setting positions limits, and also in developing strategic plans.

Jagric, T., Podobnik, Strasek and Jagric, V. (2007) tell us that, historically, just about a decade ago, investors were almost exclusively interested in assets having large annual returns or, in other words, assets capable of beating the market. But, seeing many portfolios with outstanding profits have collapsed, investors are more and more interested in the other dimension of asset performance: the risk.

Several statistical measures are used to assess risk, for instance, Coefficient Beta (β), Sharpe's, Treynor's and Jensen's indexes are commonly used.

Levišauskait (2010) clarifies that, if a portfolio is completely diversified, all of these measures (Sharpe's, Treynor's and Jensen's ratios) will agree on the ranking of the portfolios. The reason for this is that with the complete diversification total variance is equal to systematic variance. When portfolios are not completely diversified, the Treynor's and Jensen's measures can rank relatively undiversified portfolios much higher than the Sharpe´s measure does. Since the Sharpe ratio uses total risk, both systematic and unsystematic, components are included.

1.6.1. Coefficient Beta (β)

Each asset has its individual systematic-undiversified risk, measured using coefficient Beta. Coefficient Beta (β) indicates how the price of asset/ return on asset depends upon the market forces. Thus, coefficient Beta for any asset can be calculated using formula,

$$\boldsymbol{\beta_j} = \frac{\mathrm{Cov}(r_A, r_M)}{\delta^2(r_M)} \qquad [16]$$

where,

$\boldsymbol{\beta_j}$ represents Coefficient Beta (β);

$\mathbf{Cov(r_A,r_M)}$ is the covariance between returns of asset *A* and the market portfolio;

$\boldsymbol{\delta^2(r_M)}$ express variance of returns on market portfolio.

Table nº 3 - Interpretation of coefficient Beta (β)

Beta	Direction of changes in asset's return in comparison to the changes in market's return	Interpretation of β meaning
2,0	The same as market	Risk of asset is twice higher than market risk
1,0	The same as market	Asset's risk is equal to market risk
0,5	The same as market	Asset's risk twice lower than market risk
0	There is no relationship	Asset's risk is not influenced by market risk

- 0,5	The opposite from the market	Asset's risk twice lower than market risk, but in opposite direction
- 1,0	The opposite from the market	Asset's risk is equal to market risk but in opposite direction
- 2,0	The opposite from the market	Risk of asset is twice higher than market risk, but in opposite direction

Source: Levišauskait, 2010.

A very important feature of Beta to the investor is that Beta of portfolio is simply a weighted average of Betas of its component assets, where the proportions invested in the assets are the respective weights. Thus, Portfolio Beta can be calculated using formula:

$$\boldsymbol{\beta_p} = w_1 \beta_1 + w_2 \beta_2 + \ldots + w_n \beta_n = \sum_1^t w_t \beta_t \qquad [17]$$

where,

$\mathbf{w_t}$ - the proportion of the portfolio's initial value invested in asset n;

$\boldsymbol{\beta_t}$ - coefficient Beta for asset n.

1.6.2. Sharpe´s Performance Index (1966)

Sharpe (1966) introduced a measure for the performance of portfolios and proposed the term "reward-to-variability ratio" to describe it (Sharpe, 1975). As stated by Dowd (2000), in order to carry out the risk adjustment, the traditional approach uses Sharpe's ratio. To do so, let's suppose portfolio, *p*, with a return R_p, a benchmark portfolio denoted by *b*, with return R_b; *d* will be the differential return ($R_p - R_b$), and d^e will be the expected differential return. The *ex-ante* Sharpe's ratio comes as:

$$SR^{ante} = d^e / \delta^e_d \qquad [18]$$

where,

$\boldsymbol{\delta^{e}_{d}}$ is the predicted standard deviation of *d.*

This ratio captures the expected differential return per unit of risk associated with the differential return, and takes into account both the expected differential return between two portfolios and the associated differential risk. Since it gives risk estimates before decisions are actually taken, the *ex ante* Sharpe's ratio can be very useful for decision-making, for example, choosing in which assets to invest. In the same way, the *ex-post* Sharpe's ratio is defined as:

$$\boldsymbol{SR^{post}} = d / \delta_{d} \qquad [19]$$

where,

$\boldsymbol{\delta_{d}}$ is the standard deviation of *d* over a sample period. This version of this ratio takes into account both the *ex post* differential return and the associated variability of that return, and can be very useful for performance evaluation after the event.

Jagric et *al.* (2007) indicates that the annualized Sharpe's ratio is calculated by dividing the annualized excess return by the standard deviation of the return. In mathematical form,

$$\boldsymbol{SR_{h}} = (R - R_{f}) / \delta_{d} \qquad [20]$$

where,

$\boldsymbol{(R - R_{f})}$ uses the total risk or standard deviation of returns. The advantage of using annualized Sharpe's ratio for evaluating portfolios is that it does not depend on the choice of a benchmark (market index);

$\boldsymbol{\delta_{d}}$ is the standard deviation of the return *d.*

Dowd (2000) conclude that Sharpe's ratio captures both risk and return - actual or expected - in a single measure: an increase in return differential or a fall in standard deviation, are both "good" events, and leads to a rise in the Sharpe's ratio; inversely, a fall in return differential or a increase in standard deviation, are both "bad" events, leading to a fall in the Sharpe's ratio. Hence, a higher Sharpe's ratio is good, and a lower one is bad.

When choosing between two alternatives, the Sharpe's ratio criterion is therefore important when choosing the one with the higher Sharpe's ratio. If we are constructing a portfolio (deciding on investments before the event), the best is to choose assets with the highest *ex ante* Sharpe's ratio; if we are trying to evaluate traders after the event, the best is to consider the trader with the higher *ex post* Sharpe's ratio. As denoted by Dowd (2000), the higher the Sharpe's ratio, the higher the risk-adjusted return. In effect, Sharpe's ratio can be understood as a *proxy* for the risk-adjusted return.

In our empirical work, we use the simplified formula proposed by Levišauskait (2010), meaning that Sharpe's ratio shows an excess of return over risk free rate, or risk premium, by unit of total risk, measured by standard deviation,

$$\mathbf{S_r} = \frac{E(r_p) - r_f}{\delta_p^2} \quad [21]$$

where,

$\mathbf{r_p}$ represents the average return for portfolio p during some period of time;

$\mathbf{r_f}$ is the average risk-free rate of return during the same period;

$\boldsymbol{\sigma_p}$ is standard deviation of returns for portfolio *p* during the same period.

1.6.3. Treynor´s Performance Index (1965)

While the Sharpe's ratio measures portfolio premium risk over its standard deviation, Treynor's ratio, from Jack Treynor (1965), is a measure of reward (or excess return) per unit of risk, this is, compares the portfolio premium risk with the diversifiable risk of the portfolio measured by its Beta. So, the higher the Treynor's Performance Index, the better the portfolio performance. The mathematical formula proposed by Levišauskait (2010), comes as,

$$\mathbf{T_p} = \frac{r_p - r_f}{\beta_p} \qquad [22]$$

where,

$\mathbf{T_p}$ represents the Treynor Performance Index;

$(\mathbf{r_p} - \mathbf{r_f})$ is the difference between the portfolio return (r_p) and the risk-free rate of return (r_f) over a period, measuring the reward (or excess returns);

$\boldsymbol{\beta_p}$ is the Beta of the portfolio, as the unit of risk.

1.6.4. Jensen´s alpha (1968)

In Jensen's measure, *alpha* (α) is a coefficient proportional to the *excess return* of a portfolio over its required return (or its expected return), for its expected risk measured by its Beta. Hence, *alpha* is determined by the fundamental values of the company in contrast to Beta, which measures the return due to its volatility. In order to do so, CAPM determines the amount of the return which causes market volatility measured by the firm's beta. It can be positive, negative, or zero. Note that, by definition, Jensen's alpha of the market is zero. If the alpha is negative, then the portfolio is underperforming the market. The mathematical formula comes as,

$$\mathbf{J_\alpha} = E(r_p) = r_f + [E(r_M - r_f)] * \beta_p \qquad [23]$$

where,

$\mathbf{E(r_p)}$ is total portfolio expected return;

$\mathbf{r_f}$ represents the risk-free rate;

$\mathbf{E(r_M - r_f)}$ is the expected difference between market return and the risk-free rate;

$\boldsymbol{\beta_p}$ is the risk of the portfolio.

CHAPTER 2: ALTERNATIVE PORTFOLIO SELECTION PROCEDURES TO MEAN/VARIANCE ANALYSIS: THE MULTICRITERIA DECISION MAKING MODELS

As explained previously, portfolio management can be interpreted as a multicriteria problem. Multi criteria decision models has been widely used in real-life decision problems, for instance, TOPSIS, AHP, ELECTRE family, PROMETHEE family, ADELAIS and MINORA. Each one of them has distinct characteristics and distinct applications. Within these methods, literature highlights ELECTRE family, distinguished by their performance and problem resolution. As stated by Buchanan, Sheppard and Vanderpooten (1999), "experience with the methodology shows that ELECTRE was well received by the decision makers and, importantly, provided sensible and straightforward project rankings."

In this sense, this chapter provides an overview on multi criteria decision making models, in particular ELECTRE methods. The different versions of ELECTRE (which exist, why they exist, and for what kind of problematic they exist) are discussed.

But, as any other theory, ELECTRE methodology has theoretical failings, for instance, rank reversals. So, alternatives to ELECTRE family are also discussed and presented, in particular, the literature suggests AHP method and PROMETHEE family.

2.1. The multicriteria decision making models (MCDM)

A decision problem according to Roy's definition (Roy, 1991), is a representation of an element of a global decision. Zbigniew and Watróbski (2008) distinguishes decision alternatives on *realistic alternatives* (corresponding to a project, which implementation is feasible) and on *unrealistic alternatives* (which can include contradictory goals and can be only used for the discussion). The difficulty when solving multi criteria decision problems is the requirement of including alternatives' judgments (choice alternatives) from various points of view (Escobar-Toledo & López-Garcia, 2005).

To do so, Zbigniew and Watróbski (2008) consider the definition of a decision problem consists into a two-element process, ***(C, θ)***, where ***C*** represents *a set of criteria*, describing relations between properties of decision alternatives and preference levels of considered alternatives; and $\boldsymbol{\theta}$ represents *a set of meta-data of a decision situation*, consisting into the decision maker's expectations about a decision situation.

An analytic task, stated by the analyst and the decision maker, reflect particular aspects of implementation based on possible options (decision alternatives). The fundamental element of the meta-data set θ is the choice of the decision problematic situation according to the following (Roy, 1991):

a) *problematic α – the choice problematic* (finding a subset of the set *A*, which includes only the best solutions),

b) *problematic β – the sorting problematic* (assigning alternatives to defined categories),

c) *problematic γ – the ordering problematic* (constructing a ranking of alternatives in the set *A* from the best to the worst one).

Such an approach only considers a part of the decision process. Applying multi criteria methods, to analyze a decision situation, requires making a deliberate choice of a method suitable for a given decision situation. The goal of the mentioned choice is to find the multi criteria transformation *F* which fulfils, $\boldsymbol{F(C, \theta) \rightarrow \max u}$, where *u* is an indicator of a decision maker's satisfaction measured by his preferences.

But, to be applied, a multi criteria method requires some features. Brans and Mareschal (2005) summarize them into seven requisites.

- **Requisite 1:** The amplitude of the deviations between the evaluations of the alternatives within each criteria should be taken into account: $d_j(a, b) = g_j(a) - g_j(b)$

- **Requisite 2:** As the evaluations of each criteria are expressed in their own units, *the scaling effects* should be completely eliminated.

- **Requisite 3:** In the case of pair wise comparisons, an appropriate multi criteria method should provide the following information:

  - *a* is preferred to *b*;

  - *a* and *b* are indifferent;

  - *a* and *b* are incomparable.

- **Requisite 4:** Different multi criteria methods request different additional information and operate different calculation procedures so that the solutions they propose can be different. It is therefore important to develop methods being *understandable* by the decision-makers.

- **Requisite 5:** An appropriate procedure should not include technical parameters that have no significance for the decision-maker.

- **Requisite 6:** An appropriate method should provide information about the *conflicting nature* of the criteria.

- **Requisite 7:** Most of the multi criteria methods are allocating weights of criteria relative importance. An appropriate method should offer *sensitivity tools* to test easily different sets of weights.

This decision process phase, the *exploitation phase*, intends to make a representation of global preference which is the outcome of a decision maker's expectations (meta-data) and mutual local preferences between particular decision alternatives.

Zbigniew and Watróbski (2008) indicate some methods suggested by the literature and their characteristics[13]:

[13] There are other methods that can be applied, so being, **TOPSIS** (Hwang & Yoon, 1981) defined by choosing an alternative with the shortest distance to the ideal solution and the longest distance to the negative ideal solution, **ADELAIS** (Zopounidis, Despotis & Kamaratoy, 1993) provides an extensive data management capabilities and the concerned solution process provides a 'two level' interaction: interactive

- **AHP:** Transform subjective decision maker's judgments into ordered criteria weights. The procedure uses decomposition of the problem and comparison matrix of attributes to create a comprehensive estimate of a decision alternative (Saaty, 1980).
- **ELECTRE family:** The outranking is expressed by the credibility index (Roy, 1991; Figueira, Mousseau & Roy, 2005).
- **PROMETHEE family:** Based on the concepts of pseudo-criteria, elaborates an outranking relation and pair wise comparisons (Brans & Vincke, 1985).

For all this, we can easily understand that multiple criteria decision methodology has been widely used in many real-life decision problems. Let's analyze some examples.

a) *In project selection for Northern Generation, a division of the Electricity Corporation of New Zealand (ECNZ):*

Buchanan *et al.* (1999) used ELECTRE III method to rank minor projects for Northern Generation, a division of the Electricity Corporation of New Zealand (ECNZ). The authors choose this method, because it has several unique features not found in any other solution methods, in particular, the concepts of outranking and the use of indifference and preference thresholds.

This study was motivated by the fact that ECNZ pretended to introduce a more objective (and structured) method for the annual exercise of selecting minor projects to be undertaken (projects selection are conditioned by financial targets). In this sense, considering five projects, the authors assigned to each project, or alternative, a few attributes, which were then related to the criteria.

From the application of ELECTRE methodology, it follows that project nº 3 and project nº 5 were ranked together. However, Buchanan *et al.* (1999) adds that, a sensitivity or robustness analysis to final rankings should be done, for instance, by changing thresholds and weights.

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assessment of the decision maker's utility function and interactive modification of the satisfaction levels, and **MINORA** (Zopounidis, 1992), developed in order to help decision maker, and their evaluation criteria, when selecting assets to obtain the maximization of their utility.

b) *In Mass Transit Systems (MTS-s):*

Zak (2005) applied the multiple criteria decision methodology to the decision problems in mass transit systems (MTS-s). He considered three problems of strategic and tactical character: evaluation of the MTS development scenarios – problem I, ranking of the maintenance work, contractors for the MTS renovations project – problem II and selection of the transportation mode for the MTS – problem III. To solve this multiobjective ranking problem, he applied the following methods: ELECTRE, Oreste, Mappac, AHP and UTA. To do so, stakeholders/decision maker's expectations were analyzed (survey, interviews, family of criteria, among others), and results were compiled. Their features were measured by comparing final rankings generated by different methods, and by expected final rankings suggested by each of the respondents.

The results suggested that all the analyzed methods have universal character and can be applied to a wide spectrum of multiobjective ranking problems in MTS-s. Specifically, ELECTRE and AHP methods were the most reliable and users' friendly multi criteria decision methods.

c) *In natural resources management:*

Kangas, A., Kangas, J. and Pykäläinen (2001) applied ELECTRE III and PROMETHE II in natural resources management in Finland (Finnish Forest and Park Service in Kainuu, eastern Finland), because natural resources are synonymous of economic, ecological and socio-cultural sustainability. So, multi objective natural resources management planning and decision support are required. These authors applied ELECTRE III and PROMETHE II because in both methods the number of decision criteria and decision maker may be large, and the uncertainty concerning the values of criteria can be taken into account using fuzzy relations (determined by indifference and preference thresholds). Therefore, the authors considered this feature an important advantage of outranking methods.

d) *In land redevelopment in Heping Harbor Zone (Taiwan):*

Huang and Chen (2005) applied ELECTRE II to a case study based on land redevelopment in Heping Harbor Zone (in Taiwan). Considering fast changes in living environment, many cities have placed increasing expectations on land redevelopment to help urban planning problems. So, the authors defined six preliminary improvement alternatives according to the collected information.

As notice by the authors, ELECTRE method allows both quantitative and qualitative criteria to be handled. In this sense, normally discordance index is constructed using *Absolute Value of the Maximum Differentiated Performance* (A.V.M.D.P.) to evaluate benchmark as evaluation procedure. Huang and Chen (2005) conducted another benchmark procedure, namely the *Absolute Value of the Sum of Differentiated Performance* (A.V.S.D.P.) procedure. Those two benchmark evaluation procedures represent different decision maker's approaches: A.V.M.D.P. focus on discrepancies in the most important criteria, and A.V.S.D.P. focus on discrepancies in the overall criteria. The result shows that *alternative one* is the priority alternative as the discordance index evaluation benchmark for both A.V.M.D.P. and A.V.S.D.P. However, when the decision maker prefers to have more than one alternative to be taken into consideration, the alternatives after the "best" first one vary greatly. Taking the second "best" one alternative, A.V.M.D.P. indicates *alternative two* as the only choice, but A.V.S.D.P. indicates *alternative six* as the only choice. Now, taking the *third alternative*, A.V.M.D.P. gives *alternative four* as the best choice, while A.V.S.D.P. gives *alternative four* as the worse alternative.

Considering these results, Huang and Chen (2005) concluded that the two evaluation benchmarks reflect different decision maker judgment and needs criteria. Therefore, if decision makers choose the discrepancy of overall criteria as the screening benchmark for evaluation alternatives, and using the A.V.M.D.P. evaluation benchmark, the screening results would lead to serious errors.

e) *In the choice of construction equipment:*

Serdar and Aynur (2009) analyze the advantages of using concrete pumps on machine selection process. They add that concrete pump may improve productivity, may increase the quality of products and services, and may reduce the duration and the cost of the task "pouring concrete". In the long run, this can contribute to the related firms in improving their competitiveness and in outperforming their competitors in the construction industry.

In this sense, Serdar and Aynur (2009) justify the use of ELECTRE III methodology because this technique allows quantitative data to be evaluated together with qualitative data.

Gives a ranking order of alternatives rather than presenting only one option, and have flexible feature which, in turn, makes decision-makers feel more comfortable and independent. As limitation, these authors point out that this method only can be used when at least 3 and at most 13 decision criteria are available (Figueira *et al.*, 2005).

To conducted the experiment, the authors established five quantitative criteria ("selling price", "operating cost per day", "maximum pumping speed", "second hand" and "technical services"), in order to evaluate three different manufactures of concrete pumps (Z-52, X-52 and Y-52).

Considering ELECTRE III methodology, the final result obtained revealed X-52 as being the most suitable concrete pump, followed by Z-52 and Y-52. To test this final result, six independent experimental attempts of sensitivity analysis were made, for instance, the author vary each weight of each criteria separately, and the overall findings point out that the original outcome was not considerably changed.

f) *In personnel selection:*

Afshari, Mojahed, Yusuff, Hong and Ismail (2010) suggested ELECTRE method to solve personnel selection problem using multi criteria decision making process, applied in the telecommunication sector of Iran. While traditional methods for selection of human resources are mostly based on statistical analyses of test scores that are treated as accurate reflections of reality, modern approaches, however, recognize that selection is a complex process that involves a significant amount of vagueness and subjectivity. The authors firstly use ELECTRE method for pre-ranking personnel; then, after identifying the level of personnel, they apply AHP method when at least one of personnel's grades was placed in the same with another. At the end, all personnel which had been considered were sorted in different level.

The limitation of this author's study, and one of the failings of ELECTRE methodology, is that executives' judgment is ignored during the decision-making process, although some criteria could have a qualitative structure or have an uncertain structure which cannot be measured precisely.

2.1.1. The ELECTRE methodology (Elimination and Choice Translating Reality)

Within the models mentioned previously, we can highlight the ELECTRE family, from the "European school" which, as stated by Buchanan *et al.* (1999), responds to the deficiencies of the decision process methods.

Kangas *et al.* (2001), Figueira *et al.* (2005) (following the studies of Roy, 1991, Roy & Bouyssou, 1993, and Schärlig, 1985), Tervonen, Figueira, Lahdelma and Salminen (2005), Hanandeh and El-Zein (2006), Wang (2007), and Afshari *et al.* (2010), among many others, pointed out the relevance of multi criteria decision models, in particular, ELECTRE methods. So, ELECTRE methods are developed in two main phases. Firstly, the *construction of the outranking relations*, and secondly the *exploitation of those relations* to get the final ranking of the alternatives. In the *exploitation procedure*, recommendations are elaborate from the results obtained in the first phase. The nature of the recommendation depends on the problematic: choosing, ranking or sorting. Each method is characterized by its construction and exploitation procedure.

Furthermore, these authors clarify that different ELECTRE methods may differ in how the outranking relations between the alternatives are done, and in how they apply these relations to get the final ranking of the alternatives.

Being ELECTRE method based on criteria, it's important to distinct two sets of parameters: the *importance coefficients* and the *veto thresholds*. The *importance coefficients* in ELECTRE methods refer to intrinsic "weights". For a given criteria the weight, w_j, reflects its voting power when it contributes to the majority which is in favor of an outranking. The *veto thresholds* express the power attributed to a given criteria to be against the assertion "*a* outranks *b*", when the difference of the evaluation between $g(b)$ and $g(a)$ is greater than this threshold. These thresholds can be constant along a scale or they can also vary[14].

ELECTRE approach considers thresholds and outranking, defining a criteria $\boldsymbol{g_j}$, j = 1, 2, …, r and a set of alternatives $\boldsymbol{A}$ (Buchanan *et al.*, 1999). In *traditional modeling*, there are two relations for two alternatives $(a, b_h) \in A$, such that,

[14] About this topic see, Saaty (1980), Costa and J. Vansnick (1994), Keeney and Raiffa (1976), Figueira and Roy (2002), Maystre, Pictet, and Simos (1994), Mousseau (1993), Rogers and Bruen (1998, 2000), Roy and Mousseau (1996), Simos (1990), and Vansnick (1986).

a*P*b$_h$ (a is preferred to b_h) <=> $g(a) > g(b_h)$

a*I*b$_h$ (a is indifferent to b_h) <=> $g(a) = g(b_h)$

but *in ELECTRE methods*, an indifference threshold ***q***, a preference threshold ***p***, and an additional binary relation ***Q*** are introduced. So the above relations are redefined to:

a*P*b$_h$ (*a* is strongly preferred to b_h) <=> $g(a) - g(b_h) > p$

a*Q*b$_h$ (*a* is weakly preferred to b_h) <=> $q < g(a) - g(b_h) <= p$

a*I*b$_h$ (*a* is indifferent to b_h, and b to *a*) <=> $|g(a) - g(b_h)| <= q$

The definition of these thresholds will permit to outrank a relation a***S***b_h, this is, the idea is to test all the alternatives "*a* is at least as good as b_h" or "*a* is not worse than b_h", and validate, or no, the assertion a***S***b_h. So, this gives rise to one of the following four situations:

- [a***S***b_h and not(b_h***S***a)] ⇔ a***P***b (*a* is strictly preferred to *b*);
- [not(a***S***b_h) and b_h***S***a] ⇔ a***R***b (*a* is incomparable to *b*);
- [a***S***b_h and b_h***S***a] ⇔ a***I***b (*a* is indifferent to *b*);
- [not(a***S***b_h) and not(b_h***S***a)] ⇔ a***R***b (*a* is incomparable to *b*).

To test the assertion a***S***b_h (or b_h***S***a), two conditions should be verified:

- *Concordance condition*: for an outranking a***S***b_h (or b_h***S***a) be accepted, a "sufficient" majority of criteria should be in favor of this assertion;

- *No Discordance condition*: when the concordance condition holds, none of the criteria in the minority should oppose to the assertion a***S***b_h (or b_h***S***a) in a "too strong way".

As already mentioned, two types of inter-criteria preference parameters are presented in the construction of ***S***:

- A set of *weight-importance* coefficients (k_m, k = 1, 2, …, *m*) is used in the concordance test, when computing the relative importance of the coalitions of criteria being in favor of the assertion a***S***b_h, and
- A set of *veto thresholds* ($v_1(b_h)$, $v_2(b_h)$, …, $v_m(b_h)$), *h* Є B, is used in the discordance test v_j (b_h) representing the smallest difference $g_j(b_h)$ – $g_j(a)$ incompatible with assertion a***S***b_h.

Finally, to compare an alternative *a* to an attribute b_n, the relation is build through the following steps:

- Compute the partial concordance indices $c_j(a, b_h)$ and c_j (b_h, a);
- Compute the overall concordance indices $c(a, b_h)$;
- Compute the partial discordance indices $d_j(a, b_h)$ and $d_j(b_h, a)$;
- Compute the outranking relation based on the credibility indices $\delta(a, b_h)$.

As stated by Figueira *et al.* (2005), Tervonen *et al.* (2005), and Hanandeh and El-Zhein (2006), ELECTRE methods cannot be used for decision process without some external method, needed to transform the preferences into deterministic weight values. Although the innumerous weight elicitation techniques proposed (Mousseau, 1995; Hokkanen & Salminen, 1997; Figueira & Roy, 2002), Rogers and Bruen (1998b) criticized the methods available for eliciting weighting values with ELECTRE III. They highlighted the fact that due to the no compensatory nature of ELECTRE III, using weight averages does not give a true representation of the stakeholders' preferences. So, these authors approach uses pair wise comparisons to elicit the weights. They add that using weight elicitation techniques, the stability should be analyzed by using intervals for the weights, because the difficulty of expressing beliefs in mathematical terms causes inaccuracy in the evaluations.

Hereafter this general knowledge on ELECTRE methodology, it is now appropriate to present the specific features of each version.

In this sense, Kangas *et al.* (2001), José Figuier *et al.* (2005), Tervuren *et al.* (2005), Huang and Chen (2005), Hanandeh and El-Zein (2006), Wang (2007), and Afshari *et al.*

(2010) studies, guide us for each version, depending on the intended study: for *choice problem*, we can apply ELECTRE I, ELECTRE Iv, and ELECTRE IS; for *ranking problem*, we can apply ELECTRE II, ELECTRE III, ELECTRE IV, and ELECTRE-SS; and for *sorting problem* we can apply ELECTRE TRI.

2.1.1.1. Choice problematic:

A decision maker under choosing problematic must be helped in selecting a subset of actions, as small as possible, in such a way that a single action may finally be chosen.

2.1.1.1.1. ELECTRE I (Electra one):

Figueira *te al.* (2005) consider that this method does not have a significant practical interest, given the diversity nature of real world applications, which usually have a vast spectrum of quantitative and qualitative elementary consequences. This leads to the construction of a contradictory and very heterogeneous set of criteria, with both numerical and ordinal scales associated with them. In addition, a certain degree of imprecision or uncertainty is always attached to the knowledge collected from real-world problems.

The method is very simple and it should be applied only when all the criteria have been coded in numerical scales with identical ranges. In such a situation the assertion a***b***s is valid, only when two conditions hold: the strength of the concordant condition must be powerful, and no discordance against the assertion "*a* is at least as good as b_n" may occur.

The first condition, the strength of the *concordant condition*, must be understood as the sum of the weights associated to the criteria forming that condition. It can be defined by the following *concordance index*:

$$c(a,\, b) = \sum_{j: g_i(a) \geq g_j(b)} w_j \qquad [24]$$

where,

$\sum_{juju} \mathbf{w_j = 1}$, where *j* is the set of the indices of the criteria;

$\mathbf{j{:}g_j(\boldsymbol{a}) \geq g_j(\boldsymbol{b})}$, is the set of indices for all the criteria belonging to the concordance condition with the outranking relation *abs*.

In other words, the value of the concordance index must be equal or greater than a given concordance level, *s*, whose value generally falls when c(*alb*) ≥ s.

The second, and last condition, *no discordance* against the assertion "*a* is at least as good as *b*" may occur, is based on discordance measurement. The discordance is measured by a *discordance level* defined as follows:

$$d(a, b) = \max_{j:g_j(a)<g_j(b)} \left\{ g_j(b) - g_j(a) \right\} \qquad [25]$$

The power of the discordance condition tells us that, if its value surpasses a given level, *v*, the assertion is no longer valid. So, discordant condition exerts no power if d(*a, b*) ≤ v.

Both concordance and discordance indices have to be computed for every pair of actions *(a, b)* in the set *A*, where $a \neq b$. As already said, this computer procedure leads to a binary relation, where for each pair of action *(a, b)*, only one of the following situations may occur:

- a***b***s and not b***a***s ⇔ a***P***b (*a* is strictly preferred to *b*);
- b***a***se and not a***b***s ⇔ b***o***a (*b* is strictly preferred to *a*);
- a***b***s and b***a***s ⇔ a***I***b (*a* is indifferent to *b*);
- Not a***b***s and not b***a***s ⇔ a***R***b (*a* is incomparable to *b*).

One of the big disadvantages of ELECTRE I is that this framework says nothing to decision maker about how to select the best compromise action, or a subset of actions. In the construction procedure (the first procedure) of ELECTRE I method only one outranking relation *S* is matter of fact. When exploiting this outranking relation (the second procedure) in order to identify the smallest subset of actions, from which the best action could be selected, all the actions which form a cycle are considered indifferent. Because of this, ELECTRE I is criticized, giving place to ELECTRE IS, which was developed to mitigate this inconvenient.

2.1.1.1.2. ELECTRE Iv (Electra one vee):

Continuing Figueira et al. (2005) study, ELECTRE Iv, is nothing more nothing less than ELECTRE I with veto threshold (Master, Picket & Samos, 1994). The introduction of veto threshold, v_j, made possible for analysts and decision makers, to overcome the difficulties related to the heterogeneity of scales: whichever the scales type are, this method is always able to select the best compromise action or a subset of actions to be analyzed by decision makers.

In short, the concept of *veto threshold* is related to the definition of an upper bound beyond which the discordance about the assertion "*a* outranks *b*" cannot surpass, allowing an outranking. Differently from ELECTRE I, where discordance level is related to the scale of criterion g_j in absolute terms for an action *a* from *A*, in ELECTRE Iv veto threshold is related to the preference differences between $g_j(a)$ and $g_j(b)$.

The mathematic formulation little differs from ELECTRE I, in the sense that the *discordance condition* is now called *no veto condition*, which may be stated as follows:

$$G\tilde{a}(a) + v_{iu}\,(g\tilde{a}+a)) \geq g_{\tilde{a}}\,(b),\ \forall_j \in J \qquad [26]$$

Finally, to validate the assertion "*a outranks b*" it is necessary that, among the minority of criteria that are opposed to this assertion, none of them puts its veto. Despite these improvements, the problem of imperfect knowledge remains.

2.1.1.1.3. ELECTRE IS (Electra one esse):

According to Figueira et al. (2005), the main innovation of ELECTRE IS is the use of *pseudo-criteria* instead of *true-criteria*. This method takes into account, the possibility to use indifference and preference thresholds for certain criteria belonging to *F* and, correlatively, a backing up (reinforcement) of the veto effect when the importance of the concordance condition decreases. In the exploitation procedure, actions belonging to a cycle are no longer considered as indifferent as in the previous versions of ELECTRE for choice problems.

In the construction procedure, each condition is considered individually:

• *Concordance condition*:

- Condition of criteria in which ***aSb***:

$$J^S = \{ j \in J : g_j(a) + q_j(g_j(a)) \geq g_j(b) \} \quad [27]$$

- Condition of criteria in which ***bQa***:

$$J^Q = \{ j \in J: g_j(a) + q_j(g_j(a)) < g_j(b) \leq g_j(b) + p_j(g_j(b)) \} \quad [28]$$

So, concordance condition will be:

$$c(a, b) = \sum_{j \in J_S} w_j + \sum_{j \in J_Q} \varphi W_j \geq S \ , \ \varphi_j = \frac{g_j(a) + p_j(g_j(a)) - g_j(b)}{p_j(g_j(a)) - q_j(g_j(a))} \quad [29]$$

where,

the coefficient φ_j decreases linearly from *1* to *0*, when g_j describes the range $[g_j(a) + q_j(g_j(a)), g_j(a) + p_j(g_j(a))]$.

• *No veto condition:*

$$g_j(a) + v_j(g_j(a)) \geq g_j(b) + q_j(g_j(b))\ \eta_j,\ \eta_j = \frac{1 - c(aSb) - w_j}{1 - s - w_j} \qquad [30]$$

2.1.1.2. Ranking problematic:

In ranking problematic, the question lays in the way to rank of all the actions belonging to a given set of actions, from the best to the worst. There are four different ELECTRE methods to deal with this problematic: ELECTRE II, ELECTRE III, ELECTRE IV, and ELECTRE-SS.

Wang (2007) defends that there is a visible difference between ELECTRE II and ELECTRE III methods, due to the use of different types of criteria. On one hand, ELECTRE II uses the *true criteria* where no thresholds exist and the differences between criteria scores are used to determine which alternative is preferred (the indifference relation is transitive (Rogers et al., 1999). On the other hand, the criteria used by ELECTRE III are *pseudo criteria* which involve the use of two-tiered thresholds: the indifference threshold *q*, below which the decision maker shows clear indifference; and the preference threshold *p*, above which the decision maker is certain of strict preference (Rogers et al., 1999). The situation between the above two is regarded as weak preference for alternative *a* over alternative *b* which indicates the decision maker's hesitation between indifference and strict preference (Rogers, *et al.*, 1999).

2.1.1.2.1. ELECTRE II (Electre two):

Concerning the ranking problem, ELECTRE II, was the first of ELECTRE methods especially designed to deal with this problems. Besides that, it is also important to point out that ELECTRE II, as stated by Figueira *et al.* (2005), was also the first method, to use a technique based on the construction of an embedded outranking relations sequence (*a strong outranking relation* followed by a *weak outranking relation*).

Figueira *et al.* (2005), Huang and Chen (2005), Wang (2007) and Wang and Triantaphyllow (2008) clarifies that, the construction procedure is much closer to ELECTRE Iv, in the sense that it is also a *true-criteria* procedure.

The ELECTRE methods are based on the evaluation of two indices, the *concordance* index and the *discordance* index, defined for each pair of alternatives. The *concordance index* for a pair of alternatives *a* and *b* measures the strength of the hypothesis that alternative "*a* is at least as good as alternative *b*" - $a\boldsymbol{S}b_h$. The *discordance index* measures the strength of evidence against this hypothesis (Belton & Stewart, 2001). There are no unique measures of concordance and discordance indices. In ELECTRE II, the *concordance index c(a, b)* for each pair of alternatives *(a, b)* is defined as follows:

$$c(a,b) = \frac{\sum j \in Q(a,b) w_j}{\sum_{j=1}^{m} w_j} \qquad [31]$$

where,

Q (a, b) is the set of criteria for which *a* is equal or preferred to *b* (*as good as*), and

$\boldsymbol{w_j}$ is the weight of the *j-th* criteria.

and the *discordance index d (a, b)* for each pair of alternatives *(a, b)* is defined as follows:

$$d(a, b) = (\max_j (g_j(b) - (g_j(a)) / \delta \qquad [32]$$

where,

$\boldsymbol{g_j (a)}$ represents the performance of alternative *a* in terms of criteria c_j,

$\boldsymbol{g_j (b)}$ represents the performance of alternative *b* in terms of criterion c_j, and

$\boldsymbol{\delta} = \max | g_j (b) - g_j (a) |$, this is, the maximum difference on any criteria. This definition can only be used when the scores for different criteria are comparable.

After computing the concordance and discordance indices for each pair of alternatives, two types of outranking relations are built by comparing these indices with two pairs of threshold values: (c^*, d^*) and (c', d'). The pair (c^*, d^*) is defined as the concordance and discordance thresholds for the *strong* outranking relation, and the pair (c', d') is defined as the thresholds for the *weak* outranking relation where $c^* > c'$ and $d^* < d'$. The outranking relations are built according to the following two rules:

(1) If $c(a, b) \geq c^*$, $d(a, b) \leq d^*$ and $c(a, b) \geq c(b, a)$, then alternative *a* is regarded as strongly outranking alternative *b*.

(2) If $c(a, b) \geq c'$, $d(a, b) \leq d'$ and $c(a, b) \geq c(b, a)$, then alternative *a* is regarded as weakly outranking alternative *b*.

The values of (c^*, d^*) and (c', d') are decided by the decision maker for a particular outranking relation: the higher the value of c^* and the lower the value of d^*, the more severe the outranking relation becomes, that is, the more difficult it is for one alternative to outrank another (Belton & Stewart, 2001).

To determine outranking relations, descending and ascending distillation processes are applied to obtain two complete pre-orders of the alternatives, (Belton & Stewart, 2001; Rogers *et al.*, 1999). The *descending pre-order* is built up by starting with the set of “best” alternatives (those which outrank other alternatives) and going downward to the worse one. On the contrary, the *ascending pre-order* is built up by starting with the set of “worst” alternatives (those which are outranked by other alternatives) and going upward to the best one.

The last step is to combine the two complete pre-orders to get either a *partial or a complete final pre-order*. Obtaining a *partial pre-order* (not containing a relative ranking of all the alternatives) or a *complete pre-order*, depends on the level of consistency between the rankings from the two distillation procedures (Rogers *et al.*, 1999). The *partial pre-order* allows two alternatives to remain incomparable without affecting the validity of the overall ranking, which differentiates from the *complete pre-order*. A commonly used method for determining the *final pre-order* is to take the intersection of the *descending and ascending pre-orders*. The intersection of the two pre-orders is defined such that alternative *a* outranks alternative *b* (*a**S**b*) if and only if *a* outranks or is in the same class as *b* according to the two pre-orders. If alternative *a* is preferred to alternative *b* in one pre-order but *b* is preferred to *a* in the other one, then the two alternatives are incomparable in the final pre-order (Rogers *et al.*, 1999).

The main problem with this method, as stated by Huang and Chen (2005) and Wang (2007) is the occurrence of rank reversals[15]. They add that the main reason for rank reversals lies in the exploitation of the pair wise outranking relations, meaning the upward and downward distillation processes. The basic idea behind the distillation processes is to decide the rank of each alternative by the degree of how this alternative outranks all the other alternatives. When a no-optimal alternative in an alternative set is replaced by a worse one, the pair wise outranking relations related to it may be changed accordingly and the overall ranking of the whole alternative set, which depends on those pair wise outranking relations, may also be changed. The first change is reasonable when considering the fact that a no-optimal alternative has been replaced by a worse one. However, the second change is unreasonable and may cause undesirable rank reversals.

2.1.1.2.2. ELECTRE III (Electre three):

As stated by Buchanan *et al.* (1999), Figueira *et al.* (2005), Tervonen *et al.* (2005), and Serdar and Ayner (2009), ELECTRE III (from Roy, 1978), being the mostly used method, is a well-established multi criteria decision maker method that has a history of successful in real-life (Georgopoulou, Lalas & Papagiannakis 1997; Hokkanen & Salminen, 1997; Karagiannidis & Moussiopoulos, 1997; and Rogers *et al.*, 1999).

In ELECTRE III the outranking relation requires the definition of a *credibility index* (which characterizes the credibility of the assertion aSb_h - "*a outranks b*" – being defined by using the *concordance index* and a *discordance index* for each criteria g_j in *F*).

The concordance index $c_j(a, b)$ calculated for each pair of alternatives *(a, b)* in terms of each one of the decision criteria, follows the formula:

[15] Reliability and validity of ELECTRE methods is detailed on Chapter 2.1.2.1..

$$
c_j(a,b) = \begin{cases} 1 & \text{if } g_j(a) + q_j\,(g_j(a)) \geq g_j(b) \\ 0 & \text{if } g_j(a) + p_j\,(g_j(a)) \leq g_j(b) \\ g_j(a) + q_j\,(g_j(a)) < g_j(b) < g_j(a) + p_j\,(g_j(a)), & \text{otherwise} \end{cases} \qquad [33]
$$

where, $q_j\ (.)$ and $p_j\ (.)$ are the indifference and preference threshold values for criteria c_j (Belton & Stewart, 2001).

The next step is to calculate the discordance index $d_j\ (a, b)$ for all the alternatives in terms of each one of the decision criteria according to the following formula:

$$
d_j(a,b) = \begin{cases} 1 & \text{if } g_j\,(b) \geq g_j(a)+v_j(g_j(a)) \\ 0 & \text{if } g_j\,(b) \leq g_j(a)+p_j(g_j(a)) \\ g_j(a)+p_j(g_j(a)) < g_j\,(b) < g_j(a)+v_j(g_j(a)), & \text{otherwise} \end{cases} \qquad [34]
$$

where, $v_j\ (.)$ is the veto threshold for criteria c_j (Belton and Stewart, 2001). If no veto threshold is specified, then $d_j(a, b) = 0$ for all pairs of alternatives.

Finally, the credibility index $\rho(a, b)$ is defined as follows,

$$
\rho(a,b) = \begin{cases} c(a,b), & \text{if } d_j(a,b) \leq c(a,b),\ j = 1, \ldots, n \\ c(a,b) \displaystyle\prod_{j \in J(a,b)} \frac{1-d_j(a,b)}{1-c_j(a,b)}, & \text{otherwise} \end{cases} \qquad [35]
$$

where,

$$\mathbf{c(a,\ b)} = \frac{\sum_{j=1}^{m} w_j c_j(a,b)}{\sum_{j=1}^{m} w_j}$$

J* (*a, b*)** is the set of criteria for which $d_j(a,\ b)>c(a,\ b)$. The credibility index is a measure of the strength of the claim that "alternative *a* is at least as good as alternative *b*" - ***aSb. To notice that, when $d_j(a,b) = 1$, it implies that $\rho(a,\ b) = 0$, since $c(a,\ b) < 1$.

Next, the descending and ascending distillations procedures (Belton & Stewart, 2001; Rogers *et al.*, 1999) must be applied based on the credibility index, in order to construct the two pre-orders for the alternatives. Being defined the two pre-orders, they are combined to get the final overall ranking of the alternatives. The way to combine the two pre-orders follows ELECTRE II procedure.

As already point out to ELECTRE II method, the same criticism can be applied to ELECTRE III method: the occurrence of rank reversals, as stated by Kangas *et al.* (2001), Tervonen *et al.* (2005), and Wang (2007), among others[16].

2.1.1.2.3. ELECTRE IV (Electre four):

Figueira *et al.* (2005) clarifies ELECTRE IV is also a procedure based on the construction of a set of embedded outranking relations. Considering there are five different relations, $S^1, \ldots, S^5$, the S^{r+1} relation (r = 1, 2, 3, 4) accepts an outranking in a less credible circumstances than the relation S^r. ELECTRE IV exploiting procedure follows ELECTRE III.

[16] Reliability and validity of ELECTRE methods is detailed on Chapter *2.1.2.1.*.

2.1.1.2.4. ELECTRE-SS (Electre stochastic):

Hanandeh and El-Zhein (2006) proposed a modified version of ELECTRE III, called ELECTRE-SS, which uses stochastic techniques to account for uncertainty in weightings and threshold values of criteria. This method is particularly useful when a large number of decision makers are involved in the decision-making process.

In ELECTRE III, both thresholds, p and q, are treated as fixed values, and criteria weights are deterministic values. However, this involves not only the error estimation in each criteria, but also subjective input of the decision maker (Rogers & Bruen, 1998a).

In order to overcome this flaw, Hanandeh and El-Zhein (2006) introduced a new stochastic method to allow for multiple decision makers input through accepting criteria weights and thresholds as ranges, rather than deterministic values.

As ELECTRE III method, ELECTRE-SS follows similar procedures: *outranking phase* and *exploitation phase*. The outranking phase builds an outranking index by forming an outranking relation between the pairs of alternatives. The outranking index is then exploited in the second phase to produce a partial pre-order.

Considering mathematical formulation, indifference threshold q'_j and preference threshold p'_j are defined as stochastic variables, instead of being deterministic values, and can vary along the scale of the criteria value. Hence, Hanandeh and El-Zhein (2006) rewrite the preference and indifference relations as follows:

- $aSb^j \Leftrightarrow g_j(a) > g_j(b) + q'_j$

- $aIb^j \Leftrightarrow |g_j(a) - g_j(b)| \leq q'_j$

- $aP'b^j \Leftrightarrow g_j(a) > g_j(b) + p'_j$

- $aQ'b^j \Leftrightarrow q'_j < g_j(a) - g_j(b) \leq p'_j$
- $aI'b^j \Leftrightarrow |g_j(a) - g_j(b)| \leq q'_j$

This way, both values of *w* (criteria importance index – weight) and threshold values (*p* and *q*) are defined to accommodate decision maker evaluations and level of confidence in their evaluations. When the number of decision makers is significantly large, a probability distribution function can be built to represent the entire spectrum of evaluations. However, when the number of decision makers is no large enough to derive a probability distribution function, then the lowest value and the highest value are taken and a normal distribution is considered for evaluating the results between the min-max ranges. Hence,

$$- w'_j = w^{min}_j + (w^{max}_j - w^{min}_j) \times \Delta^{w}_j$$

$$- q'_j = q^{min}_j + (q^{max}_j - q^{min}_j) \times \Delta^{q}_j \qquad [36]$$

$$- p'_j = p^{min}_j + (p^{max}_j - p^{min}_j) \times \Delta^{p}_j$$

where,

Δ is the probability distribution function fit for the importance index of criteria *j*.

$\mathbf{\Delta^{q}_{j}}$ and $\mathbf{\Delta^{p}_{j}}$ are the probability distribution functions of the indifference and preference thresholds for criteria *j* respectively.

The *credibility index ρ(a,b)* for the outranking relation *aSb*, is defined using both a comprehensive *concordance index c(a,b)* and a discordance index $d_j(a, b)$ for each criteria g_j Є *G*. As ELECTRE III, partial concordance index can be defined as,

$$c'_j(a, b) = \begin{cases} 1, & g_j(a) + q'_j \geq g_j(b) \\ 0, & g_j(a) + p'_j \leq g_j(b), \text{ where } j = 1, \ldots, n \\ [p'_j + g_j(a) - g_j(b)] / [(p'_j - q'_j)], & \text{otherwise} \end{cases} \qquad [37]$$

Since Hanandeh and El-Zhein (2006) do not consider a veto threshold, the discordance index is zero for all criteria. Therefore, the credibility index *ρ' (a, b)* in this case is equal to the comprehensive concordance index *c' (a, b)*. So, the comprehensive concordance index is then calculated as follows:

$$c'(a, b) = (1/K') \sum_{j=1}^{n} w'_j \times c'_j(a, b), \text{ where } K' = \sum_{j=1}^{n} w'_j \qquad [38]$$

and to exploit the outranking matrix, two complete pre-orders are constructed:

- Z'_1, a descending distillation: $Z'_1 = \{z'_{1,1}, z'_{1,l}, \ldots, z'_{1,k}\}$
- Z'_2, an ascending distillation: $Z'_2 = \{z'_{2,1}, z'_{2,l}, \ldots, z'_{2,k}\}$

where,
$z'_{1,l}$, $z'_{2,l}$ are the number of times alternative a_i ranked in the k^{th} order in the descending and ascending distillations respectively.

Then, two complete pre-orders Z_1, Z_2 were built such that,

$$Z_1 = z'_{1,1} + \sum_{l=1}^{k} -l \times z'_{1,l} \qquad [39]$$

$$Z_2 = z'_{2,1} + \sum_{l=1}^{k} -l \times z'_{2,l} \qquad [40]$$

Finally a partial order is constructed as follows,

$$Z = Z_1 \wedge Z_2 \qquad [41]$$

To test ELECTRE-SS method, Hanandeh and El-Zhein (2006) study a case already published in Rogers *et al.* (1999). This case is about the definition of an "optimum waste strategy for the region", requested by the Federal Agency for the Environment in Switzerland. Evaluating municipal solid waste management alternatives usually involves a great deal of uncertainty, especially when considering social and environmental criteria. The region was divided into four zones for planning purposes, and eleven strategic options were identified for further assessment against eleven environmental, economic, political and technical criteria (alternative A_{ij} was evaluated in terms of criteria C_k, were i=1, …, 4, j=1, …3 and k=1, … 4). Beyond that, four major criteria categories were considered in the decision making: Environmental criteria (C1), Economic (C2), Technical (C3) and Political (C4). Each criteria is further divided into sub-criteria.

To apply ELECTRE-SS, the values of p and q are not known, but fall into a range defined by the authors, and each weights falls between the lowest and largest assigned values for each criteria used as in the original case. Running ELECTRE-SS method, Hanandeh and El-Zhein (2006) conclude that final ranking of alternatives is sensitive to both threshold and criteria weight values, and the final ranking is more sensitive to criteria weights than threshold values. Besides this, they find that average weights are not necessarily good estimates of criteria weights.

Hanandeh and El-Zhein (2006) reinforced that the new method ELECTRE-SS has the advantage of assessing the performance reliability of the selected alternative, which is not possible when using the deterministic ELECTRE III method. It also allows for close analyze of each alternative's performance, hence decisions may include alternatives that otherwise may be excluded if deterministic parameters where used. Finally, the method provides easy presentation of results in tabular format that gives the decision maker a clear ranking which can be further inspected using the graphical presentation mode.

2.1.1.3. Sorting problematic:

In sorting problematic, each action is considered independently from the others in order to determine the categories to which it seems justified to assign it, by means of comparisons to profiles (bounds, limits), norms or references. Results are expressed using the absolute notion of "assigned" or "not assigned" to a category, "similar" or "not similar" to a reference profile, "adequate" or "not adequate" to some norms. The sorting problematic refers thus to absolute judgments.

2.1.1.3.1. ELECTRE TRI (Electre tree):

ELECTRE TRI, also a very well-successful model in real life, is designed to assign a set of actions, objects or items to categories. In ELECTRE TRI categories are ordered from the worst (C_1) to the best (C_k) (Dias, Mousseau, Figueira & Clímaco, 2002; Damart, Dias & Mousseau, 2007; Xidonas, Askounis & Psarras, 2009; Coelho, Antunes & Martins, 2009; Bregar, Györkös & Juric, 2009; Sobral, 2010).

Each category must be characterized by a lower and an upper profile, where $C = \{C_1, \ldots, C_h, \ldots, C_k\}$ denote the set of categories. The assignment of a given action *a* to a certain category C_h results from the comparison of a to the profiles defining the lower and upper limits of the categories: being b_h the upper limit of category C_h, and the lower limit of category C_{h+1}, for all $h = 1, \ldots, k$. For a given category limit, b_h, this comparison rely on the credibility of the assertions $a\boldsymbol{S}b_h$ and $b_h\boldsymbol{S}a$. This credibility (index) is defined as in ELECTRE III.

After determining the credibility index, a λ - *cut* level must be introduced in order to obtain a crisp outranking relation. This level can be defined as the credibility index smallest value, compatible with the assertion $a\boldsymbol{S}b_h$.

Being P the preference, I the indifference relation and R the incomparability binary relations, action a and profile b_h may be related to each other as follows:

- aIb_h if $a\boldsymbol{S}b_h$ and $b_h\boldsymbol{S}a$

- aPb_h if $a\boldsymbol{S}b_h$ and not $b_h\boldsymbol{S}a$

- b_hPa if not $a\boldsymbol{S}b_h$ and $b_h\boldsymbol{S}a$

- aRb_h if not $a\boldsymbol{S}b_h$ and not $b_h\boldsymbol{S}a$

The objective of the exploitation procedure is to exploit the above binary relations, and propose an assignment, in particular,

1. The *conjunctive logic*, in which an action can be assigned to a category when its evaluation on each criteria is at least as good as the lower limit which has been defined on the criteria to be in this category. The action is hence assigned to the highest category fulfilling this condition.

2. The *disjunctive logic*, in which an action can be assigned to a category, if it has, on at least one criteria, an evaluation at least as good as the lower limit which has been defined on the criteria to be in this category. The action is hence assigned to the highest category fulfilling this condition.

With *disjunctive rule*, the assignment of an action is generally higher than with the *conjunctive rule*. This is why the *conjunctive rule* is usually interpreted as pessimistic while the disjunctive rule is interpreted as optimistic. This interpretation (optimistic-pessimistic) can be permuted according to the semantic attached to the outranking relation.

When no incomparability occurs in the comparison of an action a to the limits of categories, a is assigned to the same category by both the optimistic and the pessimistic procedures. When a is assigned to different categories by the optimistic and pessimistic rules, a is incomparable to all "intermediate" limits within the highest and lowest assignment categories. ELECTRE TRI is a generalization of the two above mentioned rules. The two procedures can be stated as follows,

1. *Pessimistic rule:* An action a will be assigned to the highest category C_h such that $a\boldsymbol{S}b_{h-1}$.

a) Compare a successively with b_h, h = k – 1, k – 2, . . . , 0.

b) The limit b_h is the first encountered profile such that $a\boldsymbol{S}b_h$.

2. *Optimistic rule:* An action a will be assigned to the lowest category C_h such that $b_h\boldsymbol{P}a$.

a) Compare a successively with b_h, h = 1, 2, . . . , k – 1.

b) The limit b_h is the first encountered profile such that $b_h\boldsymbol{P}a$.

2.1.2. Some test criteria for evaluating the multi criteria decision making methods

In Triantaphyllou (2000), Wang and Triantaphyllou (2004, 2008), and Wang (2007) studies, three test criteria were established to evaluate the performance of multi criteria decision making methods by testing the validity of their ranking results. These test criteria are as follows:

a) *Test Criterion #1:* **"**An effective multi criteria decision making method should not change the indication of the best alternative when a no-optimal alternative is replaced by another worse alternative (given the relative importance of each decision, criteria remains unchanged)."

Suppose that a multi criteria decision making methods has ranked a set of alternatives in some way. Next, suppose that a no-optimal alternative, say *Ak*, is replaced by another alternative, say *Ak'*, which is less desirable than *Ak.* Then, according to test criterion #1, the indication of the best alternative should not change when the alternatives are ranked again by the same method. The same should also be true for the relative rankings of the rest of the unchanged alternatives.

b) *Test Criterion #2:* "The rankings of alternatives by an effective multi criteria decision maker method should follow the transitivity property."

Suppose that a multi criteria decision maker method has ranked a set of alternatives of a decision problem in some way. Next, suppose that this problem is decomposed into a set of smaller problems, each defined on two alternatives at a time and the same number of criteria as in the original problem. Then, according to this test criteria all the rankings which are derived from the smaller problems should satisfy the transitivity property. That is, if alternative *A1* is better than alternative *A2*, and alternative *A2* is better than alternative *A3*, then one should also expect that alternative *A1* is better than alternative *A3*.

c) *Test Criterion #3:* "For the same decision problem and when using the same multi criteria decision maker method, after combining the rankings of the smaller problems that an multi criteria decision maker problem is decomposed into, the new overall ranking of the alternatives should be identical to the original overall ranking of the undecomposed problem."

As before, suppose that a multi criteria decision maker problem is decomposed into a set of smaller problems, each defined on two alternatives and the original decision criteria. Next suppose that the rankings of the smaller problems follow the transitivity property. Then, according to this test criterion when the rankings of the smaller problems are all combined together, the new overall ranking of the alternatives should be identical to the original overall ranking before the problem decomposition.

In Triantaphyllou (2000), Wang and Triantaphyllou (2004, 2008), and Wang (2007) research, these three test criteria were used to evaluate the performance of the ELECTRE II and the ELECTRE III methods. Both of them failed in terms of each one of these three test criteria.

Kangas *et al.* (2001), Tervonen *et al.* (2005), Huang and Chen (2005), Wang and Triantaphyllou (2004, 2008) and Wang (2007), demonstrate that rank reversal may occur with ELECTRE methodology, in particular, ELECTRE II and ELECTRE III.

2.1.2.1. Reliability and validity of ELECTRE methods

In Kangas *et al.* (2001) study, ELECTRE III and PROMETHEE II methods were tested. Like others authors, they identified rank reversals in both methods. In any decision making process there is uncertainty concerning not only the values of the criteria variables but also concerning, for example, the weights of the criteria. So, a sensitivity analysis with respect to the uncertain parameters used in the calculations is thus essential, as well as the application of several alternative methods to the same problem. Then, the decision maker can make the final choice among these alternative solutions (Salminen, Hokkanen, & Lahdelma, 1998).

While enhancing outranking methods advantages[17], Kangas *et al.* (2001) study confirms that if the priority of one alternative depends on other alternatives, this means that adding a new (no-optimal) alternative, a change in ranks of the initial alternatives may occur.

Tervonen *et al.* (2005) also reported problems when applying ELECTRE III: concerning preference information, if the decision makers cannot provide precise and complete weight information, or if there are multiple decision makers with conflicting preferences, ELECTRE methods cannot be used for decision process. To comprove this assertation, Tervonen *et al.* (2005) re-analyze the case study presented in Rogers *et al.* (1999): to choose the best waste incineration strategy for the Eastern Switzerland region, considering eleven alternative strategies S_i that were evaluated in terms of eleven criteria C_j.

[17] Ability to deal with uncertain, ability to deal with ordinal and other informal preference statements, and the preference estimation procedures are versatile and diverse.

To perform this analysis, Tervonen *et al.* (2005) introduce an inverse weight-space analysis into the ELECTRE III method to explore the weight space, in order to describe which weights (weight intervals) result in certain ranks for the actions, meaning that no deterministic weights are required. This will allow ELECTRE III to be used with weight information of arbitrary type. This inverse approach on ELECTRE III was motivated by:

1. this type of weight information can be provided by the existing weight elicitation techniques;
2. it allows a particular kind of easily comprehensible "robustness analysis" also in the case when the weights are deterministic, and
3. if there are multiple decision makers whose preferences need to be taken into account, the weight intervals can be determined to contain the preferences of all decision makers.

So, Tervonen *et al.* (2005) analyzed robustness with respect to the weights, but considered all the others parameters fixed (thresholds, cutting levels, among others). Tervonen *et al.* (2005) adds that, usually tests to comprove robustness of multi criteria decision method are based on sensitive analysis by changing only a discrete set of weights for a criteria, or by considering only the extremes of the feasible weight space (Dias *et al.*, 2002). In their work, they consider an inverse approach on ELECTRE III, performing an inverse weight-space analysis, to all possible weight vectors in the feasible weight space.

Executing Monte Carlo simulations, which provides sufficient accuracy for the results (according to Lahdelma, & Salminen, 2004, and Tervonen *et al.*, 2005) by changing only a single weight at a time concluded that, on one hand, in the original case study, alternatives S3.1 and S4.1 shared the best rank (based on analyzing six different sets of weights) leading to recommend S4.1 as the primary choice, and S3.1 as the secondary choice. But based on inverse analysis, alternative S4.1 seemed not to be the most adequate to "recommend" as the most favorable option, given its rank acceptability index. With 99% of those weights, it shared the first rank with alternative S3.1. On the other hand, S3.1 obtained lower rank than S4.1 with only 1% of the feasible weights, and was always ranked higher than the other alternatives (excluding S4.1).

In short, Tervonen *et al.* (2005) over this example would not “recommend” S4.1, because even small variations in the weights drop it below S3.1 in the ranking. In this sense, they would have select S3.1, and S4.1 as a ”back-up” strategy, if for some reason S3.1 could not have been chosen. Tervonen *et al.* (2005) adds that this same analysis could be done using PROMETHEE method (see Figueira *et al.*, 2005).

Huang and Chen (2005) evaluated ELECTRE II model performance, in their study about land redevelopment in Heping Harbor Zone**,** in Taiwan. The authors suggested that when evaluating ELECTRE method,

- the definition and calculation of benchmarks of concordance index and discordance index are important elements;
- the method can be applied in parallel with other evaluation methods owing to get only partial ranking (advantages and disadvantages of combining various evaluation methods, as well as their differences, should be weighted);
- the definition of weighs to be considered in the model is determined beforehand. Seeing this have a great impact upon the final ranking, particular attention shall be placed on weighting methodology when using ELECTRE evaluation method.

Wang and Triantaphyllou (2004, 2008) and Wang (2007) developed a very interesting paper about rank reversals, when using ELECTRE II and ELECTRE III to rank a set of decision alternatives.

So, to check on rank reversals using ELECTRE II, their test were based on a real-life case study, were the aim’s study was to help find the best location for a wastewater treatment plant in Ireland (Rogers *et al.*, 1999).

To check on rank reversals on ELECTRE III, their test were also based on a real-life case study, were the aim’s study were to choose the best waste incineration strategy for the eastern Switzerland region (Rogers, *et al.,* 1999).

Concerning the *first problem*, to help find the best location for a wastewater treatment plant in Ireland, Wang (2007), defined the decision problem using five alternatives and seven criteria (set as a benefit criteria: the higher the score the better the performance is) - alternative A_i was evaluated in terms of criteria C_j.

So, applying ELECTRE II methodology, Wang (2007) construct the following pre-orders. From the descending distillation, ***A2= A5> A3> A1> A4***; from the ascending distillation, ***A2>A5= A3> A1> A4***. From the intersection of the descending and ascending pre-orders, the following complete pre-order of the alternatives were obtained: ***A2>A5> A3> A1> A4*** and obviously ***A2*** is the optimal alternative at this point.

Based on this results, Wang (2007) randomly selected ***A3*** to be replaced by a worse one, ***A3'***, in order to test ranking alternatives under the first test criteria.

After applying ELECTRE II methodology, the descending and ascending distillation processes allowed to reach the following results: the descending pre-order now is ***A2= A5> A3= A1> A4***, while the ascending pre-order is ***A2=A5>A3=A1>A4***. After combining the two pre-orders together, a new complete pre-order is as follows: ***A2=A5>A3= A1> A4***. Now the best ranked alternatives are ***A2*** and ***A5*** together, a contradiction from the previous result which had ***A2*** as the only optimal alternative.

Considering the *second problem*, to choose the best waste incineration strategy for the eastern Switzerland region, Wang (2007), defined the decision problem using eleven alternatives and eleven criteria (*benefit criteria* means that the higher the score of a given criteria is, the more preferable it is; the *cost criteria* means that the lower the score of a given criteria is, the more preferable it is) -alternative A_i was evaluated in terms of criteria C_j.

So, applying ELECTRE III methodology, the pre-order obtained from the descending distillation was ***A9> A4> A7> A10> A3= A5= A8= A11> A1> A2> A6***. The pre-order obtained from the ascending distillation was ***A1= A7> A9> A4> A10> A2= A5> A3= A11> A8> A6.*** Then the two pre-orders were combined to get the final overall ranking of the alternatives, just like in ELECTRE II. Wang (2007) concluded that ***A7*** and ***A9*** are both regarded as the best-ranked alternatives because both of them are ranked first in the final partial pre-order. As a result, the rest of the alternatives were regarded as no-optimal ones.

As done to the first example, Wang (2007), selected, randomly, alternative ***A1*** and replaced it by a worse one´, ***A1'***, to test the reliability of the alternatives ranking. Applying

ELECTRE III method, Wang (2007) get the descending pre-order as ***A7> A9> A4> A10> A3 = A5 = A8= A11> A1> A2> A6***, and the ascending pre-order as ***A7> A1=A9> A4> A10> A2= A5> A3= A11> A8> A6***.

With these new results, the author concluded that the best-ranked alternative is only ***A7*** which is different from the original conclusion, which had ***A7*** and ***A9*** as the best-ranked alternatives.

Wang (2007), after analyzing both test results, explain that the reason for the contradictory results lies in the exploitation of the pair wise outranking relations, that is, the upward and downward distillation processes of ELECTRE II and ELECTRE III.

The basic idea behind the distillation processes is to decide the rank of each alternative by the degree of how this alternative outranks all the other alternatives. When a no-optimal alternative in an alternative set is replaced by a worse one, the pair wise outranking relations related to it may be changed accordingly, and the overall ranking of the whole alternative set, which depends on those pair wise outranking relations, may also be changed. The first change is reasonable when considering the fact that a no-optimal alternative has been replaced by a worse one. However, the second change is unreasonable and may cause undesirable rank reversals as confirmed by the above examples.

In short, this rank reversal happens because there is not a priori ranking of the alternatives when they are ranked by the ELECTRE II or III methods; the ranking of an individual alternative derived by these methods depends on the performance of all the other alternatives currently under consideration. This causes the ranking of the alternatives to depend on each other. Thus, it is likely that the optimal alternative may be different and the ranking of the alternatives may be distorted to some extent if one of the no-optimal alternative in the alternative set is replaced by a worse one.

In Wang's (2007) study, another example is analyzed, considering three alternatives ***A1***, ***A2***, and ***A3***. So, supposing originally that ***A1*** strongly outranks ***A3***, ***A2*** weakly outranks ***A3,*** and ***A1*** and ***A2*** are indifferent with each other, the ranking of these three alternatives will be ***A1 > A2 > A3*** when using the ELECTRE II method.

Then, let's consider the no-optimal alternative ***A3*** is replaced by a worse one. As a result of ranking process, ***A2*** may strongly outrank ***A3*** while ***A1*** is still strongly outranking

A3, and ***A1*** is still indifferent with ***A2***. Now the ranking of the three alternatives will be ***A1= A2> A3*** by using the same method since both ***A1*** and ***A2*** now strongly outranks ***A3,*** and they are indifferent with each other. It can be seen that ***A1*** and ***A2*** are ranked equally now because ***A3*** becomes less desirable.

The situation above described is exactly what happened in the *first example*: ***A2*** and ***A5*** are ranked equally after ***A3*** has been replaced by a less desirable alternative. This kind of irregular situation is undesirable for a practical decision-making problem though it is reasonable in terms of the logic of the ELECTRE II method. It could leave the ranking of a set of alternatives to be manipulated to some extent.

Wang (2007) adds that, if the number of alternatives of a decision problem is more than 3, then the situation may become worse by totally changing the indication of the best ranked alternative. As pointed out by Belton and Stewart (2001), the results of the distillations are dependent on the whole alternative set, so that the addition or removal of an alternative may change some of the preferences between the remaining alternatives. That is, even without the addition or removal of alternatives, the best ranked alternative might be another one, and the previous pre-order between the remaining alternatives might be changed to some degree by just replacing a no-optimal alternative by a worse one.

Besides this distillation problem, Wang (2007) add that there is another factor that may contribute to rank reversals. During the construction of the pair wise outranking relations, both ELECTRE II and III need to use a value or a threshold which is also dependent on the performance values of all the currently considered alternatives. For ELECTRE II, it is the parameter d (the maximum difference of any criteria) in the discordance index formula. For ELECTRE III, it is the parameter used to decide the l (*preference* relations between the alternatives during the distillations). Both d and l values may be change when a no-optimal alternative is replaced by a worse one. Then the previous outranking relations between the other unchanged alternatives may be distorted to some degree, which finally may modify the indication of the best ranked alternative or the overall ranking of the alternatives.

In this sense, and based on the tests carried out, Wang (2007) concluded that the above two factors may function together or separately to cause rank reversals. So, this author inferred that ELECTRE II and III are not reliable and robust enough to offer a firm answer to a decision problem. In this sense, decision maker should undertake some kind of sensitivity and careful when analyzing the final rankings. Because of all this, the use of other methods should be considered, for example, AHP (from Saaty, 1980) or PROMETHEE family (from Brans & Vincke, 1985).

2.1.3. Why choose ELECTRE methodology, and some successful alternatives

As already mentioned previously, a decision-making is the process of selecting one or a few alternatives that should be the most favorable one(s) to objective(s). In order to reach an optimum decision, well-defined criteria and robust solution techniques are required. Selection should also be based on extreme conditions rather than average conditions. However, each decision problem has its own particular conditions and factors, and decision makers are different from each other. Even the same decision maker may have completely opposite decisions for the same problem in different times. In addition, there are always some intangible areas to be considered, while comparing different alternatives. These are difficult to quantify, but exhibit considerable influences on the final decision, such as experience and personal relationships (assumed as qualitative information). A multicriteria decision making method, as its own name suggests, is for use in situations when more than one criteria must be considered, working all of them with the same bases: the decision matrix. So, in the decision matrix, the a_{ij} express the performance of alternative *i* according to a criteria *j*, and it's precisely the way a method works with a_{ij} that makes the difference.

Literature emphasizes some multi criteria decision models, due to their success in real-life cases, in particular, ELECTRE, AHP, PROMETHEE, MINORA and ADELAIS. Obviously these models are representations of reality, and therefore, have theoretical failings. Because of this, it is important not to focus only on one method, but to analyze the decision problem considering other methods, taking into account the specific characteristics of each one. Thus, critical mind is necessary in order to analyze the different results obtained.

In our work, which assets choose is one of the decisions regarding Portfolio Management, just like the expected return and risk exposure. In the most of the cases, in real life, for a decision maker assets are selected based on technical analysis. But, as previously exposed, we transformed portfolio management into a multicriteria problem, by defining criteria (based on financial theory).

The object of study of present work was only made with one method from ELECTRE family, in particular ELECTRE III, even we performed some investigation in order to justify the choice against another method. In order to give some knowledge about other methods, we also made an overview about the already mentioned most popular multicriteria decision making models as AHP, Promethee, Adelais and Minora. All examples given in chapter 2 (Buchanan *et al.*, 1999; Zak, 2005; Kangas *et al.*, 2001; Huang & Chen, 2005; Serdar & Aynur, 2009; Afshari et al, 2010) concluded that any method is inferior, having each one its own merits and drawbacks.

One of the first reasons that lead us to choose ELECTRE III, within all versions existent in ELECTRE family, was the possibility for taking into account indifference and preference threshold, the necessity of a quantification of the relative importance of criteria, and to be specific for ranking problems.

Another reason is that all the other methods, although being suitable for our decision problem (portfolio management) are more indicated when we want to focus in a group of decision makers, were subjectivity is presented due to qualitative variables and discussion between them. In our study we try to eliminate (at maximum) all subjectivity existent in portfolio management, by well defining criteria and threshold. Therefore, ELECTRE III is a method that allows us to eliminate subjectivity given his features already enumerated.

PROMETHEE family, AHP method, ADELAIS and MINORA methods are suggested as being successful alternatives to ELECTRE methodology, being their main features presented in the next sub-chapters.

2.1.3.1. The Analytic Hierarchy Process (AHP):

The AHP methodology is also a widely used multi criteria decision making method introduced by Satty (1980), and has been applied in numerous fields in real life, for instance, in Forman and Gass (2001), Vargas (1990) and Zahedi (1986) studies. AHP method[18] structures a decision problem into a hierarchy and evaluates multi criteria tangible and intangible factors systematically, involving four steps to solve a decision problem (Zahedi, 1986; Lin & Yang, 1996; Tam & Tummala, 2001). Afshari *et al.* (2010) summarize those four steps as follows:

- **Step 1:** *Structuring the decision problem*: Structure the hierarchy from the top (goal), through the intermediate levels (sub-sequent levels depend on criteria), to the lowest level, which usually contains the list of alternatives.
- **Step 2:** *Creating pair wise comparison matrix*: Weights are assigned to each criteria and sub criteria. These weights are assigned through a process of pair-wise comparison. The pair-wise comparisons are done in terms of which element dominates the other.
- **Step 3:** *Determining normalized weights*: Using each pair-wise comparison, matrix W is computed.
- **Step 4:** *Synthesize the priorities*: Synthesize the solution for the decision problem in order to obtain the set of priorities for alternatives. After computing the weight of alternatives in respect to sub-criteria, and then sub-criteria in respect to criteria, and also criteria in respect to goal from step 3 (in the level immediately above), they are aggregated to produce composite weights which used to evaluate decision alternatives.

After this general overview on AHP method, let's see the mathematical formulation, based on Wang (2007) study.

As already mentioned, the AHP method uses the pair-wise comparisons to determine the a_{ij} values and also the criteria weights w_j (Saaty, 1980, 1994; Saaty & Vargas, 2000). In this method, a_{ij} represents the relative value of alternative A_i when it is considered in terms of

[18] AHP method can help people setting priorities and choose the best alternatives, by reducing complex decision problems to a system of hierarchies.

criteria C_j. Assuming all the criteria are *benefit* criteria (the higher the score, the better the performance is), the best alternative is the one that satisfies the following expression:

$$P^*_{AHP} = \max_i P_i = \max_i \sum_{j=1}^{n} a_{ij}w_j, \text{ for } i = 1, 2, 3, \ldots, m. \quad [42]$$

Despite the AHP model is widely used in real life, many researchers have also criticized it for some problems. As cited in Wang (2007), Belton and Gear (1983) analyzed AHP method through a simple test: AHP method was used to rank three alternatives in a simple test problem; then a fourth alternative, identical to one of the three alternatives, was introduced in the original decision problem without changing any other data. The ranking of the original three alternatives was changed after the revised problem was ranked again, and also in AHP method rank reversal phenomenon occurred.

When Belton and Gear (1983) applied AHP method, final scores of the three alternatives were: (0.45, 0.47, 0.08), that is, *A2 > A1> A3*. Next, they add a new alternative *A4*, identical to the existing alternative *A2*, *into* the decision matrix. Now, the final AHP scores of these alternatives are: (0.37, 0.29, 0.06, 0.29), that is, the four alternatives are ranked as *A1 > A2 =A4 > A3*. Belton and Gear (1983) stated that the new result obtained contradicts the previous one in which *A2 > A1*. Their rank reversal example demonstrated that the ranking of alternatives might be changed by the addition (or deletion) of no-optimal alternatives. Also Dyer and Wendell, in 1985, studied rank reversals when the AHP was used and identical alternatives were considered in the decision problem.

In this sense, Belton and Gear (1983) proposed a revised AHP method, which was initially criticized by Saaty in 1990, but then accepted by him. So, the normalized or revised AHP method comes as follows,

$$P^*_{revised\text{ - }AHP} = \max_i P_i = \max_i \sum_{j=1}^{n} a_{ij}w_j, \text{ for } i = 1, 2, 3, \ldots, m \quad [43]$$

Subsequent studies, as Triantaphyllou and Mann (1989) concluded that even the revised AHP method suffer of some other ranking problems even without the introduction of identical alternatives. They found that the most of the problematic situations of the AHP methods are caused by the required normalization and the use of an additive formula on the data of the decision matrix for deriving the final preference values of the alternatives.

However, Bridgeman (1922), Miller and Starr (1969), and Lootsma (1991) shaded a new light on this model: the use of an additive formula is avoided by using a multiplicative expression, known as the Weighted Product Model (WPM).

In the WPM method, each alternative is compared with others in terms of a number of ratios, one for each criteria. Each ratio is raised to the power of the relative weight of the corresponding criteria. Generally, the following formula is used in order to compare two alternatives A_K and A_L:

$$R\left(\frac{A_k}{A_l}\right) = \prod_{j=1}^{n} \left(\frac{ak_j}{aL_j}\right)^{w_j} \qquad [44]$$

Wang (2007) add that, according to Triantaphyllou's (2000) study, most of the ranking irregularities which occurred when the additive variants of the AHP method were used do not occur with the multiplicative AHP method, although the existence of rank reversals still happens. In particular,

- Dyer (1990a) indicated that the sum of priorities to unity normalization makes each one dependent on the set of alternatives being compared. He also claimed that the results to individual priorities are thus arbitrary, as arbitrary sets of alternatives may be considered in the decision problem.
- Stam and Silva (1997) revealed that if the relative preference statements about alternatives were represented by judgment's intervals (this is, the pair wise preference judgments are uncertain - stochastic), rather than single values, then the rankings resulting from the traditional AHP analysis based on the single judgment values may be reversed and therefore are incorrect. Based on this statement, they developed some multivariate statistical techniques to obtain both point estimates and confidence intervals for the occurrence of certain types of rank reversal probabilities with the AHP method.
- Triantaphyllou (2000, 2001) reported another type of rank reversal with the additive AHP methods in which the indication of the optimal alternative may change when one of the no-optimal alternatives is replaced by a worse one. He also reported two new cases of ranking irregularities with the additive AHP methods: one is that the ranking

of the alternatives may be different when all the alternatives are compared two at a time and also simultaneously; another case is that the ranking of the alternatives may not follow the transitivity property when the alternatives are compared two at a time.

2.1.3.2. The PROMETHEE Method (Preference Ranking Organization Method and Enrichment Evaluations):

Brans and Mareschal (2005) presented an overview on PROMETHEE methodology. Therefore, PROMETHEE family, in particular, PROMETHEE I (partial ranking) and PROMETHEE II (complete ranking) were developed by Brans in 1982. Later, Brans and Mareschal extended PROMETHEE family by developing PROMETHEE III (ranking based on intervals) and PROMETHEE IV (continuous case). In 1988, the same authors created an interactive module called GAIA, which is a graphical representation of methodology. In 1992 and 1994, Brans and Mareschal extended, once again, PROMETHEE methodology, by developing PROMETHEE V (multi criteria decision maker method including segmentation constraints) and PROMETHEE VI (representation of the human brain). In our work we only present the most prominent versions of this method as PROMETHEE I, II, V and GAIA.

In real life, PROMETHEE methods have been applied in various fields as banking, industrial location, manpower planning, water resources, investments, medicine, chemistry, health care, tourism, dynamic management, among others. Brans and Mareschal (2005) add that his success is basically due to its mathematical properties and to its particular friendliness of use.

As in any other multi criteria decision model, decision maker expectation is to identify an alternative that optimize all the criteria. So, Brans and Mareschal (2005) starts to consider the following multi criteria problem, in order to construct an evaluation table:

$$\max \left\{ g_1(a), g_2(a), ..., g_j(a) | a \in A \right\}, \qquad [45]$$

where,

A, is a finite set of possible alternatives, and *a*, a set of evaluation criteria: some are maximized and the others minimized. For instance, in a purchasing car, price should be minimized but some car features should be maximized, as reputation, comfort, speed, reliability and, consumption.

As Brans and Mareschal (2005) pointed out, the solution of a multi criteria problem do not depend only on the basic data included in the evaluation table, but also on the decision maker himself (his preferences), requiring *additional information*. So, for each *(a, b)* Є *A*,

$$\begin{cases} \forall_j : g_j(a) \geq g_j(b) \\ \exists k : g_k(a) > g_k(b) \end{cases} \Leftrightarrow a\boldsymbol{P}b,$$

$$\forall_j : g_j(a) = g_j(b) \qquad \Leftrightarrow a\boldsymbol{I}b, \qquad [46]$$

$$\begin{cases} \exists_s : g_s(a) > g_s(b) \\ \exists r : g_r(a) < g_r(b) \end{cases} \Leftrightarrow a\boldsymbol{R}b,$$

where,

P**, **I**,* and ***R respectively stand for *preference, indifference* and *incomparability.*

Brans and Mareschal (2005) clarifies this relations interpretations, despite they have already been presented in ELECTRE methodology: an alternative is better than another if it is *at least as good as* the other on all criteria; if an alternative is better on a criteria *s* and the other one better on criteria *r*, it is impossible to decide which is the best one without additional information. Both alternatives are therefore *incomparable*! Alternatives which are not dominated by any other are called *efficient solutions.* In this sense, when alternatives are *incomparable*, additional information is necessary. The authors consider that this additional information can include, for example,

- Information between the criteria;
- Information within each criteria.

Considering *information between the criteria*, Brans and Mareschal (2005), refer that a table containing weights of relative importance must be done, considering that the set $\{w_j$, j= 1, 2, ..., k$\}$ represents weights of relative importance of the different criteria. These weights are nonnegative numbers, independent from the measurement units of the criteria. The higher the weight, the more important the criteria is. To notice that there is no objection to consider normal weights, so that $\sum_{j=1}^{k} w_j = 1$.

Concerning i*nformation within the criteria*, Brans and Mareschal (2005) clarifies that PROMETHEE is not allocating an intrinsic absolute utility to each alternative, neither globally, or on each criteria: the preference structure of PROMETHEE is based on *pair wise comparisons.* Therefore, in case of deviation between the evaluations of two alternatives, on a particular criteria, is considered,

- for small deviations, the decision maker will allocate a small preference to the best alternative;
- for negligible deviation, possibly decision maker have no preference;
- the larger the deviation, the larger decision maker's preferences.

Brans and Mareschal (2005) adds that there is no objection to consider that these preferences are real numbers varying between 0 and 1, so that,

$$P_j(a, b) = F_j[d_j(a, b)],\ \forall\ a, b \in A, \qquad [47]$$

where, $d_j(a, b) = g_j(a) - gj(b)$ and $0 \leq P_j(a, b) \leq 1$ [48]

In case of a criteria to be maximized, this function gives the preference of *a* over *b* for observed deviations between their evaluations on criteria $g_j(.)$.The function is given by:

$$P_j(a, b) > 0 \Rightarrow P_j(b, a) = 0 \qquad [49]$$

In case of a criteria to be minimized, the preference function should be reversed or alternatively given by:

$$P_j(a, b) = F_j[- d_j(a, b)] \quad [50]$$

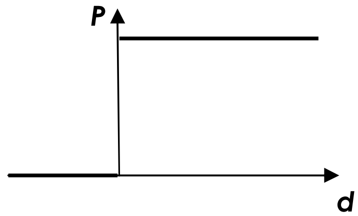
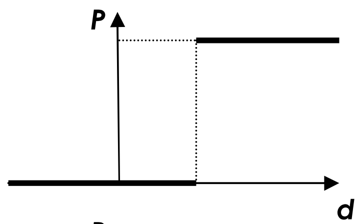
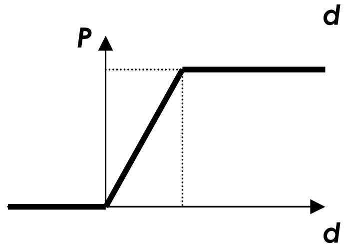
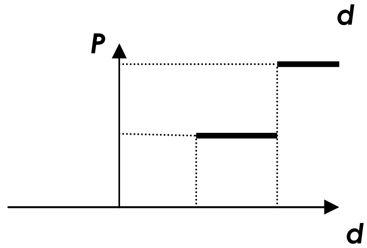
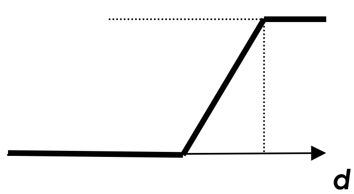
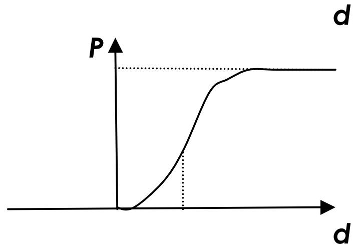
Brans and Mareschal (2005) nicknamed the pair $\{g_j(.), P_j(a, b)\}$ as *generalized criteria* associated to criteria $g_j(.)$. Such a generalized criteria has to be defined for each criteria. In order to facilitate the identification, these authors defined six types of particular preference functions, as shown in table 4.

As stated by Brans and Mareschal (2005), the q indifference threshold is the largest deviation which is considered as negligible by the decision maker, while the p preference threshold is the smallest deviation which is considered as sufficient to generate a full preference; s is an intermediate value between q and p. The identification of a generalized criteria is then limited to the selection of the appropriate parameters.

Analyzing table nº 4, Brans and Mareschal (2005) clarifies that in the case of *type 5*, a threshold of indifference q and a threshold of strict preference p have to be selected; in the case of *type 6*, the preference function remains increasing for all deviations and has no discontinuities. A parameter s has to be selected seeing he defines the inflection point of the preference function. In this particular case, authors recommend to determine first a q and a p and to fix s in between: if s is close to q the preferences will be reinforced for small deviations, while close to p they will be softened.

From the moment that the evaluation table pair $\{g_j(.)\}$, the weights w_j, and the generalized criteria pair $\{g_j(.), P_j(a, b)\}$ for i = 1, 2, ..., n, j = 1, 2, ..., k, are defined, Brans and Mareschal (2005) consider the PROMETHEE procedure can be applied.

Table nº 4 – Types of generalized criteria (*P(d)*: Preference function)

TYPE	GENERALISED CRITERION	DEFINITION	PARAMETERS TO FIX
Type 1: **Usual Criterion**		$P(d) = \begin{cases} 0 & d \leq 0 \\ 1 & d > 0 \end{cases}$	--
Type 2: **U-shape Criterion**		$P(d) = \begin{cases} 0 & d \leq q \\ 1 & d > q \end{cases}$	***q***
Type 3: **V-shape Criterion**		$P(d) = \begin{cases} 0 & d \leq 0 \\ d / p & 0 \leq d \end{cases}$	***p***
Type 4: **Level Criterion**		$P(d) = \begin{cases} 0 & d \leq q \\ 1/2 & q \leq d \leq \end{cases}$	***p, q***
Type 5: **V-shape with indiference Criterion**		$P(d) = \begin{cases} 0 & d \leq q \end{cases}$	***p, q***
Type 6: **Gaussian Criterion**		$P(d) = \begin{cases} 0 & d \leq 0 \end{cases}$	***s***

Source: Brans and Mareschal, 2005.

2.1.3.2.1. The PROMETHEE I and II Rankings

The PROMETHEE procedure is based on pair wise comparisons. So it is necessary to firstly define the aggregated preference indices and outranking flows.

Concerning aggregated preference indices, Brans and Mareschal (2005) start to define that $(a, b) \in A$ and,

$$\begin{cases} \pi(a, b) = \sum_{j=1}^{k} P_j(a, b) w_j, \\ \\ \pi(b, a) = \sum_{j=1}^{k} P_j(b, a) w_j. \end{cases} \quad [51]$$

where,

π (a, b) express with which degree *a* is preferred to *b* over all the criteria,

π (b, a) express with which degree *b* is preferred to *a* over all the criteria.

To notice that, in most of the cases, there are criteria that satisfy both ***π (a, b)*** and ***π (b, a)***, so they are usually positive, for all $(a, b) \in A$. In this sense,

$$\begin{cases} \pi(a, a) = 0, \\ 0 \leq \pi(a, b) \leq 1, \\ 0 \leq \pi(b, a) \leq 1, \\ 0 \leq \pi(a, b) + \pi(b, a) \leq 1. \end{cases} \quad [52]$$

and,

$$\begin{cases} \pi(a, b) \approx 0 \text{ implies a weak global preference of } a \text{ over } b, \\ \\ \pi(a, b) \approx 1 \text{ implies a strong global preference of } a \text{ over } b. \end{cases} \quad [53]$$

Concerning outranking flows, each alternative *a* is facing *(n-1)* other alternatives in *A*. Brans and Mareschal (2005) define the two following outranking flows:

- *the positive outranking flow* (express how an alternative *a* is *outranking* all the others). The higher $\Phi^{+}(a)$ the better the alternative:

$$\Phi^{+}(a) = \frac{1}{n-1} \sum_{x \in A} \pi\,(a, x), \qquad [54]$$

- *the negative outranking flow* (expresses how an alternative *a* is *outranked* by all the others). The lower $\Phi^{-}(a)$ the better the alternative:

$$\Phi^{-}(a) = \frac{1}{n-1} \sum_{x \in A} \pi\,(x, a). \qquad [55]$$

The PROMETHEE I Partial Ranking

Brans and Mareschal (2005) adds that PROMETHEE I partial ranking (P^{I}, I^{I}, R^{I}) is obtained from the positive and the negative outranking flows. Seeing both flows do not usually induce the same rankings, PROMETHEE I is their intersection.

$$\begin{cases} a\boldsymbol{P^{I}}b \quad \text{iff} \quad \begin{cases} \Phi^{+}(a) > \Phi^{+}(b) \text{ and } \Phi^{-}(a) < \Phi^{-}(b), \text{ or} \\ \Phi^{+}(a) = \Phi^{+}(b) \text{ and } \Phi^{-}(a) < \Phi^{-}(b), \text{ or} \\ \Phi^{+}(a) > \Phi^{+}(b) \text{ and } \Phi^{-}(a) = \Phi^{-}(b); \end{cases} \\ a\boldsymbol{I^{I}}b \quad \text{iff} \quad \Phi^{+}(a) = \Phi^{+}(b) \text{ and } \Phi^{-}(a) = \Phi^{-}(b); \\ a\boldsymbol{R^{I}}b \quad \text{iff} \quad \begin{cases} \Phi^{+}(a) > \Phi^{+}(b) \text{ and } \Phi^{-}(a) > \Phi^{-}(b), \text{ or} \\ \Phi^{+}(a) < \Phi^{+}(b) \text{ and } \Phi^{-}(a) < \Phi^{-}(b); \end{cases} \end{cases} \qquad [56]$$

where,

$\boldsymbol{P^I}$, $\boldsymbol{I^I}$, $\boldsymbol{R^I}$ respectively stands for preference, indifference and incomparability.

Brans and Mareschal (2005) conclude that, when $\boldsymbol{aP^Ib}$ (a higher power of *a* is associated to a lower weakness of *a* with regard to *b*), the information of both outranking flows is consistent and may therefore be considered as sure. When $\boldsymbol{aI^Ib}$, both positive and negative flows are equal. Finally, when $\boldsymbol{aR^Ib}$ (*a* is good on a set of criteria on which *b* is weak, and reversely *b* is good on some other criteria on which *a* is weak), the information provided by both flows is not consistent; it seems then reasonable to be careful and to consider both alternatives as incomparable, so decision maker decides which action is best in such cases.

<u>The PROMETHEE II Complete Ranking</u>

As stated by Brans and Mareschal (2005), PROMETHEE II consists of the (P^{II}, I^{II}) complete ranking, usually requested by decision maker. The *net outranking flow* can then be considered as follows (the higher the net flow the better the alternative):

$$\Phi(a) = \Phi^+(a) - \Phi^-(a) \qquad [57]$$

$$\begin{cases} aP^{II}b & \text{iff} \quad \Phi(a) > \Phi(b); \\ aI^{II}b & \text{iff} \quad \Phi(a) = \Phi(b). \end{cases} \qquad [58]$$

The authors complement the idea expressed above, saying that when PROMETHEE II is considered, all the alternatives are comparable. No incomparability remains, but the resulting information can be more disputable because more information gets lost by considering the difference. In concrete,

$$\begin{cases} -1 \leq \Phi(a) \leq 1, \\ \sum_{x \in A} \Phi(a) = 0. \end{cases} \qquad [59]$$

where,

$\Phi(a) > 0$, *a* is more outranking all the alternatives on all the criteria,

$\Phi(a) < 0$ *a* is more outranked.

The PROMETHEE I and II ranking final appreciation

Brans and Mareschal (2005) conjugated the positive and the negative outranking flows and the aggregated indices, being the result expressed below,

$$\Phi(a) = \Phi^{+}(a) - \Phi^{-}(a) = \frac{1}{n-1} \sum_{j=1}^{k} \sum_{x \in A} [P_j(a, x) - P_j(x, a)]w_j. \qquad [60]$$

Consequently,

$$\Phi(a) = \sum_{j=1}^{k} \Phi_j(a) w_j \qquad [61]$$

if,

$$\Phi_j(a) = \frac{1}{n-1} \sum_{x \in A} [P_j(a, x) - P_j(x, a)]. \qquad [62]$$

where,

$\Phi_j(a)$ is the single criteria net flow obtained when only criteria $g_j(.)$ is considered. It expresses how an alternative *a* is outranking ($\Phi_j(a) > 0$) or outranked ($\Phi_j(a) < 0$) by all the other alternatives on criteria $g_j(.)$.

According to Brans and Mareschal (2005), the global net flow of an alternative is the scalar product between the vector of weights and the profile vector of this alternative. In order to construct this scalar, knowledge's on GAIA visual interactive module are necessary.

2.1.3.2.2. PROMETHEE V: Multi criteria Decision Maker under Constraints

Brans and Mareschal (2005) clarifies that PROMETHEE I and II are appropriate to select one alternative. However, in some applications a subset of alternatives must be identified, given a set of constraints; PROMETHEE V extends PROMETHEE methods to that particular case. These authors considered a set of possible alternatives, and associate them Boolean variables $\{a_i, i = 1, 2, ..., n\}$. So,

$$w_i = \begin{cases} 1 & \text{if } a_i \text{ is selected,} \\ 0 & \text{if not.} \end{cases} \qquad [63]$$

3.1.3.2.3. The PROMETHEE Group Decision Support System (GDSS) Procedure

In order to provide decision aid to a group of decision makers, PROMETHEE GDSS was developed. This iteration process consists in 11 steps grouped in three phases, summarized by Brans and Mareschal (2005) as follows.

- PHASE I: Generation of alternatives and criteria;
- PHASE II: Individual evaluation by each *decision maker*;
- PHASE III: Global evaluation by the group.

PHASE I: Generation of Alternatives and Criteria

- **STEP 1: First contact Facilitator — decision makers**

  The analyst meets the decision makers, together or individually, in order to learn more about the problem. Usually this step takes place in the business based of each decision maker, prior to the GDSS room session.

- **STEP 2: Problem description in the GDSS room**

  The analyst describes the computer infrastructure, the PROMETHEE methodology, and introduces the problem.

- **STEP 3: Generation of alternatives**

  Each decision maker introduces possible alternatives in the computer, including their extended description. For instance, strategies, investments, locations, production schemes, marketing actions, among others depending on the problem.

- **STEP 4: Stable a set of alternatives**

  All the proposed alternatives are collected and displayed by the analyst, one by one on the video-screen, anonymously or not. An open discussion takes place, alternatives are cancelled, new ones are proposed, and combined ones are merged, until a stable set of n alternatives ($a_1, a_2, ..., a_i, ..., a_n$) is reached. This brainstorming procedure is extremely useful, because it often generates new points of view.

- **STEP 5: Comments on the alternatives**

  Once again, each decision maker introduces his comments on all the alternatives, in the computer. All these comments are collected and displayed by the analyst.

- **STEP 6: Stable set of evaluation criteria**

  The same procedure as for the alternatives is applied to define a stable set of evaluation criteria ($g_1(.), g_2(.), ..., g_j(.),...g_k(.)$). Discussion is opened, and at the end, the frame of an evaluation table is obtained. This frame consists in a matrix (n x k).

PHASE II: Individual Evaluation by each decision maker

This second phase assumes that each decision maker has a decision power given by a non-negative weight, defined as follows $\sum_{r=1}^{R} w_r = 1$.

- **STEP 7: Individual evaluation tables**

  The evaluation table (n x k) has to be completed by each decision maker. Some evaluation values are introduced in advance by the analyst, if there is an objective agreement on them (for example, prices, volumes, budgets). If not, each decision maker is allowed to introduce his own values.

  All the decision makers work on the same matrix (n x k); if some of them are not interested in particular criteria, they can simply allocate a zero weight to these criteria.

- **STEP 8: Additional PROMETHEE information**

  Each decision maker develops his own PROMETHEE analysis, under the supervision of analyst, this is, he provides PROMETHEE additional information on the weights and the generalized criteria.

- **STEP 9: Individual PROMETHEE-GAIA analysis**

  The PROMETHEE I and II rankings, the profiles of the alternatives and the GAIA plane (PROMETHEE graphical representation) as well as the net flow vector $\Phi_r(.)$ are instantaneously obtained, so that each decision maker gets his own clear view of the problem.

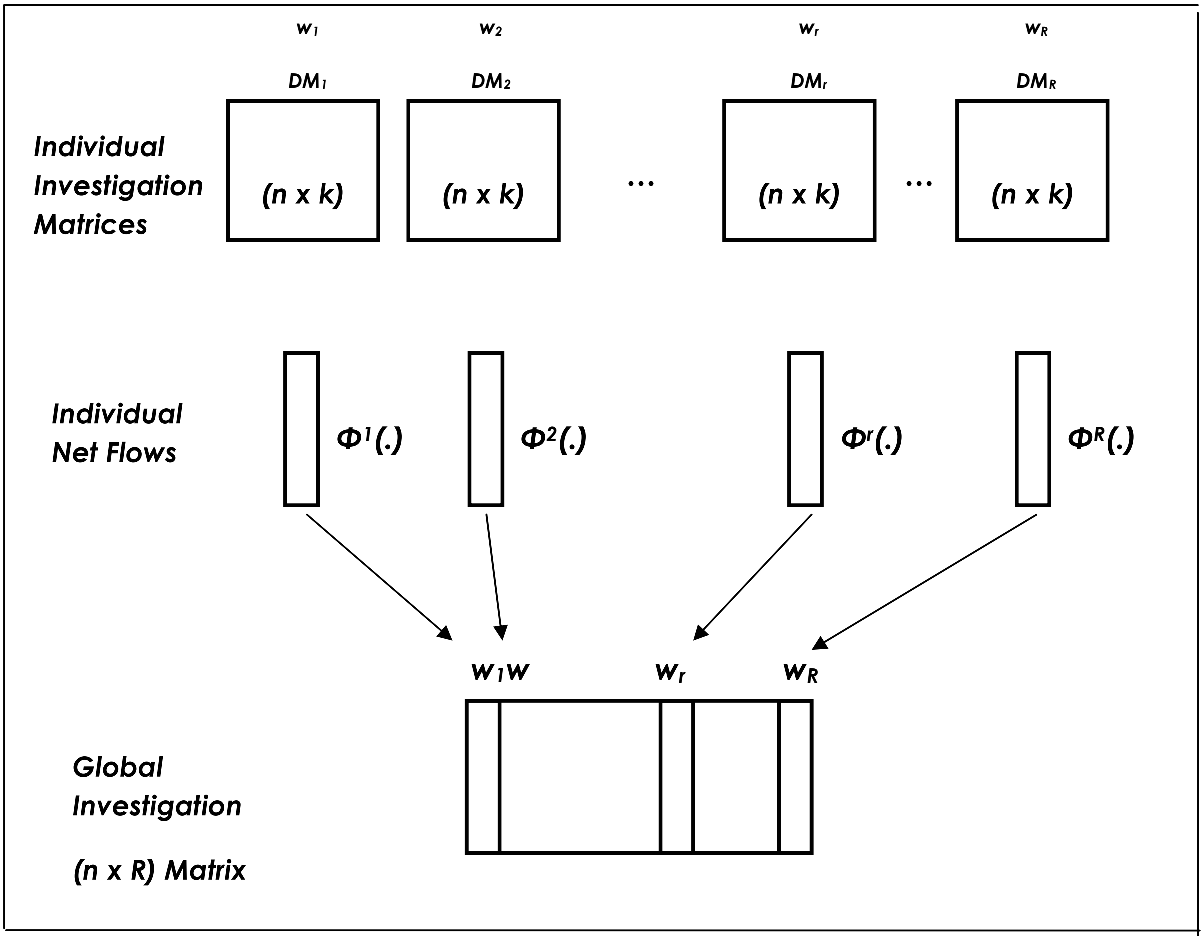
PHASE III: Global Evaluation by the Group

- **STEP 10: Display of the individual investigations**

  The rankings and the GAIA plane of each decision maker are collected and displayed by the analyst, so that the group of all decision makers' is informed of the potential conflicts.

- **STEP 11: Global evaluation**

Chart nº 8 - Overview on PROMETHEE GDSS procedure



Source: Brans and Mareschal, 2005.

The net flow vectors $\{\Phi_r(.), r = 1, 2, ..., R\}$ of all the decision makers are collected by the analyst and put into a (n x R) matrix. Each criteria of the matrix expresses the point of view of a particular decision maker. Each of these criteria has a weight w_r and an associated generalized criteria of Type 3 (p = 2), so that the preferences allocated to the deviations between the $\Phi^r_i(.)$ values will be proportional to these deviations.

Finally, a global PROMETHEE II ranking and the associated GAIA plane are then computed, allowing decision maker to see the conflicts between them. So, if the conflicts are too sensitive the following feedbacks could be considered: back to the weighting of the decision makers; back to the individual evaluations; back to the set of criteria; back to the set of alternatives; back to the starting phase and to include an additional decision maker such as a social negotiator or a government mediator.

2.1.3.3. A Multiobjective Mathematical Programming – ADELAIS

According to Zopounidis, Despotis and Kamaratoy (1993)[19] "ADELAIS is an interactive computer program developed to support the search for a satisfactory solution in multiobjective linear programming (MOLP) problems". It operates in the general form:

> max { $f_1(x)$, …, $f_n(x)$}, subject to x Є A_m where x = (x_1, …, x_m) is the vector of the decision variables, f_1 (x), …, f_n(x) are linear functions of the decision variables an A is the set of the feasible solutions (A C R^m).

The system provides extensive data management capabilities and the concerned solution process provides a 'two level' interaction: interactive assessment of the decision maker's utility function and interactive modification of the satisfaction levels.

Siskos and Despotis (1989) describe the two operational phases: a two stages preliminary phase performed once and a three stage iterative phase.

<u>1. PRELIMINARY PHASE</u>

Stage 1: Each objective function is maximized and then minimized on the feasible set *A* to obtain upper and lower bounds respectively. Particularly, if all or some of the objective functions fail to achieve a finite minimum, the lower bounds are computed with a heuristic procedure (Siskos & Despotis, 1989).

Stage 2: A starting efficient solution, a solution that is not dominated by any other feasible solution, is estimated in a way that the resulting objective values are as close as possible to the upper bounds in the minimax sense.

---

[19] Proposed by Zopounidis, C. Despotis, D. K and Kamaratoy, I. (1993). Portfolio Selection Using The Adelais Multiobjective Linear Programming System. *Computational Economics 11***:** 189–204, 1998.

2. ITERATIVE PHASE

Stage 3: At each interaction, the decision maker is provided with a new efficient solution which, except the initial one, comes from Stage 5 (see below). The system provides the attained objective values relative to the new solution, the achievement percentages with respect to the upper bounds and the satisfaction levels (the revised lower bounds) established in previous iterations. The decision maker is then asked to indicate the objectives he/she insists on increasing and if he/she intends to decrease some of the others in compensation. The decision maker's responses, combined with relative responses of previous iterations, form the basis for the establishment of new satisfaction levels. The new satisfaction levels limit the feasible set but the system provides the decision maker with the possibility to relax them if desired by analyzing the local tradeoffs among the objectives. This possibility helps in removing the consequences of previous responses that eventually contradict the decision maker's current desires by reexamining solutions that had been rejected in previous iterations. The iterative process is terminated by the decision maker within this stage when a satisfactory solution is achieved.

Stage 4: This stage constitutes a learning process of the decision maker's preferences. At first, a simple technique is set up to construct a reference set of decision alternatives (a set of vectors that might be assumed by the objective functions). Then the decision maker is asked to rank order these alternatives according to his preferences. The decision maker's subjective ranking is taken as input and a concave additive utility model is assessed by a modified version of the UTA ordinal regression algorithm (Jacquet-Lagreze & Siskos, 1982; Despotis & Yannacopoulos, 1990; Despotis, Yannacopoulos & Zopounidis, 1990). The UTA method fits ordinal utility functions of the additive form that best approximate the data (preference ranking over the set of reference alternatives) by using a linear programming formulation and post optimality analysis. The partial utilities are assumed nonlinear and are assessed in a piecewise linear form in a manner that the estimated ranking over the set of reference alternatives is as close as possible to the subjective ranking provided by the decision maker. In case of inconsistencies the decision maker is invited to interact with the model in order to improve the consistency. The utility assessment process is terminated by the system when full consistency is achieved or by the decision maker himself when acceptable consistency is achieved.

Stage 5: The assessed utility function is maximized over the set *A* of the feasible solutions as it is reduced by the satisfaction levels. A new efficient solution is obtained and the process is repeated from Stage 3. For the maximization of the utility function a piece wise linear programming technique is employed (Fourer, 1985). The flowchart of the ADELAIS system is presented in Figure 1.

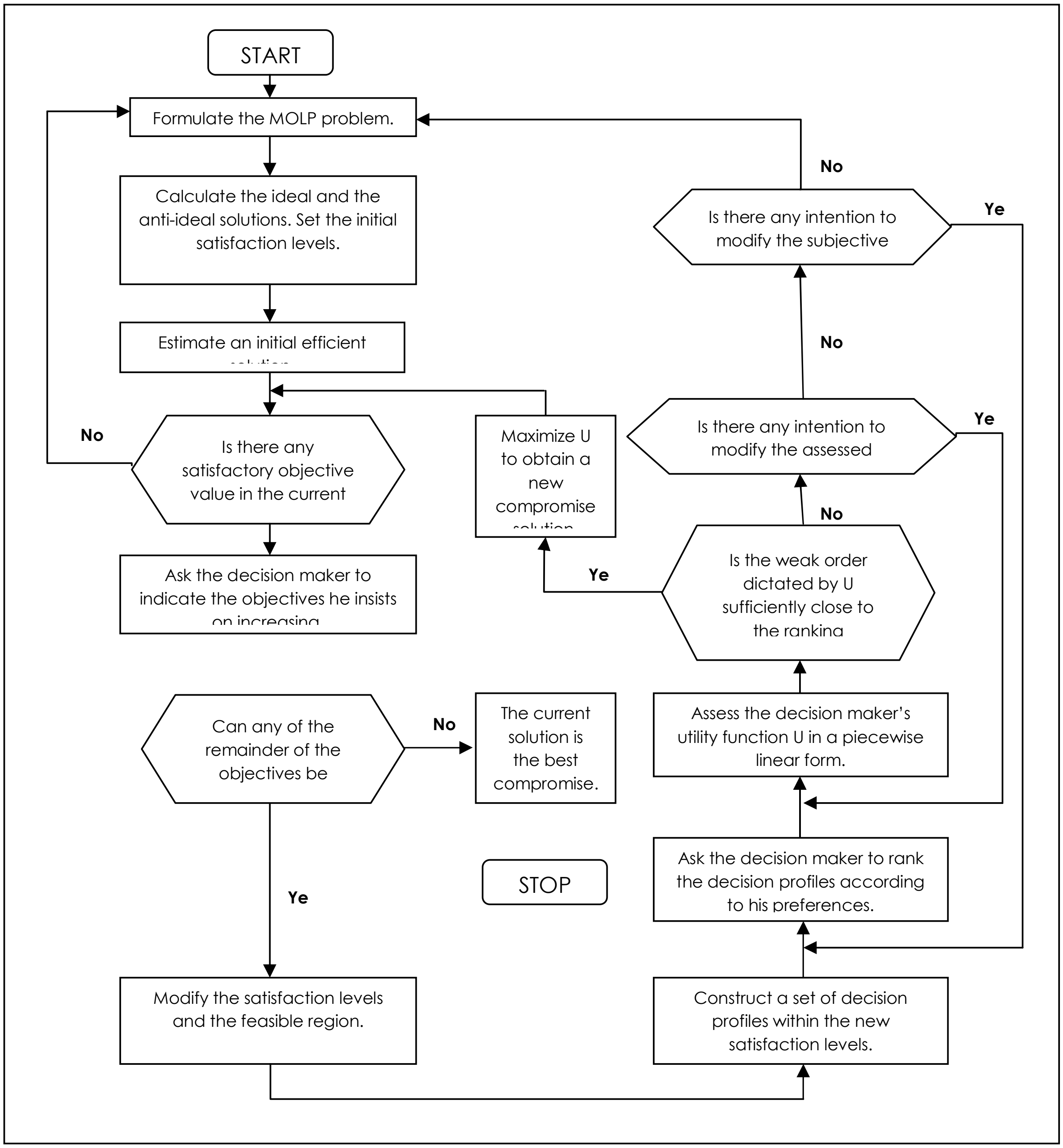


Figure nº 1: The flowchart of the ADELAIS system

Source: Zopounidis, Despotis and Kamaratoy, 1998.

2.1.3.4. The ordinal regression methodology - MINORA

Zopounidis (1992) proposed an Interactive Multicriteria model of Ordinal Regression Analysis (MINORA) to conduct the evaluation, selection and portfolio management. This model was developed in order to help decision-making agents, and their evaluation criteria, when selecting assets to obtain the maximization of their utility.

Once chosen the criteria for shares selection, this author proposed that for each share one should calculated the ratios values that the analyst wants to use. Later, the analyst, without the help of these values, must order the shares according to their preferences using his experience. Then, the values of utility are used with the ratios obtained to determine the utility function, dependent on various criteria (expressed as ratios). The utility function then varies between 0 and 1.

Zopounidis (1992) gave an empirical example of the Athens Stock Exchange, selecting 10 companies from various sectors listed in the Greek market between 1984 and 1988.

Selected in total six criteria for analysis, and asked portfolio managers to order those shares. With this information, he estimated the additive utility function more consistent with the subjective ranking of the portfolio manager.

Later Zopounidis, Godefroid and Hurson (1994) proposed that, after the selection of shares by the system MINORA, the portfolio manager could use the system ADELAIS already explained, in order to determine the optimal proportion to invest in each share.

These models have the advantage of including in the process the preferences of portfolio managers, ultimately the "users" of potential optimization methods. But, it should be noted that one should not account, in forming the optimal portfolio's correlation, with the relationships between various shares and selected sectors, besides the fact that an efficient diversification is not guarantee.

Thus, these processes must be used with some careful, because it may be produce results that could not be considered "second best". It is, however, a methodology that developed at other levels can incorporate the correlation, and thus be used as a mechanism for selecting optimal portfolios shares.

CHAPTER 3: FINANCIAL THEORY APPLIED TO PORTFOLIO MANAGMENT

When looking to capital market, equity stock is the most popular financial asset in such markets. That's why academic researchers and investors, in general, have employed various techniques to analyze the dynamics of stock prices. In particular, different methods have been developed to set certain selection criteria in choosing individual stocks with promising returns, mostly supported in three methodologies: the fundamental analysis, technical analysis, and institutional investor analysis. These methods are used to explore key factors that yield significant impacts on stock prices and the effectiveness of the factors are often individually and independently analyzed.

In Levišauskait's (2010) work, stock analysis for investment decision making is supported in an *E-I-C* analysis. This *E-I-C* analysis includes,

- ***E from Economic*** (macroeconomic) analysis: describes the macroeconomic situation in the particular country and its potential influence on the profitability of stocks. Mostly give answer to the following questions:

  i. The behavior of economics in the context of economic cycle (at what point of this cycle is the economy now: growth stage? peak? decline stage? recession stage?);
  ii. Fiscal policy of the government (financial stability, budget deficit, public debt)
  iii. Monetary policy (the stability of national currency against other foreign currencies; the ability of authorities (Central Bank) to use the money market instruments on time);
  iv. Other economic factors such as inflation/deflation; the level of unemployment; the level of consumption; investments into businesses; the possibilities to use different types of energy, their prices; foreign trade and the exchange rate of the foreign currency against national currency (devaluation? revaluation?)

- ***I from Industry*** analysis: evaluates the situation in the particular industry/ economic sector and its potential influence on the profitability of stocks. The contents of this analysis could be disclosed answeringto the following questions:

  • What is the nature of the industry? Is it monopolistic or competitive?

  • What is the level of regulation and administration inside this industry?

  • What is the situation with the self-organization of the human resources in this industry? Is there any unions to other organized structures?

  • How important and how complex is the technology for this industry?

  • What are the key factors which influence this industry?

  • What conditions in production and financial activity are important in this industry? (production resources, the perspectives for raising capital, competition from the other countries)

  • What is the stage of the industry's development cycle? (introductory? growth? maturity? decline?)

- ***C from Company*** analysis: based on fundamental analysis, is based on financial analysis of the individual companies from the shareholder approach, being the most prominent.

Within the three analysis presented, we highlight "*C from Company* analysis ", based on fundamental analysis, and the basis of our work. Therefore, the next subchapters detail the importance of financial ratios, including cash flow analysis.

3.1. The importance of financial analysis based on financial ratios

When thinking about financial theory, in particular financial ratios, is inevitable to think in what way financial data can add knowledge to our understanding of why some firms cease growing, discontinue, fail, or go into bankruptcy, the worst nightmares of investors.

Unfortunately, history is full of companies that have filed for bankruptcy, supported in all kind of motives, local and global recessions, financial crises, among many others. But, how can we protect investors from this type of loss?

Research on default prediction has been conducted for many decades and a very large number of empirical studies have been published since the pioneering work of Beaver (1966, 1968) and Altman (1968). Beaver (1966) presented empirical evidence that certain financial ratios (the most notably was cash flow to total debt) gave statistically significant signals before actual business failure. Altman (1968) extended Beaver's (1966) analysis by developing a discriminate function which combines ratios in a multivariate analysis. Altman found that his five ratios outperformed Beaver's (1966) cash flow to total debt ratio. So, the initial approach to predicted corporate failure was based on discriminate analysis, used to discriminate between failed and no failed firms (Deakin, 1972; Altman, Haldeman & Narayanan, 1977).

After that, emphasis shifted toward to probit or logit analysis. Martin (1977) and Ohlson (1980) were among the first to apply these techniques, followed by other authors, for instance, Wilcox (1973), Wiginton (1980), Zmijewski (1984), Zavgren and Friedman (1988), Aziz and Lawson (1989), Lennox (1999), Westgaard and Van der Wijst (2001), and Boritz, Kennedy and Sun (2007). As referred by Hol, Westgaard and Wijst (2002), other statistical techniques have also been introduced, such as recursive partitioning (Frydman, Altman & Kao, 1985), catastrophe theory, multidimensional scaling (Molinero & Ezzamel, 1991), neural networks (Tam & Kiang, 1992), multinominal logit models (Johnsen & Melicher, 1994), multicriteria decision aid methodology (Zopounidis & Doumpos, 1999) and rough sets (Dimitras, Slowinski, Susmaga & Zopounidis, 1999)[20].

[20] Reviews studies can be found in Jones (1987), Karels and Prakash (1987), Dimitras et al. (1999), Hol, Westgaard and Wijst (2002), Campbell, Hilscher and Szilagyi (2008), and Philosophov, L., Batten, and Philosophov, V. (2008).

Muller, Steyn-Bruwer and Hamman (2009) tested the effectiveness of four different techniques used to predict financial distress. They found that multiple discriminate analysis and recursive partitioning have the highest prediction accuracy for predicting "failed" companies. Logit analysis and neural networks were found to have the highest overall predictive accuracy.

Although, each study by itself provides a reasonable discrimination between failed and no failed firms, truly that various studies hardly show any agreement on what factors are important for failure prediction. This situation can, partly, be attributed to the fact that the studies refer to different periods, countries and industries.

Another perspective of company's financial performance was explored by Romacho and Cidrais (2007). This authors studied to what point the announcement of the accounting results would influence the behavior of investors in capital markets. They concluded, on one hand, that the most liquid assets have an increased profitability mainly in the pre-announcement of results. On the other hand, in the less liquid assets, an increase in profitability variability was verified, before and after the announcement of results (although investors have anticipated the results, they continued to feel the need to adjust their behavior after the disclosure of the same).

In short, Romacho and Cidrais (2007), concluded that investors attribute value to the publication of results, being reflected in the assets price and variability of equity returns, but also in the number of assets transacted. So, this announcement of companies' results will affect the liquidity of these assets. This idea is supported by Beaver's study (1966), where he reveled that the disclosure of accounting results have impact in assets price; if investors assign importance to that information, that announcement leave to changes on the relative positions of assets in investor's portfolio. But the impact of the results announcement on the price of assets can occur before the announcement. In this case, Ball and Brown (1968) state that, when companies publish their results, analysts have already issued their financial estimates, so an adjustment on assets price is necessary.

To define which financial ratios, among the so many found in the literature, are useful to evaluate the financial performance and financial condition of a company, the studies from Beaver (1966), Altman (1968, 2000), Yap et al. (2010), and Chen and Shimerda (1981) studies highlighted from the others. These authors' search indicates which ratios best predicts business failures, and concluded that there is no need for many ratios. For instance, Taffler's study, 1983, started with eighty potentially useful ratios, and ended up with just four. Thereafter, in our study, five ratios (table 3) were chosen among the many that had been used in previous studies with financial theory. They assess profitability, leverage and liquidity.

In our work, the choice of ratios used was based on two main criteria: in their popularity, as evidenced by their frequent of use in the finance and accounting literature, and in their good perform as showed in bankruptcy studies.

3.2. The particular case of Cash Flows

Different approach was conducted by Urbancic (2002). As already defended by other researchers, this author confirms the idea that the use of financial ratios is a very useful tool in detecting "red flags for a fraud examination" (Albrecht, 2003; Wells, 2005).

But in his study, he defends that most computed ratios usually focus only on balance sheets and income statements, forgetting that statement of cash flows can also offer useful insights from ratio analysis. He defends that cash flow ratios can add important information to the traditional income statement and balance sheet (DeFranco & Schmidgall, 1998; Geller, Ilevento, & Schmidgall, 1990; Giacomino & Mielke, 1993; Harris & Brown, 1998). On one hand, balance sheet ratios can only provide information for a static period, not providing information about investing and financing activities. On the other hand, cash-flows gives information about company's activity for a continuous period, complementing the balance sheet and income statement by providing additional information concerning an organization's ability to operate efficiently, to finance growth, and to pay its obligations.

As referred by Kisang and Shawn (2004), in almost every business cash is essential for a success, and crucial for business survival, reflecting the difference between successful operations and bankruptcy (Beck, 2004; De Franco & Schmidgall, 1998)[21]. These authors also highlight that cash flow ratios also play an important role in performance analysis in a three-dimensional perspective:

[21] Previous studies have discussed cash flow as being crucial for much business in a variety of industries: Bohannon & Edwards, 1993; Casey & Bartczak, 1985; DeFranco & Schmidgall, 1998; Epstein & Pava, 1994; Mills & Yamamura, 1998; Schmidgall, Geller, & Ilvento, 1993; Sylvestre & Urbancic, 1994.

- **Liquidity:** Cash flow information is more reliable than balance sheet or income statement information, most off all, because balance sheet data is static while the income statement contains many arbitrary noncash items as depreciation and amortization. In contrast, the cash flow statement focuses on what shareholders really care about: cash available for operations and investments (Coltman & Jagels, 2001; Mills & Yamamura, 1998). This ratio measures a company's ability to generate resources to meet current liabilities. The higher the ratio is the greater the firm's liquidity (Coltman & Jagels, 2001): current assets should exceed current liabilities on a ratio of two to one.

- **Solvency.** Solvency ratios are used to evaluate a company's ability to pay its obligations in the long term. One of the most commonly used solvency ratios, total assets to total liabilities, is calculated at a specific period of time in the balance sheet statement, while the cash flow from operations to average total liabilities ratio covers a period of time. Moreover, the total assets to total liabilities ratio ignores the varying liquidity of assets for covering various levels of debt. The cash flow from operations to total liabilities ratio gets over that deficiency by focusing directly on cash flow (Coltman & Jagels, 2001; Schmidgall, Geller, & Ilvento, 1993; Mills & Yamamura, 1998). The lower this ratio is the lower the financial flexibility and the higher the potential for default. In other words, the higher this ratio is, the better is the operations ability to pay its obligations with cash.

- **Operations.** The primary concern of management is operational activities (Jagels & Coltman, 2004; Schmidgall, Geller, & Ilvento, 1993). Two useful cash flow ratios, which focus on these activities, are cash flow margin and cash flow from operations to net income. The first, the cash flow margin, is similar to the profit margin whose purpose is to give information about the profits generated per sales. Since the customers spend cash, and not profits, the cash flow margin should be a more useful ratio than profit margin (Coltman & Jagels, 2001; Schmidgall, Geller, & Ilvento, 1993). The higher this ratio is, the better, since this ratio evaluates the company's ability to transform sales into cash. The second, the ratio of cash flow from operations to net income, is indirectly related to operations performance (Schmidgall, Geller, &

Ilvento, 1993). Net income alone cannot appropriately explain how well companies are operated due to subjective judgments in accruals, expense allocation, and valuation. The higher the ratio is the better.

Giving a closer look to statement of cash flows, he is classified into three categories: operating activities, investing activities, and financing activities. Shortly,

- *Operating activities* are presented as a reconciliation of accrual-based net income to net cash flows from operations;
- *Investing activities* are associated with capital asset and investment transactions;
- *Financing activities*, including funds obtained, as well as, cash used to pay the principal amounts of debts to creditors and dividends to stockholders.

From his research, Urbancic (2002) highlights only nine cash-flow ratios. Although recognizing that information about cash flows from investing and financing activity are important, in his opinion the centre piece of a company and its principal motive for existence is its operating activity. So, the mentioned eight cash-flow ratios are all based on operational activity. In particular,

a) Current Liability Coverage (CLC_n)

This ratio is a good indicator of a company's actual ability to meet current liabilities, being better than current ratio and quick ratio. Besides providing an indication of a company's ability to pay for debts and obligations coming due within one year, includes any *currently maturing portion of long term debt*. If a company is not generating enough cash from operations to meet its obligations, then other more costly sources for cash will be required that may increase the risk of default or bankruptcy. This ratio can be represented as,

$$CLC_n = \frac{\text{Cash flow from operations - } \textit{Cash Dividends Paid}}{\textit{Current Liabilities}} \qquad [64]$$

b) Long Term Debt Coverage ($LTDC_n$)

In this ratio the long term debt (as mortgage notes payable, bonds payable and other significant obligations) measures the solvency of a company by indicating the time it would take to pay back debts assuming that no new long term debts were incurred and that operating cash is used exclusively to repay debt. A low ratio expresses a much riskier environment because management might attempt to raise capital or other sources of financing in an effort to avoid bankruptcy. Mathematically, this ratio is expressed as,

$$LTDC_n = \frac{\text{Cash flow from operations} - \textit{Cash Dividends Paid}}{\textit{Long Term Debt}} \qquad [65]$$

c) Interest Coverage (IC_n)

Considering Interest is a tax deductible expense, it was added to cash flow from operations the amounts paid for interest and income taxes. This ratio provides a more realistic indicator of liquidity and an organization's ability to service its debt. A very low ratio signifies an increased risk that a company might not have enough cash available to meet its obligation to pay interest on its debts. This ratio is presented as,

$$IC_n = \frac{\text{Cash flow from operations} + \textit{Cash Payments for Interest and Income Taxes}}{\textit{Cash Payments for Interest}} \qquad [66]$$

d) Earnings Quality (EQ_n)

This ratio adjusts both cash flow from operations and net income with the effects of interest and income taxes providing a more realistic indication of the extent of deviation between operating cash flows and reported earnings (depreciation, amortization, losses and gains, are a typical cause for normal deviation of cash flow operations from earnings). This ratio can be represented by the formula,

$$EQ_n = \frac{\text{Cash flow from operations} + \textit{Cash Payments for Interest and Income Taxes}}{\textit{Net Income + Interest Expense + Income Tax Expense}} \qquad [67]$$

e) Asset Efficiency Ratio (AER_n)

This ratio provides an indication of how well the assets of a company are utilized to generate a cash flow return. This measure, tracked over a period of time, can provide useful insights especially when the results are compared to other companies in the same industry. Mathematically can be expressed as,

$$AER_n = \frac{\text{Cash flow from operations}}{\textit{Total Assets}} \qquad [68]$$

f) Capital Asset Ratio (CAR_n)

The capital asset ratio shows a company's ability to meet their needs for capital investment by operating activities rather than from financing activities. A ratio of 1.0 or greater means that debt financing is not necessary for capital investment. The capacity to replace or update capital assets determines whether or not a company can successfully compete with others in the same industry, behind a sign of stagnant or decreasing level of capital spending. Therefore, it is important to monitor how much a company expends on technological advances and new equipment. Mathematically,

$$\mathbf{CAR_n} = \frac{\text{Cash flow from operations} + \textit{Cash Inflows from Capital Asset Disposals} - \textit{Cash Paid for Dividends}}{\textit{Cash Outflows for Capital Asset Acquisitions}} \quad [69]$$

g) Cash Generating Power (CGP_n)

The central assumption of this ratio is based on *inflows* of cash that comes from investing and financing sections. The ratio demonstrates a company's ability to generate cash and the proportion of the cash generated by operational section compared to the total cash inflow. Year to year comparisons of the cash generating power for a company should be evaluated, as well as, comparisons with industry competitors. This ratio can be presented as,

$$\mathbf{CGP_n} = \frac{\text{Cash flow from operations}}{\text{Cash flow from operations} + \textit{Investing Cash Inflows} + \textit{Financing Cash Inflows}} \quad [70]$$

h) Operating Cash Margin (OCM_n)

Despite being similar to a traditional profit margin ratio, the operating cash margin ratio provides a more robust indicator of performance based on cash generating ability. Essentially, this ratio highlights the timing of cash flows with respect to the timing of sales. However, since cash flow margins are likely to exhibit substantial variations among companies in different industries, it is more effective to focus a comparative analysis on companies within the same industry. Expressed by,

$$\mathbf{OCM_n} = \frac{\text{Cash flow from operations}}{\textit{Sales}} \quad [71]$$

PART 2 – EMPIRICAL WORK

CHAPTER 4 – FINANCIAL RATIOS APPLIED TO PORTFOLIO MANAGEMENT: THE CASE OF PORTUGUESE STOCK INDEX

4.1. Empirical work objectives, and research questions

The scope of this work is to give to portfolio management a new perspective in an effort to support decision maker in his investment decision. So, the first scenario is to transform portfolio management into a multicriteria problem, and using ELECTRE III method, we explore the application of financial theory (financial ratios equally weighed) to select which assets (equally weighed) choose to form a certain portfolio, and test if outperforms the market (PSI-20TR). The second scenario is to also form a certain portfolio, but this time using CAPM method to select the shares (equally weighed), and also test if outperforms the market. Finally, to take some conclusions about witch method is the best to determine witch assets should be selected to form a portfolio, ELECTRE III or CAPM. To both scenarios, procedures were conducted initially made for a certain period (historical), and then monitorized for a three years holding period or follow-up period. In concrete,

- A descriptive analysis by portfolio, where average profitability and risk were analyzed, according to all periods defined in table 5 (historical and follow-up);

- A statistic analysis to follow-up period, in terms of average profitability and risk, throw parametric and nonparametric tests:

    1. Follow-up period as a unit sample approach according to periods defined in table 5;

    2. Follow-up period as a two samples approach according to tables 98 and 99.

Summing up, our empirical tests to both scenarios (ELECTRE and CAPM behavior face to market measured by PSI-20 TR) and to both analysis (descriptive and statistic) are:

- Test 1 (T_1): With ELECTRE III, portfolio average profitability is higher than market profitability?

- Test 2 (T_2): With ELECTRE, portfolio has greater profitability by unit of risk than the market (Sharpe's index)?
- Test 3 (T_3): Portfolio with assets selected by ELECTRE has a higher average profitability when compared with portfolios average profitability with assets selected by CAPM?

- Test 4 (T_4): Portfolio formed under ELECTRE assumptions has higher profitability by unit of risk than portfolio constructed under CAPM method (Sharpes' index)?

In order to test the multicriteria hypothesis using ELECTRE III, we define alternatives (table 6), criteria and thresholds (table 7), from 2000 to 2011.

4.2. Data and software used in empirical work

To carrying out the hypothesis enunciated, we start by determine witch companies were traded in Portuguese Stock Index (PSI-Geral), between 1999 and 2011. Then, we collected data referring those companies' shares: weekly closing prices, PSI-20TR weekly closing prices, annual interest treasury bonds (three months), annual financial statements of those companies, and gross dividends paid by them. Below, we present in a more detailed form, data and data source:

- From PSI-Geral, we select company's shares traded from 1999 to 2011, also named as alternatives of the model (see table 6);
- Weekly closing prices relative to shares selected, from 1999 to 2011. This data was collected from Euronext site[22];
- Weekly closing prices from PSI-20TR from 1999 to 2011. This data was collected from Euronext site[23];

[22] www.euronext.com

[23] www.euronext.com

- Monthly data from interest treasury bonds, 3M, from 1999 to 2011. This data was collected from Institute for the Management of Treasury and Public Credit, IP (IGCP, IP);
- Annual financial data from financial statements (balance sheet, income statement and annex for each company previously selected) from 1999 to 2011. This data was used to calculated financial ratios. Data was colleted from companies' web page. Specifically we collected: total asset, equity, net income, current liabilities, cash, clients and inventories;
- Gross dividends paid by each company selected, from 2000 to 2012. This information was based on annual annex.

To perform ELECTRE estimation, and construct Ranking Matrix, that gives us the best to the worst alternative, we use ELECTRE III software, kindly provided by *Université Paris Dauphine*[24].

Table nº 5 – Portfolios dates

Portfolio nº	Initial /Historical Period	Follow-up Period		
P 1	2000 – 2004	Jan05-Dec05	Jan05-Dec06	Jan05-Dec07
P 2	2001 – 2005	Jan06-Dec06	Jan06-Dec07	Jan06-Dec08
P 3	2002 – 2006	Jan07-Dec07	Jan07-Dec08	Jan07-Dec09
P 4	2003 – 2007	Jan08-Dec08	Jan08-Dec09	Jan08-Dec10
P 5	2004 – 2008	Jan09-Dec09	Jan10-Dec10	Jan10-Dec11
P 6	2005 – 2009	Jan10-Dec10	Jan11-Dec11	
P 7	2006 – 2010	Jan11-Dec11		
P 8	2007 – 2011			

Source: Own elaboration, May, 2012.

[24] A note of special thanks to Professor Luis Pacheco, from DCEE, for his prompt cooperation.

Table nº 6 – Model Alternatives

ALTERNATIVE	TICKER	SECTOR OF ACTIVITY
Banco Comercial Português	BCP	Financial services
Banco Espírito Santo e Comercial de Lisboa	BES	Financial services
Banco Português Investimento	BPI	Financial services
BRISA - Auto-estradas de Portugal, S.A.	BRI	Transportation, construction
CIMPOR - Cimentos de Portugal	CPR	Building materials
COFINA	CFN	Media segment
COMPTA	COMAE	Information Technology and Communication
Corticeira Amorim S.G.P.S., S.A.	COR	Cork industry
EDP - Energias de Portugal	EDP	Energy
ESTORIL SOL, S.G.P.S., S.A.	ESO	Game and related areas such as hospitality
FISIPE	FSP	Textile fiber producer
GRAO-PARA	GPA	Construction industry
IBERSOL	IBS	Food chain management area
Inapa - IPG, SA	INA	Paper
Jerónimo Martins SGPS, SA	JMT	Retail, manufacturing
LISGRAFICA	LIG	Printing and graphic arts
Mota-Engil	EGL	Construction industry
OREY ANTUNES	ORE	Ship industry
Portucel S.A.	PTI	Pulp and paper
Portugal Telecom	PTC	Telecommunications
REDITUS	RED	Business Consulting
Salvador Caetano	TMC	Automotive and Robotics

SEMAPA - Sociedade de Investimento e Gestão	SEM	Conglomerate
Soares da Costa SGPS, S.A.	SCOAE	Civil engineering and construction
SONAE	SON	Conglomerate holding company
SONAE Indústria	SONI	Manufacturing
SUMOL-COMPAL	SUCO	Beverage distributor
Teixeira Duarte	TDSA	Construction industry
Zon Multimédia	ZON	Media holding

Source: Own elaboration, May, 2012.

Table nº 7 – Criteria and Thresholds definition

Financial Ratios (Models Criteria)		**Models Thresholds**[25]
		Value
Return on Assets (ROA): Net Income$_N$ / Total Assets$_{N-1}$	Profitability	**P 1** (***p>*** *3,42%;* ***q≤****2,63%)* **P 2** (***p>*** *2,88%;* ***q≤****2,21%)* **P 3** (***p>*** *2,80%;* ***q≤*** *2,15%)* **P 4** (***p>*** *2,79%;* ***q≤*** *2,14%)* **P 5** (***p>*** *3,14%;* ***q≤*** *2,41%)* **P 6** (***p>*** *3,24%;* ***q≤*** *2,50%)* **P 7** (***p>*** *3,07%;* ***q≤*** *2,36%)* **P 8** (***p>*** *2,90%;* ***q≤*** *2,23%)*
Return on Equity (ROE): Net Income $_N$ / Equity $_{N-1}$	Profitability	**P 1** (***p>*** *3,42%;* ***q≤****2,63%)* **P 2** (***p>*** *2,88%;* ***q≤****2,21%)* **P 3** (***p>*** *2,80%;* ***q≤*** *2,15%)* **P 4** (***p>*** *2,79%;* ***q≤*** *2,14%)*

[25] *q*, is the indifference threshold, and *p* is the preference threshold.

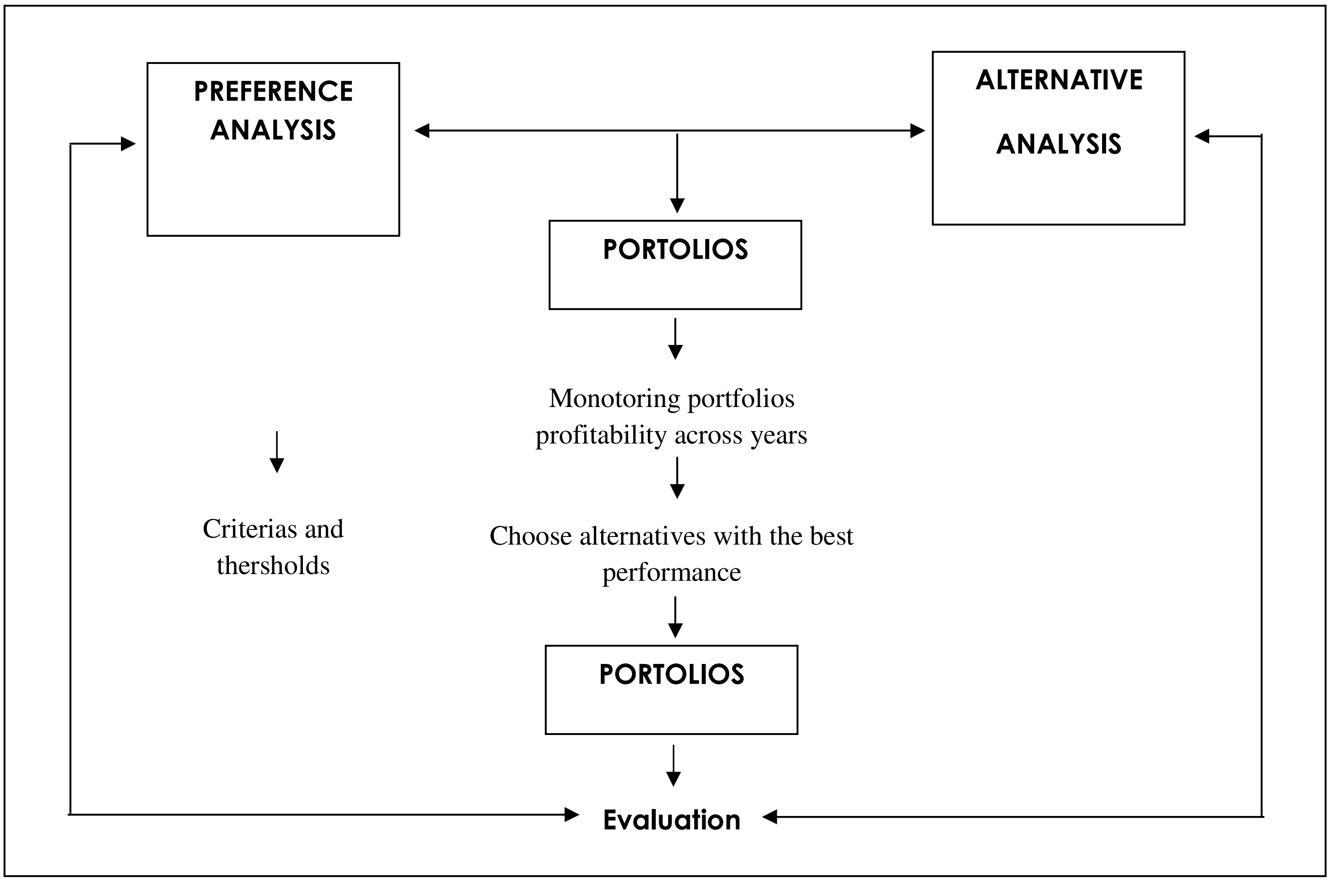
		P 5 *(**p>** 3,14%**; q≤** 2,41%)* **P 6** *(**p>** 3,24%**; q≤** 2,50%)* **P 7** *(**p>** 3,07%**; q≤** 2,36%)* **P 8** *(**p>** 2,90%**; q≤** 2,23%)*
Financial Autonomy (FA): $Equity_N$ / $Total\ Assets_N$	Leverage	***p*** > 30%; ***q*≤** 25%
General Liquidity (GL): $(Clients + Stocks + Cash)_N$ / $Current\ liabilities_N$	Liquidity	***p*** > 1,50; ***q*≤** 1,00
Reduced Liquidity (RL): $(Clients + Cash)_N$ / $Current\ liabilities_N$	Liquidity	***p*** > 1,50; ***q*≤** 1,00
Notes: Concerning ROE and ROA, **-** *q* threshold correspond to the interest treasury bonds, 3M, annual average, to the periods defined; - *p* threshold corresponds to *q* threshold plus 30%.		

Source: Own elaboration, May 2012.

4.3. Methodology and empirical work organization

Based on Spronk and Hallerbach (1997), our empirical methodology based on a multicriteria approach, can be generally presented as given in chart nº 9.

Chart nº 9: Portfolio selection process



Source: Own elaboration, based on Spronk and Hallerbach (1997), 2012.

Being characterized as a step-by-step process, we firstly selected from PSI-Geral, company's shares traded from 1999 to 2011, as expressed in *alternative analysis* box. Within this period from 1999 to 2011, eight historical/initial periods were defined, corresponding each period to a portfolio P_x, being x the portfolio number. Also, for each one of these historical periods, follow-up periods were defined, for instance, three holding periods. This procedure will allow us to see how portfolio behaved over time, as already mentioned, in a buy-and-hold strategy, where financial ratios and shares are equally weighed. A descriptive analysis was made for all periods, by portfolio. Statistically, we analyze, on one hand, all follow-up periods, as a unit sample; on the other hand, within all follow-up periods, we subdivide it into two sub periods, two samples, in order to asses country's macroeconomics evolution (the cutting point was made in year 2007, because sub prime crises started in 2008).

Secondly, and as indicated in the *preference analysis* box, the decision maker's preferences are described, reflecting the decision context and comprising the investment objectives that the investor wishes to obtain, as well the constraints imposed (thresholds). In addition, based on financial ratios, decision maker only wish to invest in a certain class of assets, so threshold must be defined for each criteria, according to the periods/portfolios previously defined. The preference structure of the decision maker, being based on a multicriteria problem, is more complex than the mean-variance approach. The decision maker may have other objectives than financial value maximization, for instance, he may want to achieve a stable rate of growth. So, based on an optimization process, processed with ELECTRE III method, possible portfolios are defined, and their performance needs to be monitored across years, from 2000 to 2011.

As *criteria* we use five financial ratios previously calculated from 1999 to 2011, in order to assess profitability, leverage and liquidity, and all of them have the same weight in the model, 20%. In order to analysis each portfolio, we calculated their average for each period defined in table 5. In particular,

- **Return on Assets (ROA):** This ratio express how much profit a company generated compared to its assets. It is expected to increase over the years. Concerning thresholds established, we consider the annual average of treasury bonds interest rate, three months. So, *q*, or indifference threshold, is equal to this value, considering each period (see table 5), and *p*, or preference threshold, is equal to *q* plus 30%. In the model have a 20% weighting. The formula used was:

$$\mathbf{ROA_N} = \text{Net Income}_N \,/\, \text{Total Assets}_{N-1} \qquad [72]$$

- **Return on Equity (ROE):** Give us the ratio between profits and shareholders' equity, and is expected to have a rate of return higher than the rate of return on treasury bonds, to be able to say that the company is really profitable. It is expected to increase over the years an amount at least equal to profits minus dividends paid. Concerning thresholds established, we consider the annual average of treasury bonds interest rate, three months. So, *q*, or indifference threshold, is equal to this value, considering each period (see table 5), and *p*, or preference threshold, is equal to *q* plus 30%. In the model have a 20% weighting. The formula used was:

$$\mathbf{ROE_N} = \text{Net Income}_N / \text{Equity}_{N-1} \qquad [73]$$

- **Financial Autonomy (FA):** This ratio related to the company's financial structure, express the extent to which the asset is being financed by equity and debt capital. This ratio is expected to increase every year or, at least, to remains stable. Threshold *q* was defined based on criteria required by investment projects subsidized by the government. In the model have a 20% weighting. The formula used was:

$$\mathbf{FA_N} = \text{Equity}_N / \text{Total Assets}_N \qquad [74]$$

- **General Liquidity (GL):** Liquidity refers to the ability to convert the asset into cash, being some items more liquid than others. So, this ratio measures the extent to which a company has cash to meet immediate and short-term obligations, or assets that can be quickly converted to do this. It is desirable that the ratio exceeds at least the value of 1, meaning that the company has at least liquid assets to meet liabilities in the short term, even without the liquidation of stocks. Threshold *q* was defined based on criteria required by investment projects subsidized by the government. In the model have a 20% weighting. The formula used was:

$$\mathbf{GL_N} = (\text{Clients + Stocks+ Cash})_N \;/ \text{Current liabilities}_N \qquad [75]$$

- **Reduced Liquidity (RL):** Measures a company's ability to meet its short-term liabilities with cash provenience of its net assets, but in a way more demanding than in the general liquidity ratio, assuming that stocks (stocks of raw materials and intermediate and finished products) will be difficult to convert into quickly cash. It is expected to exceed at least the value of 1. Threshold *q* was defined based on criteria required by investment projects subsidized by the government. In the model have a 20% weighting. The formula used was:

$$\mathbf{RL_N} = (\text{Clients} + \text{Cash})_N \ / \ \text{Current liabilities}_N \qquad [76]$$

As *thresholds*, we define a *q*, the indifference threshold, and a *p*, the preference threshold, for each criteria, from 2000 to 2011. Since we follow a buy and hold strategy, and constructed defensive portfolios, threshold *q* corresponds to the annual average of asset without risk - interest treasury bonds 3M - for the periods expressed in table 5; *p* threshold corresponds to *q* threshold plus 30% (historically, in average, market gave us profitability greater than assets without risk plus 30%) being applied to ROA and ROE criteria. Relatively to Financial Autonomy, General Liquidity and Reduced Liquidity, thresholds were defining based on the rules used in government subsidies attribution[26].

Thirdly, variables of the model must be previously treated, in particular closing prices. Concerning shares weekly closing prices, from 1999 to 2011, they were adjusted considering splits. Then, respective monthly and annual average was calculated. The same procedure was applied to PSI-20 TR closing prices.

Fourthly, seeing not all shares selected are interesting to invest, ELECTRE III software gave us the Ranking Matrix[27]. This information gave us, through an optimization process already described in Chapter 2, the best to the worst shares according to criteria and threshold defined, for each portfolio/period. Seeing ELECTRE III ranks all shares from the best to the worst, it was necessary to select the shares to use in our work. To do so, we only

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[26] In Portugal we have Operational Programme of Human Potential Organization for Enrichment Evaluations (POPH), National Strategic Reference Framework (QREN) between others.

[27] See outputs from software in annex.

selected the share(s), in minimum, 50% preferred to the others, being in ranking matrix represented by letter "P".

In fifth place, and in order to test our hypothesis, we calculated the profitability and Sharpe's index for each portfolio, and verify theirs performance face to market (PSI-20TR) across time. Profitability and Sharpe's index for both portfolio and market are calculated to all periods defined in table 5: firstly to the historical period, and then to the follow-up periods.

Finally, average profitability and Sharpe's index were calculated for each follow-up period, one year, two years and three years, for market, ELECTRE and CAPM, and then parametric tests (*t*-student test) and nonparametric tests (Mann-Whitney test) were conducted.

So, we must start by determining profitability by share (including dividends), according to the formula,

$$SP_{jt} = \left(\frac{C_{fjt}}{C_{0jt}} \right)^{\frac{1}{n^{o}\, period invested}} - 1 \qquad [77]$$

where,

$\mathbf{SP_{jt}}$ is the profitability of share *j* selected by ELECTRE for period *t*;

$\mathbf{C_{fjt}}$ is the final capital obtained with certain investment in share *j* in time *t*;

$\mathbf{C_{0jt}}$ is the initial capital invested in share *j* in time *t*.

and,

$$\mathbf{C_{fjt}} = C_0 * [1 + (\frac{CP_t}{CP_{t-1}} -1)] + Divid_t * n^{o}\ shares_{t-1} \qquad [78]$$

where,

$\mathbf{C_{fjt}}$ is the final capital obtained with certain investment in share *j* in time *t*;

$\mathbf{C_{0jt}}$ is the initial capital invested in share *j* in time *t*;

$\mathbf{CP_t}$ is the closing prices of share *j* for a certain year, from 1999 to 2011. Closing prices were determined by calculating annual average of weekly data for portfolio/period in study;

$\mathbf{Divid_t}$***nº** $\mathbf{shares_{t-1}}$ are the dividends paid in certain year, concerning the results obtained in previous year, reinvested by investor, by purchasing more shares from that company.

Then, portfolio profitability ($\mathbf{P_{pt}}$) may be defined as the average of shares profitability. To notice that, each share selected have the same weight in portfolio. Mathematically,

$$P_{pt} = \sum_{j=1}^{n} r_{jt} * w_{jt} \qquad [79]$$

where,

$\mathbf{r_{jt}}$ is the profitability of share *j* in period *t*, selected by ELECTRE;

n is the number of shares in portfolio;

$\mathbf{w_{jt}}$ is the weight of share *j* in the portfolio.

And, market profitability is calculated according to the formula,

$$MP_t = \frac{CP_t}{CP_{t-1}} - 1 \qquad [80]$$

where,

$\mathbf{MP_t}$ is the market profitability for period *t*;

$\mathbf{CP_t}$ is the PSI-20TR closing prices for a certain year, from 1999 to 2011. Closing prices were determined by calculating annual average of weekly data.

We also calculated risk measures, in particular, Treynor's index, Sharpe's index, Jensen's performance index, according to the assumptions presented in Chapter 1.

It is also important to calculate three more indicators: Beta, portfolio standard deviation, and market standard deviation. Its importance stems not only on information provided, but also because they are key-input to CAPM estimation.

On one hand, Beta gives us important information about the correlated volatility of an asset/portfolio in relation to the volatility of the benchmark (in our case PSI-20TR). Generally, if an asset/portfolio has a Beta of zero this means that its moves are not correlated with benchmark's moves. A positive Beta means that asset/portfolio generally follows the benchmark (asset/portfolio tends to move up when the benchmark moves up). A negative Beta means that asset/portfolio generally moves opposite the benchmark (asset/portfolio tends to move up when benchmark moves down, and asset/portfolio tends to move down when benchmark moves up). In short, Beta measures the part of asset's variation that cannot be removed by diversification provided by a portfolio containing many risky assets. On the other hand, Standard Deviation is used to measure dispersion or volatility of a data set. If closing prices are very close to average value (mean), standard deviation have a small value. In the cases in which closing prices are dispersed over a wide range of values, we have a large standard deviation. Thus, Beta of share, represented as $\boldsymbol{\beta_j}$, is calculated as,

$$\beta_j = \frac{\text{cov}_{jm}}{\delta_m^2} \qquad [81]$$

where,

$\mathbf{cov_{jm}}$ is covariance between average profitability of share j with market average profitability (m);

$\boldsymbol{\delta^2_m}$ is variance of market average profitability.

So, the Beta of portfolio is calculated summing all Betas of individual share, appropriately weighted by its weight in the portfolio (equal weight and correlation between assets is 0).

Concerning standard deviation of share *j*, represented by $\boldsymbol{\delta_j}$, it is calculated as,

$$\delta_j = \sqrt{\frac{1}{n-1}\sum_{t=1}^{n}(r_{jt} - \overline{r_j})^2} \quad [82]$$

and the standard deviation of portfolio, represented by $\boldsymbol{\delta_{Pj}}$ is calculated as,

$$\delta_{Pj} = \sqrt{\frac{1}{n-1}\sum_{t=1}^{n}(r_{jt} - \overline{r_j})^2 * w_{jt}} \quad [83]$$

Standard deviation of market, represented by $\boldsymbol{\delta_M}$, is calculated as,

$$\delta_m = \sqrt{\frac{1}{n-1}\sum_{t=1}^{n}(r_{mt} - \overline{r_m})^2} \quad [84]$$

Lastly, to draw conclusions, tables and charts are made in order to compare the performance of shares and Sharpe's index selected by ELECTRE with results obtained by the market. Therefore, information concerning profitability (for portfolio and market), Beta and risk measures (in particular Sharpe's index) are compiled and presented by portfolio (historical and follow-up periods).

Despite the importance given to the multicriteria methodology, which is the bases of our work, it is nevertheless interesting to see the results obtained with the CAPM methodology[28], and compares them with the ELECTRE methodology.

In this sense, CAPM by share is calculated as,

$$\mathbf{CAPMS_{jt}} = Rf_t + Beta*(RM_t - Rf_t) \quad [85]$$

where,

[28] See Chapter 1.

$CAPMS_t$ is the CAPM results for share *j* in period *t*;

Rf_t is the risk free corresponding to the average of monthly interest treasury bonds, 3M, for period *t*;

RM_t is the market profitability at period *t*.

To determine in witch shares to invest, CAPM result is compared with share profitability, individually, for all shares selected from PSI-Geral traded between 1999 and 2011. Only when profitability of share is positive and higher than CAPM, investment in that share should be made. This analysis was made for all periods, historical and follow-up, defined in table 5. Having shares selected according to this procedure, we must calculate:

- Profitability of each share selected;
- Profitability of portfolio;
- Market profitability;
- Betas of shares;
- Beta of portfolio;
- Standard deviation of each share selected;
- Standard deviation of portfolio;
- Standard deviation of market;
- Calculate risk measures, using assumption defined in Chapter 1.

As made to ELECTRE method, we must compare performance of each portfolio composed by shares selected with CAPM, in terms of profitability and Sharpe's index, with results of market (PSI-20TR). In order to do so, and draw conclusions, tables and charts are made in order to compare both performances. Therefore, information about profitability (portfolio and market), Beta and risk measures (in particular Sharpe's index) are compiled, and presented by portfolio (historical and follow-up periods).

This analysis will also permit to take conclusions about the use of a multicriteria model (ELECTRE III), and the use of a traditional model (CAPM). These two models do not replace each other, but instead, complement, seeing they are based on completely different assumptions.

4.4. Portfolios composition under ELECTRE and CAPM assumptions

Starting with ELECTRE methodology, we process alternatives, criteria and threshold, with ELECTRE III software, being the results obtained with a distillation process based on Credibility Matrix and Ranking Matrix. So, following a descending distillation, this is, from the best alternatives to the worst ones according to their financial performance measured by financial ratios, main results are displayed in table 8. To notice that results are present by portfolio, and each of them represents a specific period (see table 5), as explained previously.

Table nº 8 – Shares selected by ELECTRE

P1 (00-04)		P2 (01-05)		P3 (02-06)		P4 (03-07)		P5 (04-08)		P6 (05-09)		P7 (06-10)		P8 (07-11)	
1	Orey	**1**	Cimpor	**1**	Cimpor	**1**	Cimpor	**1**	Cimpor	**1**	Cimpor	**1**	PT	**1**	PT
2	Ibersol	**2**	Orey	**2**	Ibersol	**2**	Zon	**2**	Zon	**2**	PT	**2**	Brisa	**2**	Jerónimo Martins
3	Brisa	**3**	Ibersol	**3**	PT	**3**	Ibersol		PT	**3**	Jerónimo Martins	**3**	Jerónimo Martins	**3**	Brisa
4	Cimpor		Brisa		Semapa		PT		Reditus	**4**	Zon	**4**	Ibersol	**4**	Ibersol
5	Semapa	**4**	Semapa	**4**	Teixeira Duarte	**4**	Teixeira Duarte	**3**	BPI		BPI	**5**	Zon		Portucel
6	BPI	**5**	BPI		Orey		Sonae		Ibersol		Portucel	**6**	Portucel	**5**	Zon
	Cofina		Cofina		Cofina	**5**	Reditus		Jerónimo Martins	**5**	Ibersol		Cimpor	**6**	Cimpor
7	PT	**6**	Teixeira Duarte	**5**	Brisa		Semapa	**4**	Orey	**6**	Reditus	**7**	Mota Engil	**7**	EDP
	Soane	**7**	PT	**6**	Zon		Cofina		Semapa	**7**	Mota Engil		EDP	**8**	Mota Engil
	BCP		BCP		BPI	**6**	BPI		Sonae		EDP	**8**	Semapa	**9**	Semapa
8	Portucel	**8**	Mota Engil		Sonae		Orey	**5**	Portucel	**8**	Teixeira Duarte	**9**	BPI	**10**	BPI
9	Reditus		Portucel	**7**	Portucel	**7**	Jerónimo Martins		Brisa		Brisa		Reditus		Orey
10	Toyota	**9**	Reditus		BCP		Portucel	**6**	Mota Engil	**9**	Semapa			**11**	Corticeira Amorim
	BES		BES	**8**	Jerónimo Martins	**8**	Brisa	**7**	Soares da Costa	**10**	Soares da Costa			**12**	Cofina
					BES	**9**	Mota Engil							**13**	Soane

Source: From ELECTRE III software output estimation, May 2012

Giving a closer look to the results obtained by ELECTRE, we can easily see that shares selected are always the same in the different periods. This can be explained by the companies' good financial performance across years.

Concerning results obtained with CAPM, summarized in table 9, they express those shares that had a good performance in stock market. Therefore, besides having a positive profitability in the stock market, their profitability is higher than CAPM estimation. As happened with ELECTRE results, most of the shares are common to almost periods/portfolios.

Table nº 9 – Shares selected by CAPM

P1 (00-04)	P2 (01-05)	P3 (02-06)	P4 (03-07)	P5 (04-08)	P6 (05-09)	P7 (06-10)	P8 (07-11)
BES	BES	BES	BES	BPI	BRISA	CIMPOR	CIMPOR
BRISA	BPI	BPI	BPI	BRISA	CIMPOR	EDP	JERÓNIMO MARTINS
CIMPOR	BRISA	BRISA	BRISA	CIMPOR	EDP	IBERSOL	PORTUCEL
COFINA	CIMPOR	CIMPOR	CIMPOR	CORTIC. AMORIM	IBERSOL	JERÓNIMO MARTINS	PT
MOTA ENGIL	COFINA	COFINA	COFINA	EDP	JERÓNIMO MARTINS	PORTUCEL	REDITUS
OREY ANTUNES	CORTIC. AMORIM	CORTIC. AMORIM	CORTIC. AMORIM	GRÃO-PARA	MOTA ENGIL	PT	
PORTUCEL	IBERSOL	EDP	EDP	IBERSOL	OREY ANTUNES	REDITUS	
PT	MOTA ENGIL	IBERSOL	IBERSOL	JERÓNIMO MARTINS	PORTUCEL	SEMAPA	
REDITUS	OREY ANTUNES	JERÓNIMO MARTINS	JERÓNIMO MARTINS	MOTA ENGIL	PT	SOARES DA COSTA	
SEMAPA	PORTUCEL	MOTA ENGIL	MOTA ENGIL	PORTUCEL	REDITUS	SONAE	
SUMOL-COMPAL	SALVADOR CAETANO	OREY ANTUNES	OREY ANTUNES	PT	SEMAPA		
	SEMAPA	PORTUCEL	PORTUCEL	REDITUS	SOARES DA COSTA		
		PT	REDITUS	TOYOTA	SONAE		
		REDITUS	TOYOTA	SEMAPA			
		SALVADOR CAETANO	SEMAPA	SOARES DA COSTA			
		SEMAPA	SOARES DA COSTA	SONAE			
		SOARES DA COSTA	SONAE	TEIXEIRA DUARTE			
		SONAE	TEIXEIRA DUARTE				
		SONAE INDÚSTRIA	ZON				
		TEIXEIRA DUARTE					
		ZON					

Source: Own elaboration, May 2012.

4.4.1. Descriptive analysis: by portfolio individually

4.4.1.1. Main results obtained with Portfolio 1

Table nº 10 – Portfolio 1 (ELECTRE results)

FOLLOW-UP PORTFOLIO P1 - Main Results (ELECTRE METHODOLOGY)				
ASSETS SELECTED	**HISTORICAL**	**FOLLOW-UP (MONTHLY)**		
	2000-2004	**Jan05-Dec05**	**Jan05-Dec06**	**Jan05-Dec07**
PROFITABILITY (INCLUDES DIVIDENDS):				
OREY	24,20%	3,05%	0,79%	0,63%
IBERSOL	-7,60%	2,16%	3,04%	2,51%
BRISA	9,90%	1,05%	1,73%	1,57%
CIMPOR	6,28%	1,11%	2,19%	1,45%
SEMAPA	4,36%	5,07%	3,76%	2,55%
BPI	-1,41%	2,41%	3,24%	1,98%
COFINA	24,41%	1,05%	1,31%	0,56%
PT	2,41%	0,26%	0,93%	0,95%
SONAE	-9,84%	3,28%	3,07%	2,87%
BCP	-14,60%	2,03%	2,04%	1,53%
PORTUCEL	5,47%	1,75%	2,45%	1,59%
REDITUS	0,42%	-1,50%	-0,71%	2,24%
S. CAETANO	-4,19%	0,67%	2,27%	2,84%
BES	1,69%	0,54%	1,48%	1,34%
PSI-20 TR	**-6,52%**	**1,06%**	**1,62%**	**1,51%**
PROFITABILITY P1	**3,05%**	**1,78%**	**2,13%**	**1,84%**
BETA P1	**0,80**	**0,67**	**0,77**	**0,78**
STANDARD DEVIATION P1	**1,96**	**1,87**	**2,27**	**2,80**
STANDARD DEVIATION PSI-20 TR	**2,43**	**0,29**	**1,18**	**2,01**
SHARPE'S INDEX				
P1	0,22%	0,88%	0,86%	0,59%
PSI-20 TR	-3,77%	3,15%	1,22%	0,66%
TREYNOR'S INDEX				
P1	0,53%	2,45%	2,53%	2,11%
PSI-20 TR	-9,15%	0,91%	1,44%	1,32%
JENSEN'S INDEX				
P1	0,42%	1,64%	1,95%	1,64%
PSI-20 TR	-9,15%	0,91%	1,44%	1,32%

Source: Own elaboration, July 2012.

Concerning T_1, from the table above, we easily verify that, generally, P1 performed better than the market (3,05% face to -6,52%), and this behavior continued in the follow-up period. In particular, in the first follow-up period we could obtain with P1 profitabilities of 1,78% face to 1,06%, in the second, profitabilities of 2,13% face to 1,62%, and in the third follow-up period, profitabilities of 1,84% face to 1,51%. So, apparently, portfolio selection based on companies' financial performance is a good tool. To this good behavior, mostly contribute Orey Antunes and Cofina, with profitability in historical period of 24,20% and 24,41% respectively. In follow-up periods, almost every share a positive contributes.

Relatively to T_2, when analyzing Sharpe's index, which captures the expected differential return (portfolio profitability minus risk free) per unit of risk, only in historical period his value is higher than market (0,22% face to -3,77%), being in every follow-up periods lower than the market (0,88% face to 3,15%, 0,86% face to 1,22% and 0,59% face to 0,66%). So, only in historical period premium risk by unit invested is positive and higher than market. Concerning Treynor's index, and being a measure of excess of return per unit of risk, in all periods, historical and follow-up, portfolio had better values than the market: 0,53%, 2,45%, 2,53% and 2,11% for portfolio, and -9,15%, 0,91%, 1,44% and 1,32% for market. For Jensen's index, values show us that portfolio had an extraordinary performance face to market. So, in all periods, P1 over performed the market with positive and higher values, in particular in historical period were P1 had 0,42% face to -9,15% for the market.

Concerning the dichotomy portfolio's risk versus market risk, coefficient Beta tells us in what way both are correlated. So, seeing all Betas are between 0,67 and 0,80, and according to table 3, both risks are equal. However, profitabilities obtained in historical period indicate the contrary: P1 obtained a positive profitability, and market obtained a negative one.

Table nº 11 – Portfolio 1 (CAPM results)

FOLLOW-UP PORTFOLIO P1 - Main Results (CAPM METHODOLOGY)				
ASSETS	HISTORICAL	FOLLOW-UP (MONTHLY)		
	2000-2004	Jan05-Dec05	Jan05-Dec06	Jan05-Dec07
PROFITABILITY (INCLUDES DIVIDENDS):				
BES	1,69%	0,54%	1,48%	1,34%
BRISA	9,90%	1,05%	1,73%	1,57%
CIMPOR	6,28%	1,11%	2,19%	1,45%
COFINA	24,41%	1,05%	1,31%	0,56%
MOTA ENGIL	4,27%	3,82%	4,34%	2,93%
OREY ANTUNES	24,20%	3,05%	0,79%	0,63%
PORTUCEL	5,47%	1,75%	2,45%	1,59%
PT	2,41%	0,26%	0,93%	0,95%
REDITUS	0,42%	-1,50%	-0,71%	2,24%
SEMAPA	4,36%	5,07%	3,76%	2,55%
SUMOL-COMPAL	3,28%	0,05%	0,89%	0,39%
PSI-20 TR	**-6,52%**	**1,06%**	**1,62%**	**1,51%**
PROFITABILITY P1	**7,88%**	**1,48%**	**1,74%**	**1,47%**
BETA P1	**0,68**	**0,68**	**0,72**	**0,75**
STANDARD DEVIATION P1	**2,11**	**1,95**	**2,28**	**2,78**
STANDARD DEVIATION PSI-20 TR	**2,43**	**0,29**	**1,18**	**2,01**
SHARPE'S INDEX				
P1	2,48%	0,68%	0,69%	0,46%
PSI-20 TR	-3,77%	3,15%	1,22%	0,66%
TREYNOR'S INDEX				
P1	7,76%	1,97%	2,17%	1,71%
PSI-20 TR	-9,15%	0,91%	1,44%	1,32%
JENSEN'S INDEX				
P1	5,25%	1,33%	1,56%	1,28%
PSI-20 TR	-9,15%	0,91%	1,44%	1,32%

Source: Own elaboration, July 2012.

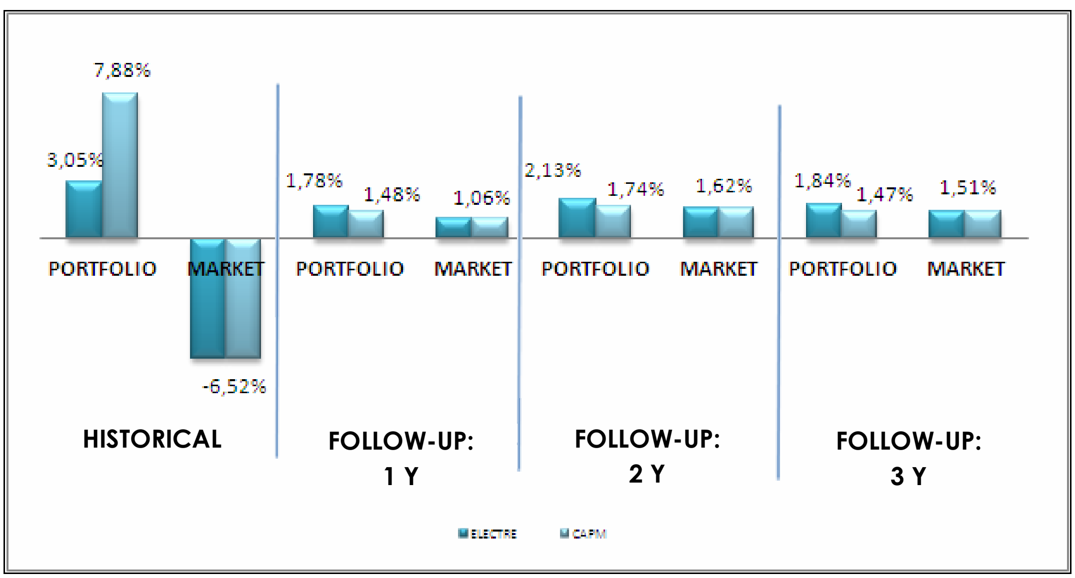
Now, looking to CAPM results, P1 performed better than the market, in particular, P1 obtained a profitability of 7,88% face to -6,52%. This good performance can mostly be identified in the follow-up period (1,48% face to 1,06%, 1,74% face to 1,62% and 1,47% face to 1,51%). From this we can conclude that, besides this shares performed better than estimations made by CAPM, the respective portfolio performed better than de market. Main responsible to this situation was Orey Antunes and Cofina, in historical period. In follow-up periods, almost every share gave a positive contribution to the profitability obtained.

Concerning Sharpe’s index, the conclusions taken to ELECTRE results can be extrapolated to CAPM, because only in historical period premium risk by unit invested is positive and higher than market (2,48% face to -3,77%), being lower in every follow-up periods (in particular in first and second periods). Analyzing Treynor's index, the excess of return per unit of risk, in all periods, in higher in portfolio, in particular for historical period where index for portfolio had a value of 7,76% and market had -9,15%. Finally, Jensen's index tells us that, in historical period, P1 had an extraordinary performance face to the market, over performed it, with positive and higher values, except in the last follow-up period: in historical period for P1 we had 5,25%, and market had -9,15%.

Looking to coefficient Beta, and having this indicator values between 0,68 and 0,75, both risks, portfolio and market, are equal. But, as concluded to ELECTRE, also with CAPM P1 obtained a positive profitability face to a negative obtained by the market, in historical period.

From table 10 and table 11 we can infer that portfolio composed by shares selected with ELECTRE III, and portfolio composed by shares selected with CAPM both performed better than the market, which had a bad performance in historical period (-6,52%). This behavior was maintained in follow-up periods, for both portfolios. But, which portfolio behaved better? The portfolio constructed using ELECTRE or using CAPM?

Chart nº 10 – Profitability of Portfolio 1

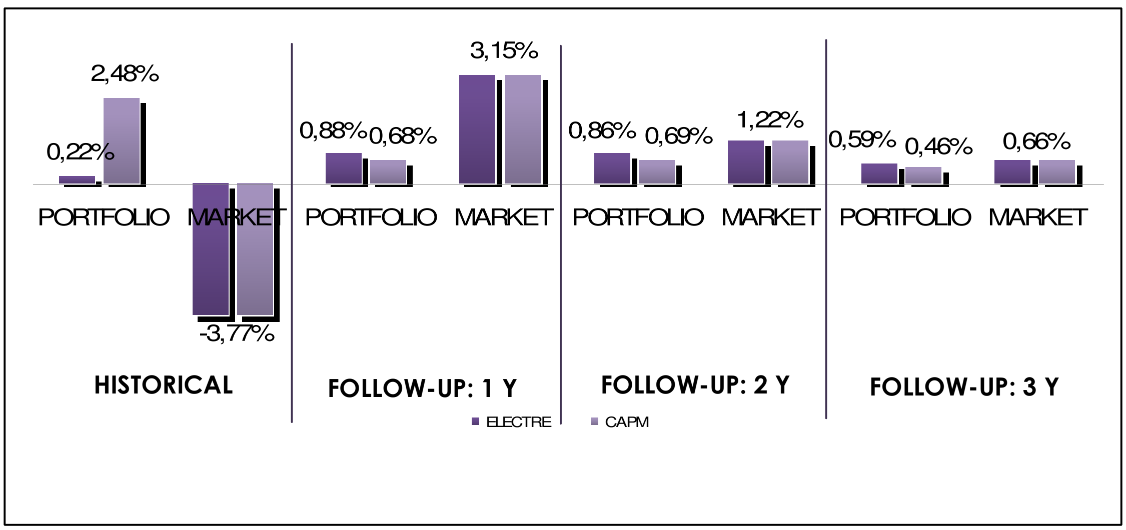


Source: Own elaboration, July 2012.

In order to answer to *T3*, from chart 10 we promptly can see that, in historical period, P1 profitability obtained with ELECTRE III was 3,05%, and with CAPM was 7,88%. Clearly, CAPM performed better than ELECTRE III, in historical period. But, this large difference is only visible in historical period. In follow-up periods differences in profitabilities are very small, for instance, 1,78% face to 1,48%, 2,13% face to 1,74% and 1, 84% face to 1, 47%. So, we can conclude that in a holding period of 3 years, to P1, ELECTRE III behaved better than CAPM.

From chart nº 11, which sustain answer to *T4*, we can see that only in historical period CAPM had high profitability by unit of risk than ELECTRE. In all other periods, follow-up periods, ELECTRE (0,88%, 0,86% and 0,59%) had greater profitability by unit of risk than CAPM (0,68%, 0,69% and 0,46%).

Chart nº 11 – Sharpe's Index of Portfolio 1



Source: Own elaboration, July 2012.

4.4.1.2. Main results obtained with Portfolio 2

Table nº 12 – Portfolio 2 (ELECTRE results)

FOLLOW-UP PORTFOLIO P2 - Main Results (ELECTRE METHODOLOGY)				
ASSETS SELECTED	HISTORICAL	FOLLOW-UP (MONTHLY)		
	2001-2005	Jan06-Dec06	Jan06-Dec07	Jan06-Dec08
PROFITABILITY (INCLUDES DIVIDENDS):				
CIMPOR	2,22%	2,11%	1,51%	-0,34%
OREY	30,06%	-1,61%	-0,50%	-0,68%
IBERSOL	2,12%	3,47%	2,76%	0,31%
BRISA	10,56%	2,09%	1,83%	-0,37%
SEMAPA	9,30%	1,63%	1,27%	0,05%
BPI	1,85%	1,68%	1,69%	-1,77%
COFINA	6,06%	-0,04%	0,17%	-3,42%
TEIXEIRA DUARTE	-2,70%	2,43%	2,12%	-2,02%
PT	-3,53%	1,30%	1,35%	0,23%
BCP	-14,74%	1,51%	1,24%	-2,40%
MOTA ENGIL	8,40%	3,30%	2,13%	-0,73%
PORTUCEL	1,86%	2,79%	1,44%	0,07%
REDITUS	-10,53%	-0,40%	4,33%	2,19%
BES	0,23%	1,62%	1,74%	-1,11%
PSI-20 TR	**-8,26%**	**2,18%**	**1,74%**	**-0,85%**
PROFITABILITY P2	**2,94%**	**1,56%**	**1,65%**	**-0,71%**
BETA P2	**0,66**	**0,87**	**0,98**	**0,92**
STANDARD DEVIATION P2	**1,87**	**2,55**	**3,00**	**2,95**
STANDARD DEVIATION PSI-20 TR	**1,11**	**0,65**	**1,46**	**1,95**
SHARPE'S INDEX				
P2	0,39%	0,53%	0,48%	-0,32%
PSI-20 TR	-9,47%	3,02%	1,05%	-0,55%
TREYNOR'S INDEX				
P2	1,10%	1,55%	1,46%	-1,03%
PSI-20 TR	-10,47%	1,97%	1,53%	-1,08%
JENSEN'S INDEX				
P2	0,72%	1,35%	1,44%	-0,95%
PSI-20 TR	-10,47%	1,97%	1,53%	-1,08%

Source: Own elaboration, July 2012.

Concerning P2, and relatively to T_1, composed by shares selected by ELECTRE III, we can also concluded that behaved better than the market, in the historical period (2,94% face to -8,26%), and this behavior not continued in follow-up periods: in the first follow-up period we could obtain with P2 a profitability of 1,56% face to 2,18%, and in the second, a profitability of 1,65% face to 1,74%. Therefore, although being not as robust as results obtained with P1, portfolio selection based on companies' financial performance still being a good tool. Orey Antunes gave a very interesting contribution to this profitability, in historical period; in follow-up periods, almost every share gave an important contribution.

Concerning T_2, and as happened with P1, also in P2 Sharpe's index indicates that premium risk by unit invested is higher in historical period, with 0,39% face to -9,47%. In follow-up periods, market beat portfolio, being the most significant difference in first follow-up period with 0,53% and market 3,02%. For Treynor's index, and only in historical and third follow-up periods, P2 had an excess of return per unit of risk, in particular in historical period: P2 had a value of 1,10% and market had -10,47%. Jensen's index confirms conclusions already taken, seeing portfolio over performed the market with positive and higher values, in historical period and in the last follow-up period. Once again, the most significant difference remains in historical period were P2 had 0,72% and market had -10,47%.

From table 12, we can also see that P2 and the market also have the same risk, seeing Betas' values are between 0,66 and 0,98. Once again, in historical period, P2 and market profitabilities show us a different tendency, seeing P2 obtained a positive profitability, and market achieved a negative one. In follow-up periods, tendency is the same.

Table nº 13 – Portfolio 2 (CAPM results)

FOLLOW-UP PORTFOLIO P2 - Main Results (CAPM METHODOLOGY)				
ASSETS	HISTORICAL	FOLLOW-UP (MONTHLY)		
	2001-2005	Jan06-Dec06	Jan06-Dec07	Jan06-Dec08
PROFITABILITY, BY ASSET (INCLUDES DIVIDENDS):				
BES	0,23%	1,62%	1,74%	-1,11%
BPI	1,85%	1,68%	1,69%	-1,77%
BRISA	10,57%	2,09%	1,83%	-0,37%
CIMPOR	2,22%	2,11%	1,51%	-0,34%
COFINA	6,06%	-0,04%	0,17%	-3,42%
CORTIC. AMORIM	1,22%	1,42%	2,81%	-0,61%
IBERSOL	2,12%	3,47%	2,76%	0,31%
MOTA ENGIL	8,40%	3,30%	2,13%	-0,73%
OREY ANTUNES	30,05%	-1,61%	-0,50%	-0,68%
PORTUCEL	1,86%	2,79%	1,44%	0,07%
SALVADOR CAETANO	1,41%	3,10%	6,94%	2,17%
SEMAPA	9,28%	1,63%	1,27%	0,05%
PSI-20 TR	**-8,26%**	**2,18%**	**1,74%**	**-0,85%**
PROFITABILITY P2	**6,27%**	**1,80%**	**1,98%**	**-0,54%**
BETA P2	**0,49**	**0,71**	**0,76**	**0,78**
STANDARD DEVIATION P2	**1,75**	**2,47**	**3,07**	**3,04**
STANDARD DEVIATION PSI-20 TR	**1,11**	**0,65**	**1,46**	**1,95**
SHARPE'S INDEX				
P2	2,32%	0,64%	0,58%	-0,25%
PSI-20 TR	-9,47%	3,02%	1,05%	-0,55%
TREYNOR'S INDEX				
P2	8,22%	2,21%	2,32%	-0,99%
PSI-20 TR	-10,47%	1,97%	1,53%	-1,08%
JENSEN'S INDEX				
P2	4,06%	1,58%	1,77%	-0,77%
PSI-20 TR	-10,47%	1,97%	1,53%	-1,08%

Source: Own elaboration, July 2012.

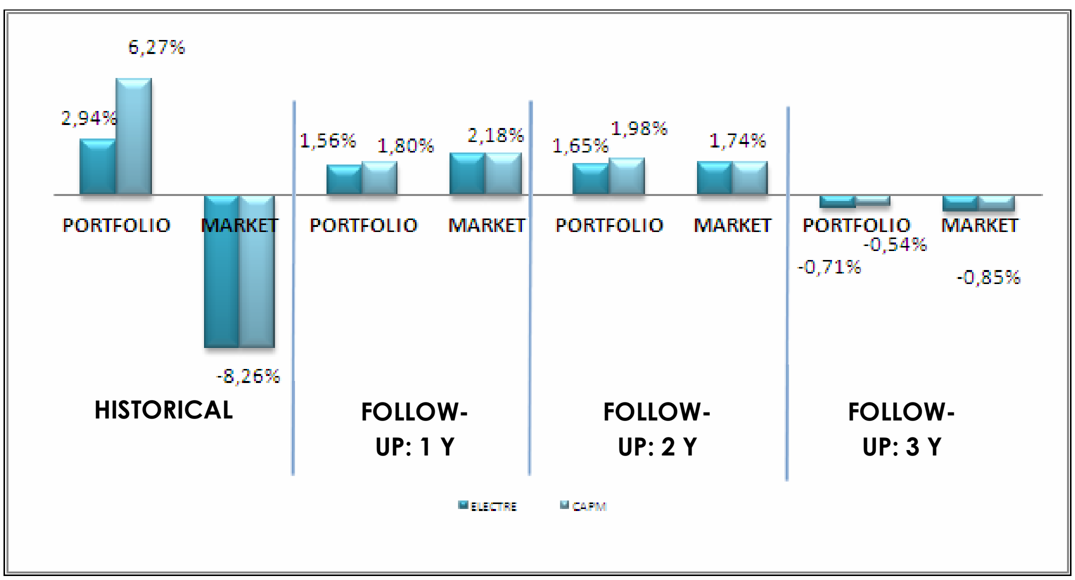
Portfolio P2, formed with shares selected by CAPM, performed better than the market. For instance, P2 obtained a profitability of 6,27% (see Orey Antunes profitability of 30,05%) face to -8,26%. This good performance kept in second and third follow-up periods (1,98% face to 1,74% and -0,54% face to -0,85%), where all shares, in general, contribute with positive profitability, except in the third follow-up period.

In this case, results are again similar to the others, were expected differential return per unit of risk for portfolio, measured by Sharpe's index, is significantly higher than market, in historical period (2,32% face to -9,47%). Excess of return per unit of risk, measured by Treynor's index, is better for portfolio in all periods: in historical period portfolio had a value of 8,22% face to -10,47%, and in follow-up periods portfolio had 2,21%, 2,32% and -0,99% face to 1,97%, 1,53% and -1,08% respectively. Finally, P2 over performance the market in almost every period. But in historical period portfolio difference is most significant: 4,06% for P2 and -10,47% for market.

From table 13, value of Beta in historical period is less than 0,5, telling us that risk of P2 is twice lower than market risk, but in opposite direction. So, looking to profitability tendency, market had negative profitability, and portfolio a positive profitability. In follow-up periods, results are different, being Betas between 0,71 and 0,78, meaning that portfolio and market have the same risk.

From table 12 and table 13, we can stat that portfolio composed by shares selected with ELECTRE III, and portfolio composed by shares selected with CAPM both performed better than the market, in all periods, historical and follow-up. However, it's important to realize which portfolio performed better, portfolio constructed using ELECTRE or portfolio constructed using CAPM. In order to obtain an answer to tests *T2* and *T4*, let's look to charts below.

Chart nº 12 – Profitability of Portfolio 2

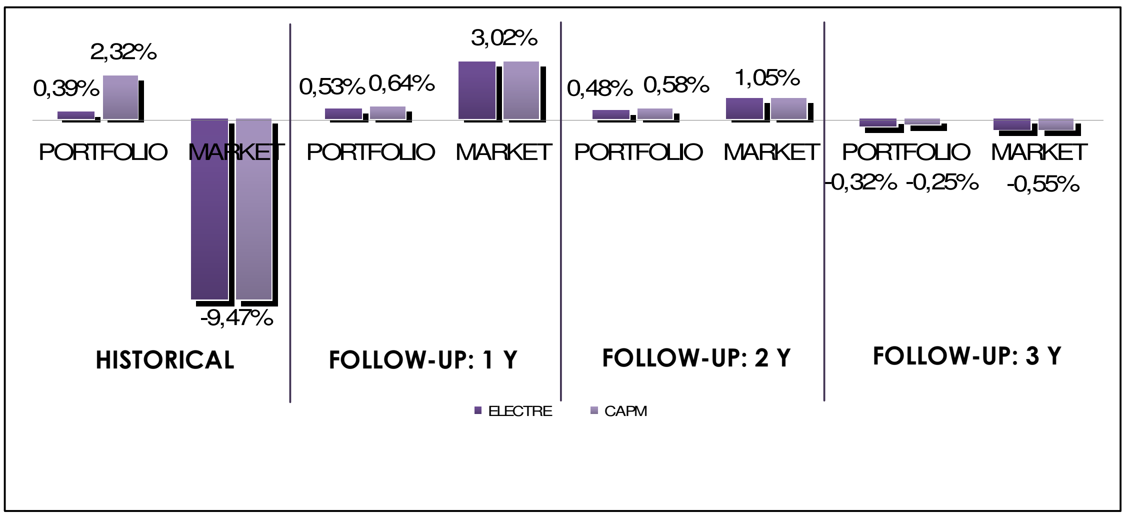


Source: Own elaboration, July 2012.

Sustained on chart 12, we can see that portfolio constructed with CAPM performed better than the one constructed with ELECTRE, answering this way to *T3*. And, we also can sustain that this difference is very visible especially in the historical, were ELECTRE III portfolio obtained 2,94% and CAPM obtained 6,27%. So, we can conclude that in a holding period of 3 years, to P2, CAPM behaved better than ELECTRE III.

From chart nº 13, which sustain answer to *T4*, we can see that in every periods CAPM had greater profitability by unit risk than ELECTRE: CAPM had 2,32%, 0,64%, 0,58% and -0,25%, ELECTRE had 0,39%, 0,53%, 0,48% and -0,32%.

Chart nº 13 – Sharpe's Index of Portfolio 2



Source: Own elaboration, July 2012.

4.4.1.3. Main results obtained with Portfolio 3

Table nº 14 – Portfolio 3 (ELECTRE results)

FOLLOW-UP PORTFOLIO P3 - Main Results (ELECTRE METHODOLOGY)				
ASSETS SELECTED	HISTORICAL	FOLLOW-UP (MONTHLY)		
	2002-2006	Jan07-Dec07	Jan07-Dec08	Jan07-Dec09
PROFITABILITY, BY ASSET (INCLUDES DIVIDENDS):				
CIMPOR	3,54%	-0,09%	-0,71%	1,32%
IBERSOL	19,52%	1,44%	-0,16%	0,75%
PT	4,42%	0,84%	0,72%	1,81%
SEMAPA	17,54%	0,13%	0,24%	0,81%
TEIXEIRA DUARTE	7,14%	0,45%	-3,48%	-0,80%
OREY	24,42%	0,39%	1,10%	-0,20%
COFINA	19,22%	-1,26%	-4,52%	-0,48%
BRISA	13,69%	0,80%	-0,65%	0,49%
ZON	9,43%	0,20%	-2,28%	-1,02%
BPI	18,70%	-0,58%	-3,24%	-1,53%
SONAE	16,51%	2,46%	-2,97%	0,02%
PORTUCEL	15,94%	-0,23%	-0,13%	0,72%
BCP	15,63%	0,16%	-2,14%	-0,96%
JERÓNIMO MARTINS	15,96%	3,99%	2,17%	3,04%
BES	6,13%	1,06%	-1,47%	-0,57%
PSI-20 TR	**2,36%**	**1,30%**	**-2,33%**	**-0,76%**
PROFITABILITY P3	**13,85%**	**0,65%**	**-1,17%**	**0,23%**
BETA P3	**0,63**	**0,89**	**0,89**	**0,90**
STANDARD DEVIATION P3	**2,50**	**3,65**	**3,34**	**3,17**
STANDARD DEVIATION PSI-20 TR	**1,51**	**0,64**	**2,31**	**2,54**
SHARPE'S INDEX				
P3	4,67%	0,12%	-0,42%	0,00%
PSI-20 TR	0,14%	1,71%	-1,11%	-0,39%
TREYNOR'S INDEX				
P3	18,65%	0,49%	-1,59%	0,01%
PSI-20 TR	0,21%	1,09%	-2,57%	-0,98%
JENSEN'S INDEX				
P3	11,70%	0,43%	-1,41%	0,01%
PSI-20 TR	0,21%	1,09%	-2,57%	-0,98%

Source: Own elaboration, July 2012.

Concerning *T1*, analysis to table 14 tell us that, in historical period, profitability obtained with this P3 is positive and much better than market profitability (13,85% face to 2,36%), were almost every shares highlighted due to their contribution. In follow-up periods, first and third follow-up periods, profitability was better than the market, in particular in third follow-up period: P3 obtained a profitability of 0,23% and market -0,76%. Once again, companies' financial performance can be very useful to decision maker, allowing him to make a good selection within all possible alternatives.

For P3, and answering to *T2*, Sharpe's index results tell us that expected differential return (portfolio profitability minus risk free) per unit of risk, is higher than market, except in first follow-up period. We found the most significant value in historical period were portfolio had 4,67% and the market had 0,14%. Treynor's index results, which measures excess of return per unit of risk, revels that except for the first follow-up period, P3 obtained better results than market: in historical period 18,65% face to 0,21% to the market, and in follow-up periods 0,49%, -1,59% and 0,01% face to 1,09%, -2,57% and -0,98% for the market. In all periods, historical and follow-up, portfolio had better values than the market. Finally, P3 over performed the market with higher values, except in first follow-up period. These results are supported by Jensen's index calculation, for instance, in historical period P3 had 11,70% and market had 0,21%, a significant difference.

Table 14 also contains information about risk between P3 and the market, measured by coefficient Beta. In this case, value goes from 0,63 to 0,90, meaning that both have the same risk. Except for the last follow-up period, in which portfolio have a positive profitability and market a negative one, tendency is the same.

Table nº 15 – Portfolio 3 (CAPM results)

FOLLOW-UP PORTFOLIO P3 - Main Results (CAPM METHODOLOGY)				
ASSETS	**HISTORICAL**	**FOLLOW-UP (MONTHLY)**		
	2002-2006	**Jan07-Dec07**	**Jan07-Dec08**	**Jan07-Dec09**
PROFITABILITY, BY ASSET (INCLUDES DIVIDENDS):				
BES	6,13%	1,44%	-2,84%	-1,49%
BPI	18,70%	0,55%	-4,59%	-2,45%
BRISA	13,69%	1,35%	-4,59%	-0,45%
CIMPOR	3,54%	1,31%	-2,10%	0,37%
COFINA	19,22%	0,36%	-5,85%	-1,40%
CORTIC. AMORIM	15,63%	0,47%	-3,48%	-1,92%
EDP	6,30%	1,89%	-1,17%	-0,24%
IBERSOL	19,52%	0,99%	-1,55%	-0,18%
JERÓNIMO MARTINS	15,96%	4,29%	0,75%	2,08%
MOTA ENGIL	31,53%	0,70%	-3,13%	-0,58%
OREY ANTUNES	24,42%	0,41%	-0,31%	-1,13%
PORTUCEL	15,94%	1,00%	-1,52%	-0,22%
PT	4,42%	1,30%	-0,68%	0,86%
REDITUS	7,58%	5,78%	3,11%	2,08%
SALVADOR CAETANO	16,07%	3,56%	1,30%	-0,76%
SEMAPA	17,54%	1,63%	-1,15%	-0,13%
SOARES DA COSTA	5,43%	11,57%	-0,30%	1,69%
SONAE	16,51%	3,23%	-4,32%	-0,91%
SONAE INDÚSTRIA	9,65%	1,18%	-6,38%	-2,95%
TEIXEIRA DUARTE	7,14%	4,03%	-4,82%	-1,72%
ZON	9,43%	1,01%	-3,64%	-1,92%
PSI-20 TR	**2,36%**	**1,30%**	**-2,33%**	**-0,76%**
PROFITABILITY P3	**13,54%**	**2,29%**	**-2,13%**	**-0,54%**
BETA P3	**0,69**	**0,94**	**0,87**	**0,93**
STANDARD DEVIATION P3	**2,32**	**3,39**	**3,19**	**3,04**
STANDARD DEVIATION PSI-20 TR	**1,51**	**0,64**	**2,31**	**2,54**
SHARPE'S INDEX				
P3	4,91%	0,61%	-0,74%	-0,25%
PSI-20 TR	0,14%	1,71%	-1,11%	-0,39%
TREYNOR'S INDEX				
P3	16,43%	2,20%	-2,72%	-0,82%
PSI-20 TR	0,21%	1,09%	-2,57%	-0,98%
JENSEN'S INDEX				
P3	11,39%	2,07%	-2,37%	-0,76%
PSI-20 TR	0,21%	1,09%	-2,57%	-0,98%

Source: Own elaboration, July 2012.

Table 15 express CAPM results, and after analyzing it we can infer that P3 is better than the market in all periods: 13,54% face to 2,36%, 2,29% face to 1,30%, -2,13% face to -2,33% and -0,54% face to -0,76%. The huge difference remains in historical period, were portfolio had a profitability of 13,54%, and market had 2,36%. To achieve this result, almost every share gave a positive contribution.

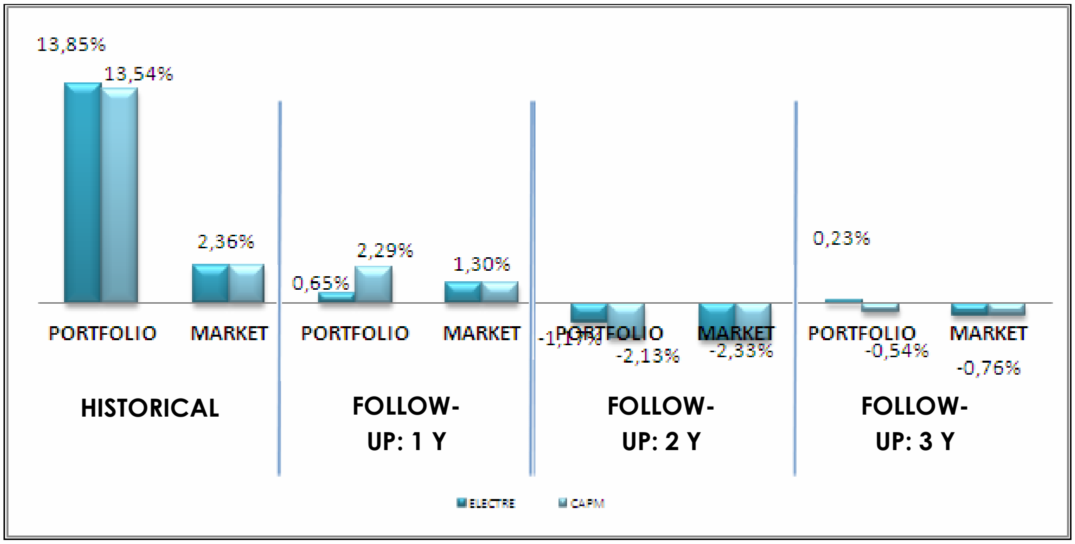
In line with results obtained with the others portfolios, also P3 achieved a higher expected differential return per unit of risk when compared with the market. This information, measure by Sharpe's index, is clearly visible in historical period, and in second and third follow-up periods. Although, the most significant difference remains, once again, in historical period were P3 had 4,91% and market had 0,14%. Concerning excess of return per unit of risk, Treynor's index indicates us that, generally, portfolio behaved better than market, being the most significant difference in historical and first follow-up periods: 16,43% and 2,20% for P3, and 0,21% and 1,09% for market. Finally, Jensen's index show us if portfolio had an extraordinary performance face to market. So, in all periods, P3 over performed the market with higher values, for instance, in historical period portfolio had 11,39% face to 0,21%.

As happened with previous portfolios, information about risk between P3 and the market, revels that both have the same risk, seeing Betas are comprehended between 0,69 to 0,94. Both profitability signs revels the same tendency.

From table 14 and table 15, we can concluded that portfolio composed by shares selected with ELECTRE III, and portfolio composed by shares selected with CAPM both performed better than the market in all periods. Besides all this, is also important to check witch portfolio behaved better, the portfolio constructed using ELECTRE or using CAPM.

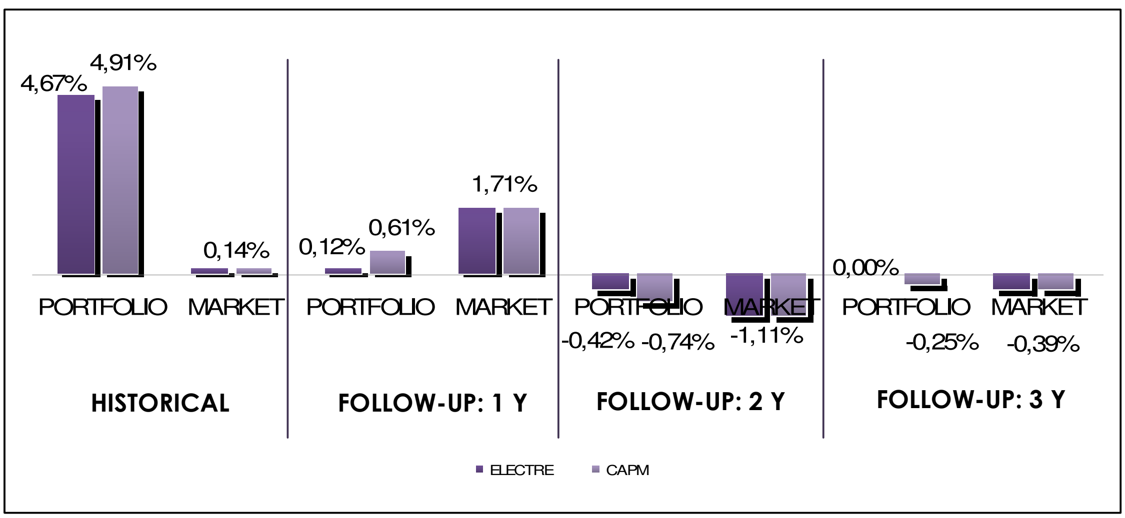
Looking to chart nº 14, in all periods, ELECTRE III generally performed better than CAPM, in particular in historical period, and in second and third follow-up periods. The most significant difference remains e the third follow-up period, were ELECTRE III obtained a positive profitability, and CAPM and the market both obtained negative profitability. So, to *T3*, we can conclude that in a holding period of 3 years, to P3, ELECTRE III behaved better than CAPM.

Chart nº 14 – Profitability of Portfolio 3



Source: Own elaboration, July 2012.

Chart nº 15 – Sharpe's Index of Portfolio 3



Source: Own elaboration, July 2012.

From chart nº 15, which sustain answer to *T4*, we can see that in historical and first follow-up period, CAPM had greater profitability by unit of risk than ELECTRE, but in second and third follow-up periods was ELECTRE to have higher values.

4.4.1.4. Main results obtained with Portfolio 4

Table nº 16 – Portfolio 4 (ELECTRE results)

FOLLOW-UP PORTFOLIO P4 - Main Results (ELECTRE METHODOLOGY)				
ASSETS SELECTED	HISTORICAL	FOLLOW-UP (MONTHLY)		
	2003-2007	Jan08-Dec08	Jan08-Dec09	Jan08-Dec10
PROFITABILITY, BY ASSET (INCLUDES DIVIDENDS):				
CIMPOR	12,45%	-4,03%	0,47%	-0,27%
ZON	23,64%	-7,29%	-3,08%	-2,72%
IBERSOL	27,29%	-4,44%	-1,01%	-0,89%
PT	14,06%	-2,05%	0,69%	0,72%
TEIXEIRA DUARTE	25,41%	-9,82%	-2,80%	-2,40%
SONAE	36,14%	-10,63%	-2,67%	-2,02%
REDITUS	27,80%	-1,94%	-0,95%	-0,77%
SEMAPA	26,44%	-2,40%	-0,37%	0,04%
COFINA	17,24%	-10,21%	-1,49%	-2,19%
BPI	25,41%	-8,41%	-3,49%	-3,51%
OREY	11,23%	-0,96%	-1,95%	-1,10%
JERÓNIMO MARTINS	30,12%	-2,24%	1,21%	2,28%
PORTUCEL	18,87%	-2,73%	-0,38%	0,22%
BRISA	16,64%	-4,74%	-1,23%	-1,57%
MOTA ENGIL	37,50%	-6,26%	-1,04%	-2,80%
PSI-20 TR	**13,44%**	**-5,83%**	**-1,78%**	**-1,45%**
PROFITABILITY P4	**23,35%**	**-5,21%**	**-1,21%**	**-1,13%**
BETA P4	**0,67**	**0,88**	**0,97**	**0,96**
STANDARD DEVIATION P4	**2,79**	**2,72**	**2,64**	**2,77**
STANDARD DEVIATION PSI-20 TR	**2,56**	**1,95**	**1,71**	**1,43**
SHARPE'S INDEX				
P4	7,61%	-2,02%	-0,54%	-0,47%
PSI-20 TR	4,41%	-3,14%	-1,17%	-1,14%
TREYNOR'S INDEX				
P4	31,85%	-6,22%	-1,48%	-1,36%
PSI-20 TR	11,30%	-6,11%	-2,01%	-1,63%
JENSEN'S INDEX				
P4	21,21%	-5,48%	-1,44%	-1,32%
PSI-20 TR	11,30%	-6,11%	-2,01%	-1,63%

Source: Own elaboration, July 2012.

Concerning *T1*, results obtained with P4 expressed in table 16, show us that in every period this portfolio had better profitability than the market. However, only in historical period profitability are positive (23,35% to P4 and 13,44% to market), being negative in all follow-up periods: -5,21% face to -5,83%, -1,21% face to -1,78% and -1,13% face to -1,45%. Generally, financial performance and market are aligned, despite the bad performance founded in follow-up periods. To achieve this profitability, positive and negative, almost every share gave their contribution.

Analyzing Sharpe's index, and giving answer to *T2*, we see that P4 behaved better than the market in every period, historical and follow-up, being the most significant difference in historical period, where portfolio had 7,61% and market had 4,41%. So, in historical period, premium risk by unit invested was clearly high in P4. Looking to Treynor's index, we easily see that excess of return per unit of risk is more significant for portfolio than for market, in all periods. Jensen's index show us that portfolio highlighted in every periods, over performing market with higher values. The most significant difference remains in historical period where P4 obtained 21,21% face to 11,30%.

To confirm tendency already identified in previous portfolios, also in P4 we found equal risk with the market. To asses this results, it's enough to look to Betas' values, which are between 0,67 and 0,97, and see tendency evidence by both profitabilities (in all periods tendency is the same).

Table nº 17 – Portfolio 4 (CAPM results)

FOLLOW-UP PORTFOLIO P4 - Main Results (CAPM METHODOLOGY)				
ASSETS	**HISTORICAL**	**FOLLOW-UP (MONTHLY)**		
	2003-2007	**Jan08-Dec08**	**Jan08-Dec09**	**Jan08-Dec10**
PROFITABILITY, BY ASSET (INCLUDES DIVIDENDS):				
BES	14,06%	-7,31%	-2,80%	-3,08%
BPI	25,41%	-9,36%	-3,49%	-3,51%
BRISA	16,64%	-4,73%	-1,23%	-1,57%
CIMPOR	12,45%	-4,44%	0,47%	-0,27%
COFINA	17,24%	-9,13%	-1,49%	-2,19%
CORTIC. AMORIM	22,57%	-6,53%	-2,99%	-1,28%
EDP	21,14%	-2,85%	-1,32%	-1,37%
IBERSOL	27,29%	-4,79%	-1,01%	-0,89%
JERÓNIMO MARTINS	30,12%	-1,40%	1,21%	2,28%
MOTA ENGIL	37,50%	-5,43%	-1,04%	-2,80%
OREY ANTUNES	11,23%	-1,12%	-1,95%	-1,10%
PORTUCEL	18,87%	-2,40%	-0,38%	0,22%
REDITUS	27,80%	-1,86%	-0,95%	-0,77%
TOYOTA	24,53%	0,39%	-3,08%	-3,38%
SEMAPA	26,44%	-1,32%	-0,37%	0,04%
SOARES DA COSTA	34,56%	-8,74%	-2,16%	-3,61%
SONAE	36,14%	-9,41%	-2,67%	-2,02%
TEIXEIRA DUARTE	25,41%	-9,28%	-2,80%	-2,40%
ZON	23,64%	-7,37%	-3,08%	-2,72%
PSI-20 TR	**13,44%**	**-5,83%**	**-1,78%**	**-1,45%**
PROFITABILITY P4	**23,84%**	**-5,11%**	**-1,64%**	**-1,60%**
BETA P4	**0,80**	**0,78**	**0,90**	**0,87**
STANDARD DEVIATION P4	**2,79**	**2,82**	**2,65**	**2,65**
STANDARD DEVIATION PSI-20 TR	**2,56**	**1,95**	**1,71**	**1,43**
SHARPE'S INDEX				
P4	7,79%	-1,91%	-0,71%	-0,67%
PSI-20 TR	4,41%	-3,14%	-1,17%	-1,14%
TREYNOR'S INDEX				
P4	27,15%	-6,93%	-2,08%	-2,04%
PSI-20 TR	11,30%	-6,11%	-2,01%	-1,63%
JENSEN'S INDEX				
P4	21,70%	-5,38%	-1,87%	-1,78%
PSI-20 TR	11,30%	-6,11%	-2,01%	-1,63%

Source: Own elaboration, July 2012.

Results obtained with CAPM are very close to the ones obtained with ELECTRE III. On one hand, in historical period, profitability of P4 is higher than the market, 23,84% face to 13,44%, where almost every shares gave a positive contribution. On the other hand, in follow-up periods, profitability of P4 is near the market, being all negative (-5,11% face to -5,83%, -1,64% face to -1,78% and -1,60% face to -1,45%). Once again, almost every share is responsible for these results.

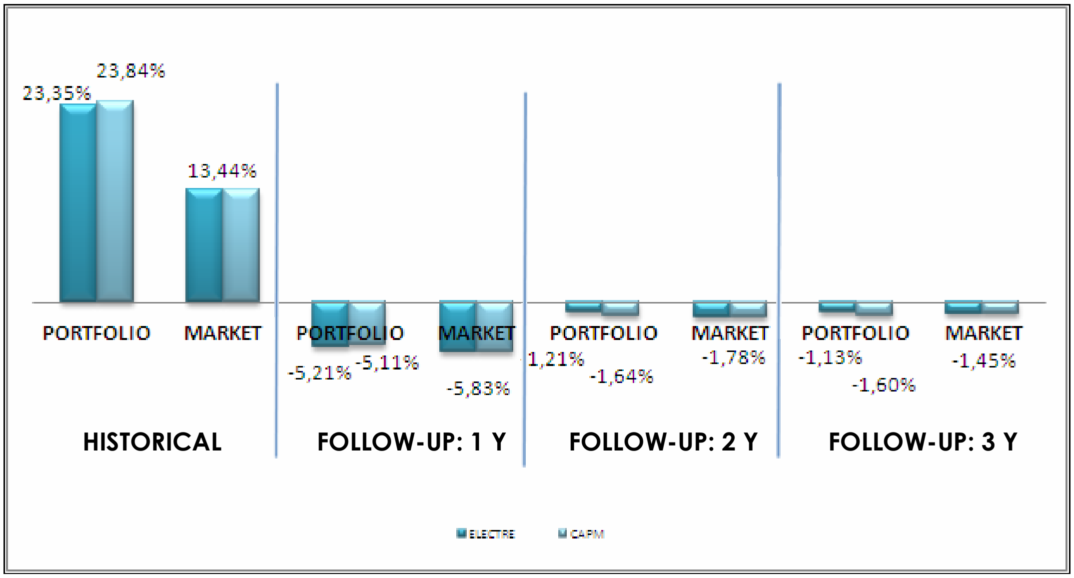
In all periods, Sharpe's index indicates that portfolio behaved better than market, having high expected differential return per unit of risk. For instance, in historical period, P4 obtained 7,79% and market 4,41%, and in first follow-up period, P4 had -1,91% and market -3,14%. For Treynor's index, P4 behaved better only in historical, obtaining a value of 27,15% face to the 11,30% from the market. In all follow-up periods, portfolio had lower values, despite differences are not very significant. Finally, Jensen's index shows us that, generally, portfolio had an extraordinary performance face to market. The most significant difference remains in historical period, were P4 had 21,70% and market had 11,30%.

From table 17, calculation of Betas confirm tendency already identified in previous portfolios, this is, P4 and market have the same risk, seeing the less value of Beta is 0,78 and the high value is 0,90. Besides that, in all periods profitabilities, portfolio and market, have the same tendency.

Tables 16 and 17 allow us to conclude that, in general, portfolios composed by shares selected with ELECTRE III, and composed by shares selected with CAPM both performed better than the market. We can also infer, from chart nº 16, that portfolio constructed using ELECTRE III performed better in second and third follow-up periods, and CAPM performed better in historical and first follow-up periods, despite differences are not very significant. So, we can conclude that in a holding period of 3 years, to P1, ELECTRE III behaved better than CAPM, and this way answering to *T3*.

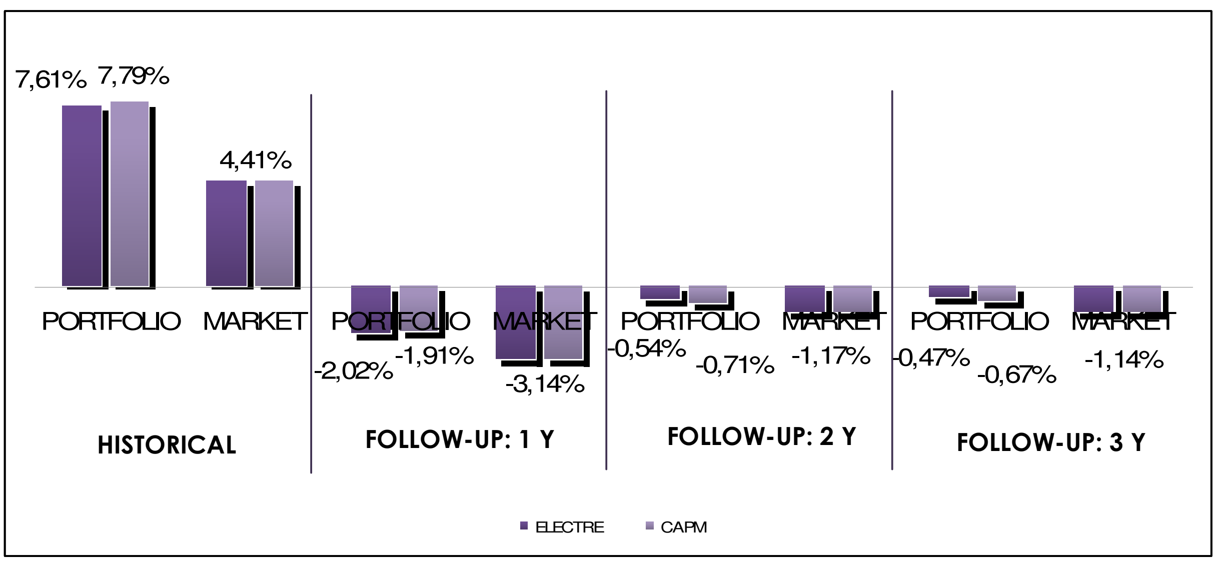
From chart nº 17, which sustain answer to *T4*, we can see that only in first follow-up period CAPM had less profitability by unit of risk than ELECTRE: -1,91% for CAPM and -2,02% for ELECTRE. In all other period, ELECTRE had more than CAPM.

Chart nº 16 – Profitability of Portfolio 4



Source: Own elaboration, July 2012.

Chart nº 17 – Sharpe's Index of Portfolio 4



Source: Own elaboration, July 2012.

4.4.1.5. Main results obtained with Portfolio 5

Table nº 18 – Portfolio 5 (ELECTRE results)

FOLLOW-UP PORTFOLIO P5 - Main Results (ELECTRE METHODOLOGY)				
ASSETS SELECTED	**HISTORICAL**	**FOLLOW-UP (MONTHLY)**		
	2004-2008	**Jan09-Dec09**	**Jan09-Dec10**	**Jan09-Dec11**
PROFITABILITY, BY ASSET (INCLUDES DIVIDENDS):				
CIMPOR	13,79%	5,51%	2,01%	1,55%
ZON	18,63%	1,69%	1,98%	1,85%
PT	13,55%	4,17%	3,04%	0,15%
REDITUS	35,62%	0,06%	-0,18%	-1,72%
BPI	9,13%	2,01%	-0,76%	-3,38%
IBERSOL	20,35%	2,60%	1,00%	-1,26%
JERÓNIMO MARTINS	37,16%	5,02%	4,86%	3,55%
OREY	22,17%	1,23%	0,04%	0,09%
SEMAPA	28,29%	1,99%	1,57%	-0,10%
SONAE	29,72%	6,36%	2,95%	0,59%
PORTUCEL	16,39%	2,45%	1,99%	0,95%
BRISA	16,83%	2,83%	0,49%	-1,67%
MOTA ENGIL	30,66%	4,73%	-0,73%	-1,71%
SOARES DA COSTA	32,84%	5,81%	-0,56%	-1,25%
PSI-20 TR	**9,09%**	**2,45%**	**0,82%**	**-0,39%**
PROFITABILITY P5	**27,13%**	**3,32%**	**1,27%**	**-0,17%**
BETA P5	**0,99**	**0,93**	**1,00**	**0,97**
STANDARD DEVIATION P5	**2,67**	**2,63**	**2,86**	**3,04**
STANDARD DEVIATION PSI-20 TR	**2,04**	**0,95**	**0,71**	**0,84**
SHARPE'S INDEX				
P5	9,27%	1,19%	0,39%	-0,10%
PSI-20 TR	3,28%	2,40%	0,97%	-0,64%
TREYNOR'S INDEX				
P5	25,02%	3,36%	1,13%	-0,33%
PSI-20 TR	6,68%	2,27%	0,68%	-0,54%
JENSEN'S INDEX				
P5	24,72%	3,13%	1,13%	-0,31%
PSI-20 TR	6,68%	2,27%	0,68%	-0,54%

Source: Own elaboration, July 2012.

Concerning *T1*, portfolio P5 obtained in all periods higher profitability than the market, being the most significant different founded in historical period. In concrete, in historical period P5 obtained 27,13% face to 9,09% obtained by the market; in follow-up periods, P5 reached profitabilities of 3,32%, 1,27% and -0,17% face to the market with 2,45%, 0,82% and -0,39%. In order to obtain these results, we cannot highlight any share in particular; they all contribute with positive and negative profitabilities.

Not different from other results, and answering to *T2*, Sharpe's index shows us that P5 had, in certain periods, a high expected differential return per unit of risk when compared with market. And this results are mostly visible in historical period, were P5 had 9,27% face to 3,28%. In first follow-up period, we also have a significant difference, but this time in favor of market: 1,19% to P5 and 2,40% to market. Also Treynor's index is in line with previous results, seeing in every periods P5 had better results than market, meaning that the excess of return per unit of risk is higher in P5 than in market: 25,02%, 3,36%, 1,13% and -0,33% for P5, and 6,68%, 2,27%, 0,68% and -0,54% for market. Jensen's index also shows us that P5 over performed market in all periods, especially in historical period. So, in historical period P5 had 24,72% and market had 6,68%.

Betas presented in table 18 tell us that risk between P5 and market is the same: the lower value of Beta is 0,93 and the higher value of Beta is 1,00. If we look to the sign of both profitabilities, in all periods portfolio and market have the same sign.

Table nº 19 – Portfolio 5 (CAPM results)

FOLLOW-UP PORTFOLIO P5 - Main Results (CAPM METHODOLOGY)				
ASSETS SELECTED	HISTORICAL	FOLLOW-UP (MONTHLY)		
	2004-2008	Jan09-Dec09	Jan09-Dec10	Jan09-Dec11
PROFITABILITY, BY ASSET (INCLUDES DIVIDENDS):				
BPI	4,97%	1,70%	-0,76%	-3,38%
BRISA	12,38%	1,75%	0,49%	-1,67%
CIMPOR	9,45%	3,19%	2,01%	1,55%
CORTIC. AMORIM	14,11%	0,62%	2,08%	2,01%
EDP	17,52%	1,35%	0,35%	0,34%
GRÃO-PARA	4,30%	1,52%	0,76%	-0,24%
IBERSOL	15,77%	2,75%	1,00%	-1,26%
JERÓNIMO MARTINS	31,93%	3,94%	4,86%	3,55%
MOTA ENGIL	25,68%	4,52%	-0,73%	-1,71%
PORTUCEL	11,95%	1,84%	1,99%	0,95%
PT	9,23%	3,59%	3,04%	0,15%
REDITUS	48,77%	-0,06%	-0,18%	-1,72%
TOYOTA	30,45%	-5,02%	-4,13%	-1,48%
SEMAPA	23,41%	0,71%	1,57%	-0,10%
SOARES DA COSTA	27,78%	5,82%	-0,56%	-1,25%
SONAE	24,78%	5,97%	2,95%	0,59%
TEIXEIRA DUARTE	10,32%	4,42%	1,61%	-2,81%
PSI-20 TR	**9,09%**	**2,45%**	**0,82%**	**-0,39%**
PROFITABILITY P5	**18,99%**	**2,27%**	**0,96%**	**-0,38%**
BETA P5	**0,92**	**1,03**	**0,93**	**0,88**
STANDARD DEVIATION P5	**2,59**	**2,55**	**2,74**	**2,89**
STANDARD DEVIATION PSI-20 TR	**2,04**	**0,95**	**0,71**	**0,84**
SHARPE'S INDEX				
P5	6,40%	0,82%	0,30%	-0,18%
PSI-20 TR	3,28%	2,40%	0,97%	-0,64%
TREYNOR'S INDEX				
P5	18,02%	2,04%	0,88%	-0,60%
PSI-20 TR	6,68%	2,27%	0,68%	-0,54%
JENSEN'S INDEX				
P5	18,02%	2,09%	0,82%	-0,53%
PSI-20 TR	6,68%	2,27%	0,68%	-0,54%

Source: Own elaboration, July 2012.

Portfolio constructed under CAPM assumptions only behaved better than the market in historical period, although differences in follow-up periods are not very significant. For instance, in historical period P5 profitability was 18,89% and market obtained 9,09%, an huge difference; in follow-up periods, P5 reach 2,27%, 0,96% and -0,38% face to 2,45%, 0,82% and -0,39%. Once again, P5 performed better than the market, and we cannot point out any share in particular.

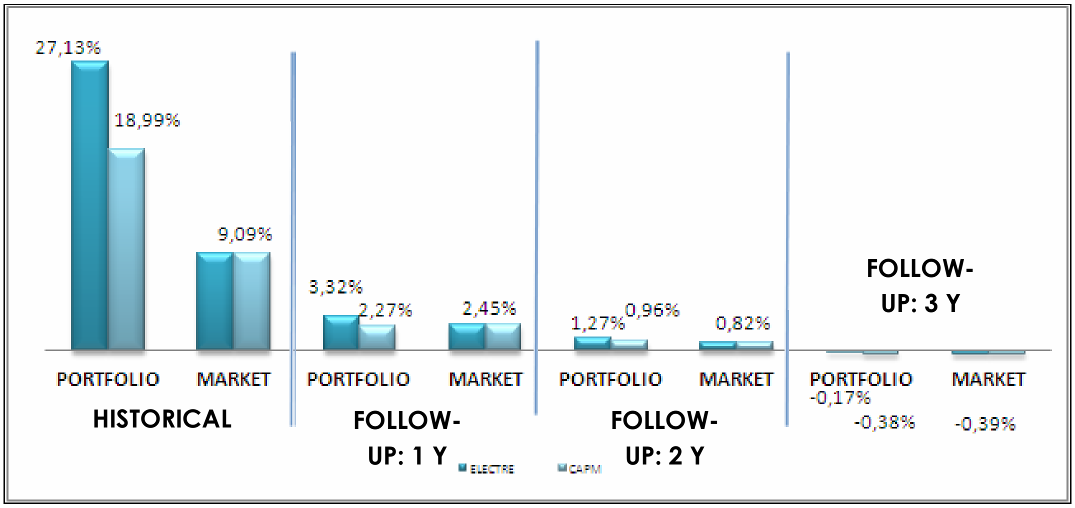
Although results obtained with CAPM are different from results obtained with ELECTRE, in general they are in line with conclusions already presented. In this sense, when analyzing Sharpe's index results, which captures the expected differential return (portfolio profitability minus risk free) per unit of risk, portfolio performed better in historical and third follow-up periods. In concrete, P5 obtained 6,40% and -0,18% in historical and third follow-up periods respectively, and market obtained 3,28% and -0,64% for the same periods. Looking to Treynor's index results, only in historical and second follow-up periods, portfolio was better than market: 18,02% face to 6,68%, a huge difference in historical period. In first and third follow-up periods differences are not very significant but exist. Interpreting Jensen's index results, we easily see that the big difference remains in historical period, were P5 obtained 18,02% and market 6,68%. This means that P5 over performed market in historical period, maintaining this tendency in almost every follow-up periods, second and third.

As happened with previous portfolios, P5 and market have the same risk: Betas vary between 0,92 and 1,03, and profitabilities tendencies corroborated this conclusion.

Comparing results obtained with table 18 and table 19, witch are visually expressed in chart nº 18, we can concluded that portfolio composed by shares selected with ELECTRE III, and portfolio composed by shares selected with CAPM both performed better than the market in all periods. We can also infer, from this chart, that portfolio constructed using ELECTRE III performed better than portfolio using CAPM, in all periods. So, we can conclude that in a holding period of 3 years, to P5, ELECTRE III behaved better than CAPM, allow us to answer to *T3*.

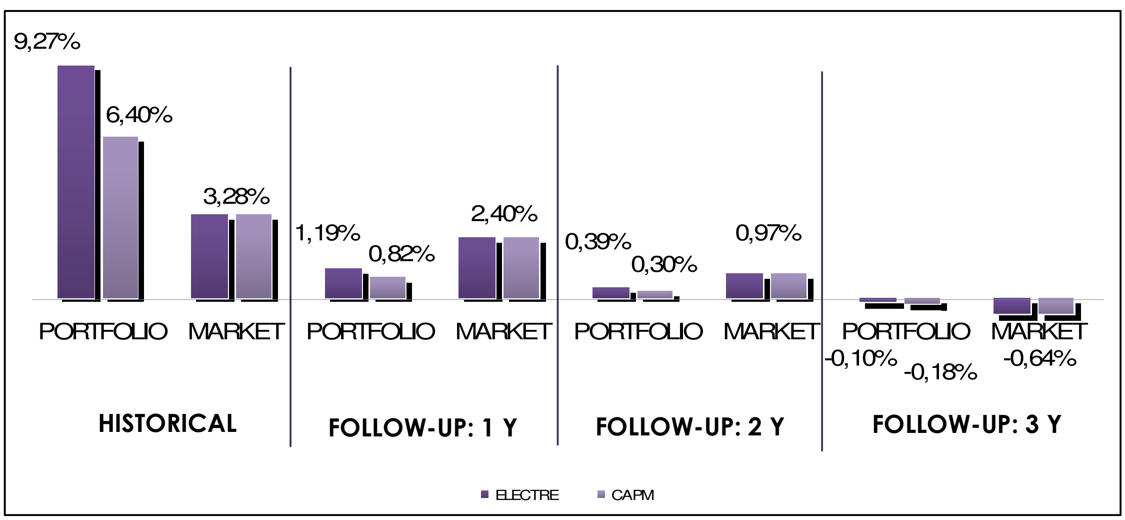
From chart nº 19, which sustain answer to *T4*, we can see that in all periods, ELECTRE had high profitability by unit of risk than CAPM: for ELECTRE, 9,27%, 1,19%, 0,39% and -0,10%, and to CAPM, 6,40%, 0,82%, 0,30% and -0,18%.

Chart nº 18 – Profitability of Portfolio 5



Source: Own elaboration, July 2012.

Chart nº 19 – Sharpe's Index of Portfolio 5



Source: Own elaboration, July 2012.

4.4.1.6. Main results obtained with Portfolio 6

Table nº 20 – Portfolio 6 (ELECTRE results)

FOLLOW-UP PORTFOLIO P6 - Main Results (ELECTRE METHODOLOGY)			
ASSETS SELECTED	**HISTORICAL**	**FOLLOW-UP (MONTHLY)**	
	2005-2009	**Jan10-Dec10**	**Jan10-Dec11**
PROFITABILITY, BY ASSET (INCLUDES DIVIDENDS):			
CIMPOR	22,31%	-1,53%	-0,51%
PT	21,30%	1,54%	-2,04%
JERÓNIMO MARTINS	43,49%	4,57%	2,72%
ZON	0,54%	-1,83%	-2,24%
BPI	9,16%	-3,54%	-6,02%
PORTUCEL	23,01%	1,44%	0,05%
IBERSOL	30,83%	-0,62%	-3,16%
REDITUS	47,21%	-0,41%	-2,59%
MOTA ENGIL	33,69%	-6,06%	-4,96%
EDP	25,68%	-1,21%	-0,56%
TEIXEIRA DUARTE	9,70%	-1,54%	-6,44%
BRISA	18,11%	-2,05%	-3,99%
SEMAPA	31,66%	0,99%	-1,26%
SOARES DA COSTA	33,75%	-6,69%	-4,67%
PSI-20 TR	**-0,08%**	**-0,78%**	**-1,79%**
PROFITABILITY P6	**25,03%**	**-1,21%**	**-2,55%**
BETA P6	**0,98**	**0,96**	**0,90**
STANDARD DEVIATION P6	**2,78**	**2,93**	**3,13**
STANDARD DEVIATION PSI-20 TR	**2,04**	**0,35**	**0,80**
SHARPE'S INDEX			
P6	8,12%	-0,44%	-0,85%
PSI-20 TR	-1,26%	-2,49%	-2,39%
TREYNOR'S INDEX			
P6	22,99%	-1,36%	-2,96%
PSI-20 TR	-2,57%	-0,87%	-1,91%
JENSEN'S INDEX			
P6	22,54%	-1,30%	-2,67%
PSI-20 TR	-2,57%	-0,87%	-1,91%

Source: Own elaboration, July 2012.

Relatively to *T1*, from table 20 we can conclude that P6 obtained, in historical period, profitability much higher than the market, 25,03% face to -0,08%. In follow-up period, both profitabilities are negative, being more negative for P6. From all shares that compose this portfolio, in historical period we cannot highlight any share in particular: in general all had good profitability.

Despite conclusions are similar to the ones taken previously, and answering to *T2*, results for Sharpe's index evidence more significant differences, in favor of portfolio, for all periods. In concrete, portfolio had 8,12%, -0,44% and -0,85% for historical and follow-up periods respectively, and market had -1,26%, -2,49% and -2,39% for the same periods. In short, portfolio had an expected differential return per unit of risk more significant than market. In Treynor's index, only in historical period P6 had a high excess of return per unit of risk when compared with market: 22,99% face to -2,57%. In follow-up periods, market obtained a high excess of return per unit of risk, despite being negative. Finally, Jensen's index indicates that portfolio over performed market in historical period, were P6 obtained 22,54% and market -2,57%. In follow-up we found an inverse situation, were market over performed portfolio, although being negative.

Table nº 21 – Portfolio 6 (CAPM results)

FOLLOW-UP PORTFOLIO P6 - Main Results (CAPM METHODOLOGY)			
ASSETS	**HISTORICAL**	**FOLLOW-UP (MONTHLY)**	
	2005-2009	**Jan10-Dec10**	**Jan10-Dec11**
PROFITABILITY, BY ASSET (INCLUDES DIVIDENDS):			
BRISA	19,87%	-2,48%	-3,99%
CIMPOR	24,13%	-2,42%	-0,51%
EDP	27,55%	-1,64%	-0,56%
IBERSOL	32,78%	-1,64%	-3,16%
JERÓNIMO MARTINS	45,63%	3,76%	2,72%
MOTA ENGIL	35,68%	-5,57%	-4,96%
OREY ANTUNES	23,38%	0,44%	-1,27%
PORTUCEL	24,85%	1,69%	0,05%
PT	23,11%	2,20%	-2,04%
REDITUS	49,41%	-0,41%	-2,59%
SEMAPA	33,63%	0,61%	-1,26%
SOARES DA COSTA	35,74%	-5,08%	-4,67%
SONAE	24,63%	-0,54%	-2,33%
PSI-20 TR	**-0,08%**	**-0,78%**	**-1,79%**
PROFITABILITY P6	**30,80%**	**-0,85%**	**-1,89%**
BETA P6	**0,88**	**0,80**	**0,82**
STANDARD DEVIATION P6	**2,70**	**3,02**	**3,20**
STANDARD DEVIATION PSI-20 TR	**2,04**	**0,35**	**0,80**
SHARPE'S INDEX			
P6	10,48%	-0,31%	-0,63%
PSI-20 TR	-1,26%	-2,49%	-2,39%
TREYNOR'S INDEX			
P6	32,29%	-1,18%	-2,47%
PSI-20 TR	-2,57%	-0,87%	-1,91%
JENSEN'S INDEX			
P6	28,31%	-0,94%	-2,02%
PSI-20 TR	-2,57%	-0,87%	-1,91%

Source: Own elaboration, July 2012.

Results obtained with CAPM estimation are better than the market, in historical period, and close in follow-up periods, despite negative performance. This difference is largely identified in historical period, were P6 obtained a profitability of 30,80% and market obtained -0,08%. In order to obtain these results, no shares stood out, this is, in historical period in general all obtained good performance, and in follow-up period almost obtained negative performances.

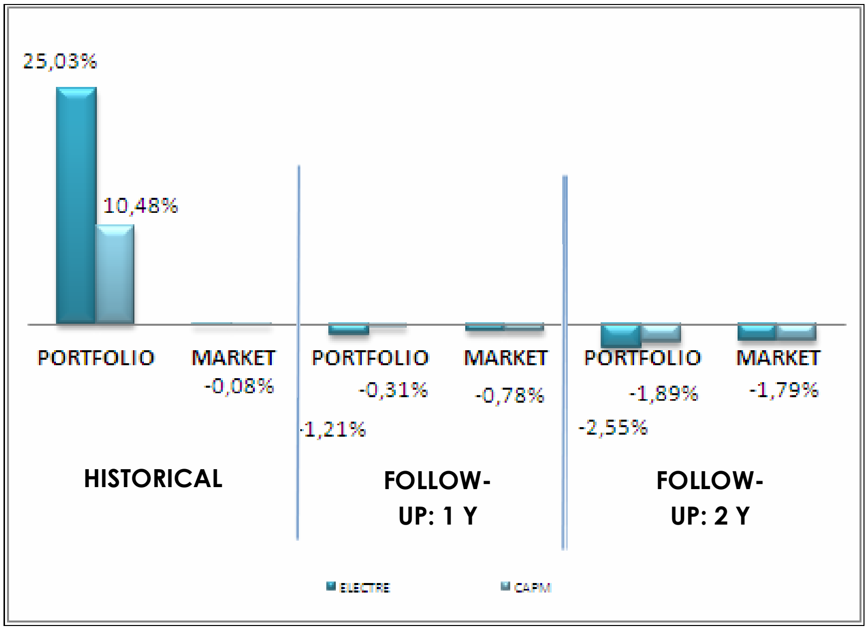
Concerning Sharpe's index, portfolio behaved better than the market in all periods, with significant differences. This means that expected differential return per unit of risk in better for portfolio in all periods, for example, P6 obtained 10,48% and market -1,26%. In follow-up periods, differences are also very significant: -0,31% face to -2,49%, and 0,63% face to -2,39%. Treynor's index, measuring excess of return per unit of risk, in bigger for portfolio in historical period because this one obtained 32,29% and market -2,57%. In follow-up periods, P6 behaved worst than market. Finally, Jensen's index show us that portfolio had an extraordinary performance face to market in historical period (28,31% face to -2,57%), but this tendency do not verify in follow-up periods.

As happened with previous portfolios, P6 and the market have the same risk in all periods (the lower value of Beta is 0,80, and the higher value is 0,88), although in historical period results revels the opposite: profitability of the market is negative, and profitability of P6 is very positive.

As expressed in chart 20, and in tables 20 and table 21, we can concluded that portfolio composed by shares selected with ELECTRE III, and portfolio composed by shares selected with CAPM both performed better than the market in historical period. We can also see, from this chart, that portfolio constructed using CAPM performed better than portfolio using ELECTRE III in follow-up periods, but not in historical period, were ELECTRE III obtained a very good performance. So, we can conclude that in a holding period of 3 years, to P6, CAPM behaved better than ELECTRE III, answering to *T3*.

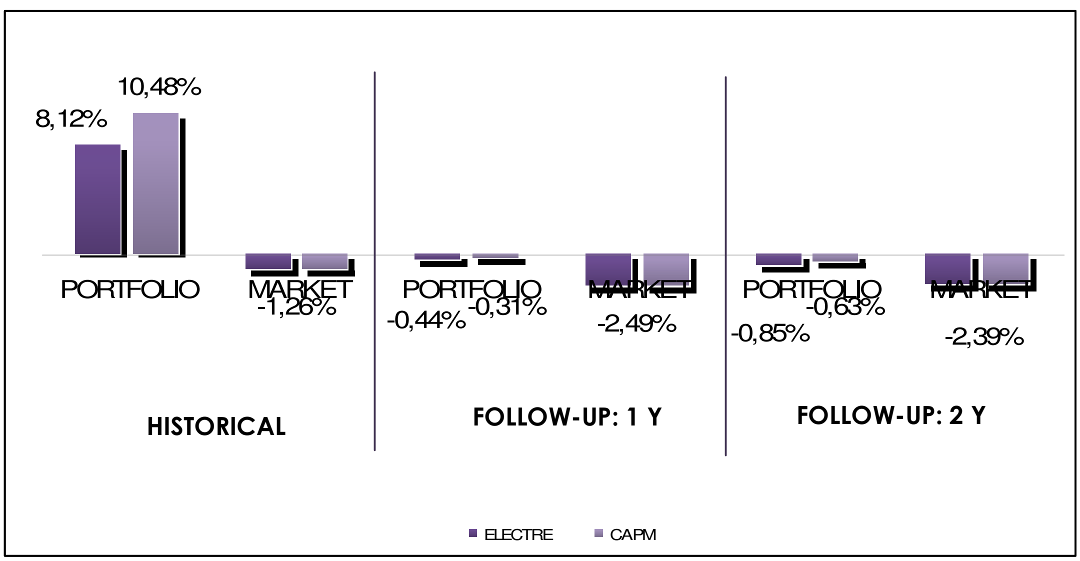
From chart nº 21, which sustain answer to *T4*, we can see that in all periods, CAPM had high profitability by unit of risk than ELECTRE, for instance, ELECTRE had 8,12%, -0,44% and -0,85%, and CAPM had 10,48%, -0,31% and -0,63%.

Chart nº 20 – Profitability of Portfolio 6



Source: Own elaboration, July 2012.

Chart nº 21 – Sharpe's Index of Portfolio 6



Source: Own elaboration, July 2012.

4.4.1.7. Main results obtained with Portfolio 7

Table nº 22 – Portfolio 7 (ELECTRE results)

FOLLOW-UP PORTFOLIO P7 - Main Results (ELECTRE METHODOLOGY)		
ASSETS SELECTED	**HISTORICAL**	**FOLLOW-UP (MONTHLY)**
	2006-2010	**Jan11-Dec11**
PROFITABILITY, BY ASSET (INCLUDES DIVIDENDS):		
PT	9,21%	-5,40%
BRISA	-4,13%	-5,85%
JERÓNIMO MARTINS	25,52%	0,93%
IBERSOL	4,22%	-5,63%
ZON	-17,95%	-2,94%
PORTUCEL	5,87%	-1,27%
CIMPOR	2,01%	0,56%
MOTA ENGIL	-3,57%	-4,25%
EDP	2,88%	0,17%
SEMAPA	6,96%	-3,42%
BPI	-13,73%	-8,42%
REDITUS	10,99%	-4,72%
PSI-20 TR	**-0,52%**	**-2,78%**
PROFITABILITY P7	**2,36%**	**-3,35%**
BETA P7	**0,77**	**0,66**
STANDARD DEVIATION P7	**2,55**	**3,19**
STANDARD DEVIATION PSI-20 TR	**2,08**	**0,96**
SHARPE'S INDEX		
P7	0,00%	-1,10%
PSI-20 TR	-1,38%	-3,07%
TREYNOR'S INDEX		
P7	0,00%	-5,35%
PSI-20 TR	-2,88%	-2,94%
JENSEN'S INDEX		
P7	0,00%	-3,52%
PSI-20 TR	-2,88%	-2,94%

Source: Own elaboration, July 2012.

From table nº 22, and answering to *T1*, P7 obtained a profitability of 2,36% face to -0,52% obtained by the market, being the shares from Jerónimo Martins the most responsible by these results with a profitability of 25,52%.

Concerning *T2*, once again Sharpe's index indicates P7 behaved better than the market in all periods: 0,00% face to -1,38% in historical period, and -1,10% face to -3,07% in follow-up period. With this information we can say that expected differential return per unit of risk is high in P7 than in market. Treynor's index give information about excess of return per unit of risk, and results show us that P7 had better values than market in historical period (0,00% face to -2,88%, a significant difference). But in follow-up period results are very negative to portfolio with -5,33% and to market with -2,94%, despite being less negative. Concerning performance, in the sense of obtaining extraordinary performance, Jensen's index tells us that in historical period portfolio over performed market (0,00% face to -2,88%), but in follow-up period situation inverted, becoming market the one with better performance (-3,52% for portfolio and -2,94% for market).

Looking to Betas expressed in the same table, 0,66 in historical period, and 0,77 in follow-up period, meaning P7 and the market have the same risk, despite the tendency founded in historical period: profitability of the market is negative, and profitability of P6 is very positive.

Table nº 23 – Portfolio 7 (CAPM results)

FOLLOW-UP PORTFOLIO P7 - Main Results (CAPM METHODOLOGY)		
ASSETS	HISTORICAL	FOLLOW-UP (MONTHLY)
	2006-2010	Jan11-Dec11
PROFITABILITY, BY ASSET (INCLUDES DIVIDENDS):		
CIMPOR	5,80%	0,66%
EDP	6,70%	-0,34%
IBERSOL	8,09%	-4,08%
JERÓNIMO MARTINS	30,18%	2,17%
PORTUCEL	9,81%	-1,02%
PT	13,27%	-4,66%
REDITUS	15,12%	-6,31%
SEMAPA	10,94%	-2,45%
SOARES DA COSTA	22,77%	-5,10%
SONAE	2,95%	-3,99%
PSI-20 TR	**-0,52%**	**-2,78%**
PROFITABILITY P7	**12,56%**	**-2,51%**
BETA P7	**0,93**	**0,82**
STANDARD DEVIATION P7	**2,95**	**3,56**
STANDARD DEVIATION PSI-20 TR	**2,08**	**0,96**
SHARPE'S INDEX		
P7	3,46%	-0,75%
PSI-20 TR	-1,38%	-3,07%
TREYNOR'S INDEX		
P7	10,92%	-3,27%
PSI-20 TR	-2,88%	-2,94%
JENSEN'S INDEX		
P7	10,20%	-2,67%
PSI-20 TR	-2,88%	-2,94%

Source: Own elaboration, July 2012.

Results obtained with CAPM estimation are largely better than the market, in historical period: P7 obtained a profitability of 12,56% and market obtained -0,52%. All shares selected gave an important (positive) contribution, in particular Jerónimo Martins' shares.

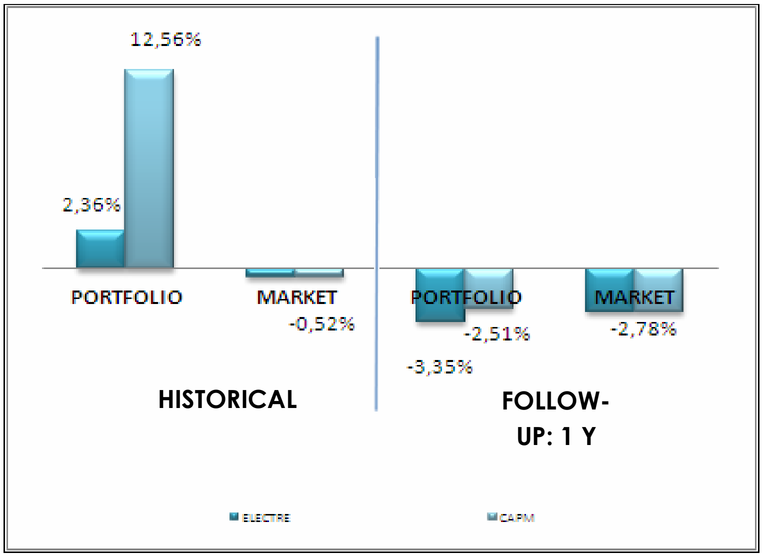
Interpreting Sharpe's index results, we can say that P7 had a better expected differential return (portfolio profitability minus risk free) per unit of risk when compared with market (3,46% face to -1,38% for historical period, and -0,75% face to -3,07% in follow-up period). Concerning Treynor's index, only in historical period P7 had a high a measure of excess of return per unit of risk (10.91% face to -2,88%). In follow-up period, this conclusion do not verify (-3,27% for P7 and -2,94% for market). Finally, Jensen's index shows us that portfolio had an extraordinary performance over market in every period: 10,20% and -2,67% in historical and follow-up periods respectively and -2,88% and -2,94% for the same periods.

Betas indicated in table 23 express the same risk between P7 and market: 0,93 for historical period and 0,82 for follow-up period. But, in historical period tendency founded in profitabilities revels the opposite, seeing profitability of the market is negative, and profitability of P7 is positive.

Analyzing chart 22, we can concluded that portfolio composed by shares selected with ELECTRE III, and portfolio composed by shares selected with CAPM both performed better than the market. We can also see, from this chart, that portfolio constructed using CAPM performed better than portfolio using ELECTRE, existing a large difference between both profitabilities in historical period: CAPM obtained 12,56% and ELECTRE obtained 2,36%. So, we can conclude that in a holding period of 3 years, to P7, CAPM behaved better than ELECTRE III, answering to *T3*.

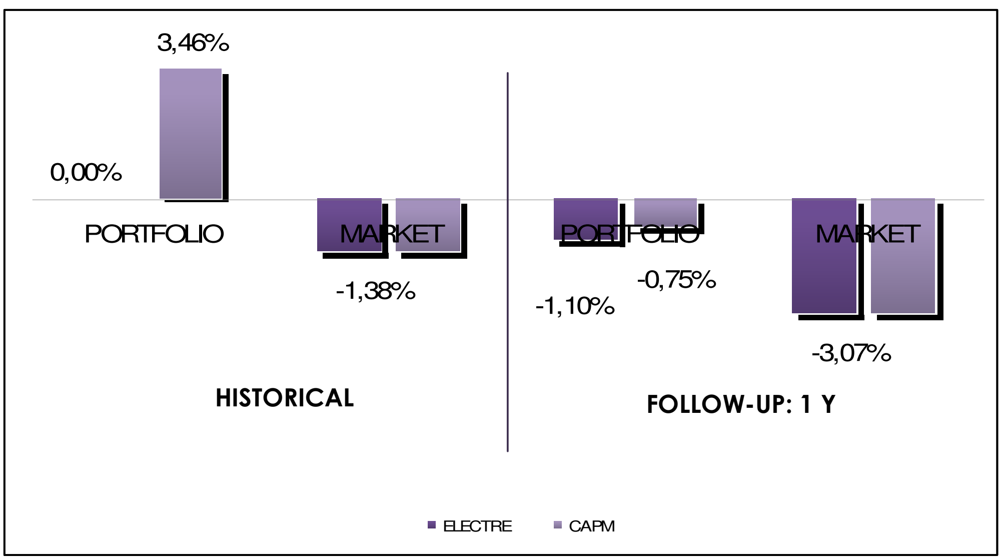
From chart nº 23, which sustain answer to *T4*, we can see that in historical period CAPM had greater profitability by unit of risk than ELECTRE: 3,46% to CAPM face to 0,00% to ELECTRE. In follow-up period, ELECTRE had more than CAPM: -0,75% for CAPM and -1,10% for ELECTRE.

Chart nº 22 – Profitability of Portfolio 7



Source: Own elaboration, July 2012.

Chart nº 23 – Sharpe's Index of Portfolio 7



Source: Own elaboration, July 2012.

4.4.1.8. Main results obtained with Portfolio 8

Table nº 24 – Portfolio 8 (ELECTRE results)

FOLLOW-UP PORTFOLIO P8 - Main Results (ELECTRE METHODOLOGY)	
ASSETS SELECTED	**HISTORICAL**
	2007-2011
PROFITABILITY, BY ASSET (INCLUDES DIVIDENDS):	
PT	-18,00%
JERÓNIMO MARTINS	8,21%
BRISA	-32,57%
IBERSOL	-25,06%
PORTUCEL	-18,88%
ZON	-36,04%
CIMPOR	-18,61%
EDP	-20,83%
MOTA ENGIL	-35,33%
SEMAPA	-22,29%
BPI	-42,32%
OREY	-27,81%
CORTIC. AMORIM	-24,48%
COFINA	-38,19%
SONAE	-28,11%
PSI-20 TR	**-8,82%**
PROFITABILITY P8	**-25,36%**
BETA P8	**0,74**
STANDARD DEVIATION P8	**2,99**
STANDARD DEVIATION PSI-20 TR	**2,43**
SHARPE'S INDEX	
P8	-9,21%
PSI-20 TR	-4,54%
TREYNOR'S INDEX	
P8	-37,23%
PSI-20 TR	-11,05%
JENSEN'S INDEX	
P8	-27,58%
PSI-20 TR	-11,05%

Source: Own elaboration, July 2012.

To the last portfolio studied, and answering to *T1*, P8 obtained a profitability of -25,36% face to -8,82% obtained by the market, in which almost every share gave a negative contribution except Jerónimo Martins, which contribute with a positive profitability of 8,21%.

Concerning *T2*, and for the last portfolio, results are different from the others. In historical period, and using Sharpe's index, P8 had an expected differential return per unit of risk very lower, and negative, when compared with market: -9,21% face to -4,54%. Concerning Treynor's index, the excess of return per unit of risk in P8 is much lower than market, being the difference very significant (-37,23% face to -11,05%). Finally, Jensen's index show us that portfolio underperformed the market with negative and lower values, in concrete, -27,58% face to -11,05%.

Betas expressed in table 24 shows that P8 and the market have the same risk, with a value of 0,74.

Table nº 25 – Portfolio 8 (CAPM results)

FOLLOW-UP PORTFOLIO P8 - Main Results (CAPM METHODOLOGY)	
ASSETS	**HISTORICAL**
	2007-2011
PROFITABILITY, BY ASSET (INCLUDES DIVIDENDS):	
CIMPOR	1,39%
JERÓNIMO MARTINS	34,80%
PORTUCEL	1,06%
PT	2,15%
REDITUS	5,18%
PSI-20 TR	**-8,82%**
PROFITABILITY P8	**8,91%**
BETA P8	**0,66**
STANDARD DEVIATION P8	**2,53**
STANDARD DEVIATION PSI-20 TR	**2,43**
SHARPE'S INDEX	
P8	2,64%
PSI-20 TR	-4,54%
TREYNOR'S INDEX	
P8	10,16%
PSI-20 TR	-11,05%
JENSEN'S INDEX	
P8	6,68%
PSI-20 TR	-11,05%

Source: Own elaboration, July 2012.

Results obtained with CAPM estimation are better than the market: P8 obtained a profitability of 8,91% and market obtained -8,82%. Once again Jerónimo Martins highlighted with a profitability of 34,80%.

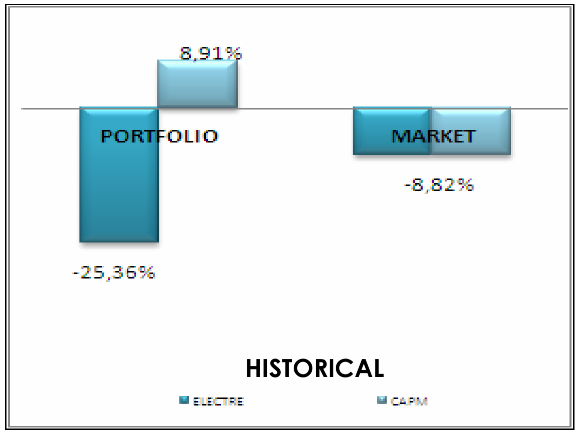
With CAPM, Sharpe's index results are in line with results previously obtained, this is, portfolio had a higher expected differential return per unit of risk, for instance, P8 had 2,64% and market had -4,54%. For Treynor's index results, P8 presented a high excess of return per unit of risk: 10,16% for P8 and -11,05% for market. Finally, Jensen's index results tell us that P8 had an extraordinary performance face to market: 6,68% face to -11,05%.

As founded to ELECTRE III, coefficient Beta calculated to CAPM, express the same conclusion: a value of 0,66 revels the same risk between portfolio and market. However, tendency founded in profitabilities shows another tendency: profitability of P8 is positive and market profitability is negative.

Analyzing chart 24, we can concluded that portfolio composed by shares selected with ELECTRE III behaved worst than the market, and portfolio composed by shares selected with CAPM performed better than the market. We can also see, from this chart, that portfolio constructed using CAPM performed much better than portfolio using ELECTRE, existing a large difference between both profitabilities: CAPM obtained 8,91% and ELECTRE obtained -25,36%. In order to answer to *T3*, we can conclude that in a holding period of 3 years, to P8, CAPM behaved better than ELECTRE III.

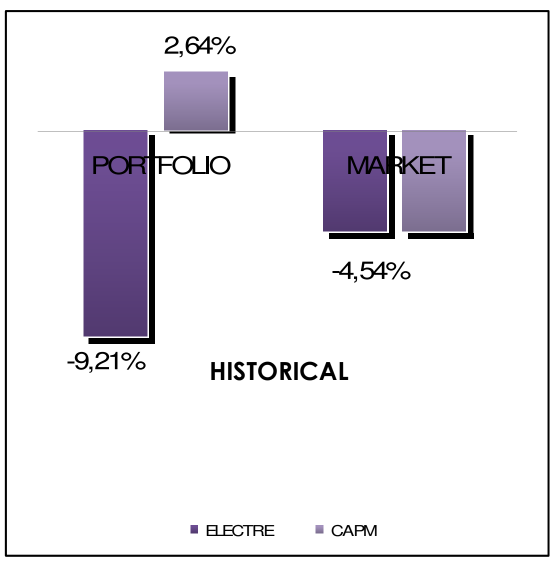
From chart nº 25, which sustain answer to *T4*, we can see that CAPM had high profitability by unit of risk than ELECTRE, for instance, -9,21% for ELECTRE and 2,64% for CAPM.

Chart nº 24 – Profitability of Portfolio 8



Source: Own elaboration, July 2012.

Chart nº 25 – Sharpe's Index of Portfolio 8



Source: Own elaboration, July 2012.

4.4.2. Statistic analysis: parametric and nonparametric tests results

Parametric and nonparametric testes are useful to test the significance of assumptions or factors that may influence the behavior of variable of reference, where the intend is to test whether or not that assumption or factor had a significant effect. To do so, there have two methods: *parametric tests* (requiring that the shape of the sampling distribution is known, usually Normal distribution), and *nonparametric tests* (they do not require the knowledge of sampling distribution). Whenever possible, one should use parametric tests in place of nonparametric tests, because first tests allow us to obtain more robust results (Maroco, 2007).

In order to apply parametric and nonparametric tests, we calculate monthly average profitability and monthly Sharpe's index, by holding period, for PSI, ELECTRE and CAPM, and compare means between them. So, we form three groups: group 1 – PSI, group 2 – ELECTRE, and group 3 – CAPM.

4.4.2.1. Statistics results obtained with follow-up periods - unit sample

After making a descriptive analysis by portfolio, in order to answer to empirical tests enunciated at the beginning of this chapter, it's time to make the same tests but this time using a statistic analysis. In order to do so, we calculate average profitability and Sharpe's index for each holding period, being results expressed in tables and chart below. This procedure was applied only to holding/follow-up periods, because these are the ones that, actually, allow us to monitorized behavior of PSI, ELECTRE and CAPM, in the long term (three years).

Table nº 26 – Follow-up results, by group (1 year)

Follow-up Results - 1 y			
	PSI-1Y	ELECTRE-1Y	CAPM-1Y
AVERAGE PROF.	-0,2060%	0,1093%	0,4407%
SHARPE'S INDEX	-6,77%	-1,27%	4,58%
Nº OBSERVATIONS	84	84	84

Source: Own elaboration, September 2012.

Table nº 27 – Follow-up results, by group (2 years)

Follow-up - 2 y			
	PSI-2Y	ELECTRE-2Y	CAPM-2Y
AVERAGE PROF.	-0,2060%	0,2264%	0,2236%
SHARPE'S INDEX	-6,8551%	0,6226%	0,6244%
Nº OBSERVATIONS	84	144	144

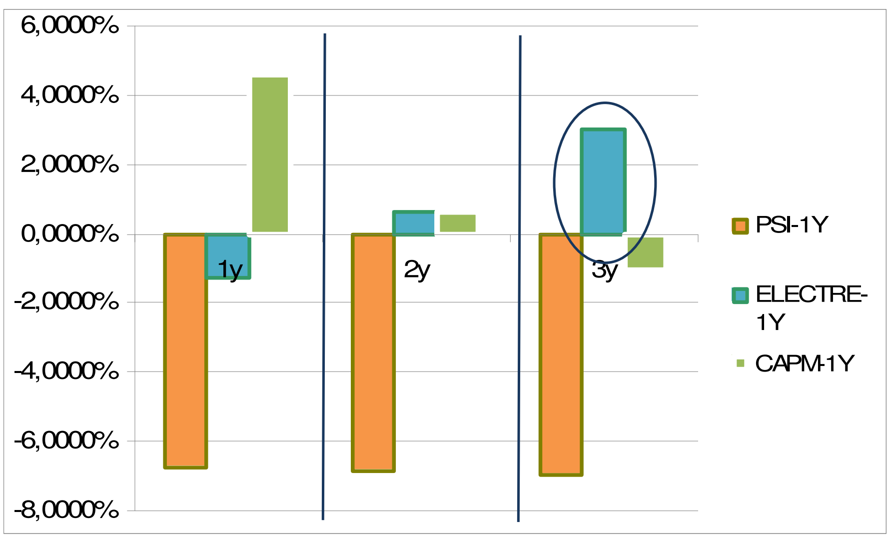
Source: Own elaboration, September 2012.

Table nº 28 – Follow-up results, by group (3 years)

Follow-up - 3 y			
	PSI-3Y	ELECTRE-3Y	CAPM-3Y
AVERAGE PROF.	-0,2060%	0,3902%	0,1331%
SHARPE'S INDEX	-6,9733%	3,0272%	-1,0422%
Nº OBSERVATIONS	84	180	180

Source: Own elaboration, September 2012.

Chart nº 26 – Sharpe's Index results, by period/group



Source: Own elaboration, September 2012.

From tables 26, 27 and 28 jointly interpreted with chart nº 26, we can easily see that in all follow-up holding periods ELECTRE III and CAPM had better Sharpe's index results than market. But, between ELECTRE III and CAPM results are different depending on the period analyzed. So, monitoring a holding period of 1 year, CAPM behaved much better than ELECTRE (4,58% face to -1,27%), and average profitability confirms that. In a holding period of 2 years, results are very similar, with a slight difference in favor of CAPM (0,6226% for ELECTRE and 0,6244% to CAPM). Concerning average profitability, there is also a very small difference in favor of ELECTRE. But the big difference remains in a holding period of 3 years, were ELECTRE III clearly is able to obtain differential return per unit of risk much high than CAPM: 3,0272% for ELECTRE III and -1,0422% for CAPM. Average profitability confirms these conclusions. These results show us that there is no doubt that, in long run, select assets to invest based on a multicriteria method, ELECTRE III in our study, proved to be a good choice, better than traditional methods as CAPM, and better than the market (PSI-20TR).

4.4.2.1.1. Parametric tests

In order to apply this kind of tests, two conditions must verify: (1) depend variable have normal distribution, and (2) population variance is homogeneous, when we are comparing two or more variables.

To test normality, the most used test is the Kolmogorov-Smirnov test[29], or alternatively, Shapiro-Wilk test (Shapiro and Wilk, 1965). To homogeneity in variance, usually Levene test (Levene, 1969) is the most used. We start by testing if PSI-20 TR, ELECTRE and CAPM follows, or not, a normal distribution.

[29] Usually referred as K-S test.

a) Normality Test:

a.1.) Follow-up - 1 year

Table nº 29 – Tests of normality for 1st follow-up period (Average profitability)

Tests of Normality							
METHOD		Kolmogorov-Smirnov[a]			Shapiro-Wilk		
		Statistic	Df	Sig.	Statistic	Df	Sig.
PROFITABILITY	**PSI**	0,117	84	0,006	0,917	84	0,000
	ELECTRE	0,052	84	0,200*	0,978	84	0,159
	CAPM	0,074	84	0,200*	0,975	84	0,100
a. Lilliefors Significance Correction							
*. This is a lower bound of the true significance.							

Source: SPSS output, September 2012.

Table nº 30 – Tests of homogeneity for 1st follow-up period (Average profitability)

Test of Homogeneity of Variance				
PROFITABILITY	**Levene Statistic**	**df1**	**df2**	**Sig.**
Based on Mean	0,064	2	249	0,938
Based on Median	0,110	2	249	0,896
Based on Median and with adjusted df	0,110	2	246,852	0,896
Based on trimmed mean	0,100	2	249	0,905

Source: SPSS output, September 2012.

From tables above, we can conclude that only ELECTRE and CAPM has a normal distribution for average profitability, according to K-S test, with Lilliefors correction, both with *p-value* > α = 0,05. Concerning homogeneity of variance, all have homogenous variances, seeing all *p-value* > α = 0,05.

Table nº 31– Tests of normality for 1st follow-up period (Sharpe's Index)

Tests of Normality							
METHOD		Kolmogorov-Smirnov[a]			Shapiro-Wilk		
		Statistic	df	Sig.	Statistic	df	Sig.
SHARPE	PSI	0,104	84	0,025	0,972	84	0,066
	ELECTRE	0,100	84	0,039	0,982	84	0,282
	CAPM	0,044	84	0,200*	0,992	84	0,870

a. Lilliefors Significance Correction
*. This is a lower bound of the true significance.

Source: SPSS output, September 2012.

Table nº 32 – Tests of homogeneity for 1st follow-up period (Sharpe's Index)

Test of Homogeneity of Variance				
SHARPE	Levene Statistic	df1	df2	Sig.
Based on Mean	23,925	2	249	0,000
Based on Median	21,664	2	249	0,000
Based on Median and with adjusted df	21,664	2	185,078	0,000
Based on trimmed mean	23,728	2	249	0,000

Source: SPSS output, September 2012.

From tables above, we can conclude that only CAPM has a normal distribution for average Sharpe's index, according to K-S test, with Lilliefors correction, with *p-value* > α = 0,05. Concerning homogeneity of variance, none have homogenous variances, seeing all *p-value* < α = 0,05.

a.2.) Follow-up - 2 years

Table nº 33 – Tests of normality for 2nd follow-up period (Average Profitability)

Tests of Normality							
METHOD		Kolmogorov-Smirnov[a]			Shapiro-Wilk		
		Statistic	Df	Sig.	Statistic	Df	Sig.
PROFITABILITY	**PSI**	0,117	84	0,006	0,917	84	0,000
	ELECTRE	0,067	144	0,200*	0,966	144	0,001
	CAPM	0,076	144	0,040	0,966	144	0,001

a. Lilliefors Significance Correction

*. This is a lower bound of the true significance.

Source: SPSS output, September 2012.

Table nº 34 – Tests of homogeneity for 2nd follow-up period (Average profitability)

Test of Homogeneity of Variance				
PROFITABILITY	**Levene Statistic**	**Df1**	**df2**	**Sig.**
Based on Mean	0,540	2	369	0,583
Based on Median	0,648	2	369	0,523
Based on Median and with adjusted df	0,648	2	367,428	0,523
Based on trimmed mean	0,630	2	369	0,533

Source: SPSS output, September 2012.

Looking for estimation results for second follow-up period, only ELECTRE has a normal distribution, for average profitability, according to K-S test, with Lilliefors correction, with *p-value* > α = 0,05. Concerning homogeneity of variance, all have homogenous variances, seeing all *p-value* > α = 0,05.

Table nº 35 – Tests of normality for 2nd follow-up period (Sharpe's Index)

Tests of Normality							
METHOD		Kolmogorov-Smirnov[a]			Shapiro-Wilk		
		Statistic	df	Sig.	Statistic	df	Sig.
SHARPE	PSI	0,104	84	0,025	0,972	84	0,066
	ELECTRE	0,057	144	0,200*	0,991	144	0,474
	CAPM	0,048	144	0,200*	0,995	144	0,872

a. Lilliefors Significance Correction

*. This is a lower bound of the true significance.

Source: SPSS output, September 2012.

Table nº 36 – Tests of homogeneity for 2nd follow-up period (Sharpe's Index)

Test of Homogeneity of Variance				
SHARPE	Levene Statistic	df1	df2	Sig.
Based on Mean	27,156	2	369	0,000
Based on Median	25,964	2	369	0,000
Based on Median and with adjusted df	25,964	2	309,843	0,000
Based on trimmed mean	27,197	2	369	0,000

Source: SPSS output, September 2012.

Looking for estimation results for second follow-up period, only ELECTRE and CAPM have a normal distribution, for average Sharpe's index, according to K-S test, with Lilliefors correction, with *p-value* > α = 0,05. Concerning homogeneity of variance, no one have homogenous variances, seeing all *p-value* < α = 0,05.

a.3.) Follow-up - 3 years

Table nº 37 – Tests of normality for 3^{rd} follow-up period (Average profitability)

Tests of Normality							
METHOD		Kolmogorov-Smirnov[a]			Shapiro-Wilk		
		Statistic	df	Sig.	Statistic	df	Sig.
PROFITABILITY	PSI	0,117	84	0,006	0,917	84	0,000
	ELECTRE	0,070	180	0,031	0,964	180	0,000
	CAPM	0,067	180	0,049	0,966	180	0,000
a. Lilliefors Significance Correction							

Source: SPSS output, September 2012.

Table nº 38 – Tests of homogeneity for 3^{rd} follow-up period (Average profitability)

Test of Homogeneity of Variance				
PROFITABILITY	Levene Statistic	df1	df2	Sig.
Based on Mean	0,693	2	441	0,500
Based on Median	0,854	2	441	0,426
Based on Median and with adjusted df	0,854	2	436,443	0,426
Based on trimmed mean	0,845	2	441	0,430

Source: SPSS output, September 2012.

Results estimation for third follow-up period revels that no way has a normal distribution, for average profitability, according to K-S test, with Lilliefors correction, because all *p-value* < α = 0,05. Concerning homogeneity of variance, all have homogenous variances, seeing all *p-value* > α = 0,05.

Table nº 39 – Tests of normality for 3rd follow-up period (Sharpe's Index)

Tests of Normality							
METHOD		Kolmogorov-Smirnov[a]			Shapiro-Wilk		
		Statistic	df	Sig.	Statistic	Df	Sig.
SHARPE	PSI	0,104	84	0,025	0,972	84	0,066
	ELECTRE	0,059	180	0,200*	0,993	180	0,602
	CAPM	0,070	180	0,034	0,991	180	0,281

a. Lilliefors Significance Correction

*. This is a lower bound of the true significance.

Source: SPSS output, September 2012.

Table nº 40 – Tests of homogeneity for 3rd follow-up period (Sharpe's Index)

Test of Homogeneity of Variance				
SHARPE	Levene Statistic	df1	df2	Sig.
Based on Mean	26,573	2	441	0,000
Based on Median	24,717	2	441	0,000
Based on Median and with adjusted df	24,717	2	377,686	0,000
Based on trimmed mean	26,514	2	441	0,000

Source: SPSS output, September 2012.

Results estimation for third follow-up period revels that only ELECTRE has a normal distribution, for average Sharpe's index, according to K-S test, with Lilliefors correction, because *p-value* > α = 0,05. Concerning homogeneity of variance, no way have homogenous variances, seeing all *p-value* < α = 0,05.

b) *t-student* test for individual method:

The *t*-student test, tests if the mean of a single variable differs from a specified constant, assuming that the data is normally distributed. However, this test is fairly robust to departures from normality. So, we want to test if the average profitability and Sharpe's index for PSI, ELECTRE and CAPM differ from zero, at 95% confidence level, for each follow-up/holding period. Our test hypotheses are:

H_o : μ = 0% (average profitability/Sharpe's index of *i* isn't significant different from zero, with i = PSI, ELECTRE and CAPM).

H_1 : μ ≠, < , > 0% (average profitability/Sharpe's index of *i* is significant different from zero, with i = PSI, ELECTRE and CAPM).

Table nº 41 – *t*-student results for average profitability (all follow-up periods)

PROFITABILITY	t*			Sig. (2-tailed)*		
	1 y	2 y	3 y	1 y	2 y	3 y
PSI	-0,327	-0,327	-0,327	0,745	0,745	0,745
ELECTRE	0,173	0,439	0,810	0,863	0,661	0,419
CAPM	0,718	0,468	0,302	0,475	0,640	0,763
*Test Value = 0% with Confidence interval = 95%.						

Source: Own elaboration based on SPSS outputs, 2012.

Table nº 42 – *t*-student results for Sharpe's Index (all follow-up periods)

SHARPE'S INDEX	t*			Sig. (2-tailed)*		
	1 y	2 y	3 y	1 y	2 y	3 y
PSI	0,934	-0,180	-1,563	0,353	0,857	0,120
ELECTRE	-0,121	-0,453	0,304	0,904	0,652	0,761
CAPM	-0,269	-0,554	-0,926	0,789	0,581	0,356
*Test Value = 0% with Confidence interval = 95%.						

Source: Own elaboration based on SPSS outputs, 2012.

b.1.) Follow-up period – 1 year

Regarding average profitability, we can conclude that to first follow-up period, with a 95% confidence interval, for ELECTRE *p-value* =0,863 > α = 0,05, meaning that we cannot reject H_o. So, with an error probability of 5%, ELECTRE average profitability isn't significantly different from 0%. For CAPM, similar results were obtained, CAPM *p-value* = 0,475 > α = 0,05, meaning that we cannot reject H_o. So, with an error probability of 5%, CAPM average profitability isn't significantly different from 0%. Finally to PSI, *p-value* =0,745 > α = 0,05, meaning that we cannot reject H_o. So, with an error probability of 5%, PSI average profitability isn't significantly different from 0%. All these results are confirmed by *t* value: for ELECTRE 0,173 < 1,96, for CAPM 0,718 < 1,96 and for PSI -1,96 < -0,327. Generally, we conclude that every way of determining profitability isn't significantly different from 0%.

Concerning average Sharpe's index, we can conclude that to first follow-up period, with a 95% confidence interval, for ELECTRE *p-value* =0,904 > α = 0,05, meaning that we cannot reject H_o. So, with an error probability of 5%, ELECTRE average Sharpe's index isn't significantly different from 0%. For CAPM, similar results were obtained, CAPM *p-value* = 0,789 > α = 0,05, meaning that we cannot reject H_o. So, with an error probability of 5%, CAPM average Sharpe's index isn't significantly different from 0%. Finally to PSI, *p-value* =0,353 > α = 0,05, meaning that we cannot reject H_o. So, with an error probability of 5%, PSI average Sharpe's index isn't significantly different from 0%. All these results are confirmed by t value: for ELECTRE -1,96 < -0,121, for CAPM -1,96 < -0,269 and for PSI 0,934 < 1,96. Generally, we conclude that every way of determining Sharpe's index isn't significantly different from 0%.

b.2.) Follow-up period – 2 years

Now, looking to results obtained to second follow-up period to average profitability, with an 95% confidence interval, for ELECTRE *p-value* =0,661 > α = 0,05, meaning that we cannot reject H_o. So, with an error probability of 5%, ELECTRE average profitability isn't significantly different from 0%. For CAPM, similar results were obtained, CAPM *p-value* = 0,640 > α = 0,05, meaning that we cannot reject H_o. So, with an error probability of 5%, CAPM average profitability isn't significantly different from 0%. Finally to PSI, *p-value* =0,745 > α = 0,05, meaning that we cannot reject H_o. So, with an error probability of 5%, PSI average profitability isn't significantly different from 0%. All these results are confirmed by t value: for ELECTRE 0,439 < 1,96, for CAPM 0,468 < 1,96 and for PSI -1,96 < -0,327. Generally, we conclude that every way of calculating average profitability isn't significantly different from 0%.

Analyzing results obtained to second follow-up period relatively to average Sharpe's index, with an 95% confidence interval, for ELECTRE *p-value* =0,652 > α = 0,05, meaning that we cannot reject H_o. So, with an error probability of 5%, ELECTRE average Sharpe's index isn't significantly different from 0%. For CAPM, similar results were obtained, CAPM *p-value* = 0,581 > α = 0,05, meaning that we cannot reject H_o. So, with an error probability of 5%, CAPM average Sharpe's index isn't significantly different from 0%. Finally to PSI, *p-value* =0,857 > α = 0,05, meaning that we cannot reject H_o. So, with an error probability of 5%, PSI average Sharpe's index isn't significantly different from 0%. All these results are confirmed by t value: for ELECTRE -1,96 < -0,453, for CAPM -1,96 < -0,554 and for PSI -1,96 < -0,180. Generally, we conclude that every way of calculating average Sharpe's index isn't significantly different from 0%.

b.3.) Follow-up period - 3 years

Relatively to average profitability, results obtained to third follow-up period, with a 95% confidence interval, for ELECTRE *p-value* =0,419 > α = 0,05, meaning that we cannot reject H_o. So, with an error probability of 5%, ELECTRE average profitability isn't significantly different from 0%. For CAPM, similar results were obtained, CAPM *p-value* =

0,763 > α = 0,05, meaning that we cannot reject H_o. So, with an error probability of 5%, CAPM average profitability isn't significantly different from 0%. Finally to PSI, *p-value* = 0,745 > α = 0,05, meaning that we cannot reject H_o. So, with an error probability of 5%, PSI average profitability isn't significantly different from 0%. All these results are confirmed by *t* value: for ELECTRE 0,810 < 1,96, for CAPM 0,302 < 1,96 and for PSI -1,96 < -0,327. Generally, we conclude that every way of determining average profitability isn't significantly different from 0%.

Concerning average Sharpe's index, results obtained to third follow-up period, with a 95% confidence interval, for ELECTRE *p-value* =0,761 > α = 0,05, meaning that we cannot reject H_o. So, with an error probability of 5%, ELECTRE average Sharpe's index isn't significantly different from 0%. For CAPM, similar results were obtained, CAPM *p-value* = 0,356 > α = 0,05, meaning that we cannot reject H_o. So, with an error probability of 5%, CAPM average Sharpe's index isn't significantly different from 0%. Finally to PSI, *p-value* = 0,120 > α = 0,05, meaning that we cannot reject H_o. So, with an error probability of 5%, PSI average Sharpe's index isn't significantly different from 0%. All these results are confirmed by *t* value: for ELECTRE 0,304 < 1,96, for CAPM -1,96 < -0,926 and for PSI -1,96 < -1,563. Generally, we conclude that every way of determining average Sharpe's index isn't significantly different from 0%.

c) ANOVA

In order to obtain more robust results than the ones previous obtained we must conduct a One-way ANOVA. This test allows us to determine if exists differences among the average profitabilities and Sharpe's index, and which means differ. In order to do so, inputs from PSI, ELECTRE and CAPM should come from populations with equal variances (see Levene's homogeneity-of-variance test). Conducting a one-way ANOVA post hoc test, range tests and pair wise multiple comparisons can determine which means differ. On one hand, range tests identify homogeneous subsets of means that are not different from each other. On the other hand, pair wise multiple comparisons test the difference between each pair of means and yield a matrix where asterisks indicate significantly different group means at an alpha level of 0,05. So, estimation was based on Tukey's test (Tukey, 1953), and Scheffé's test (Sheffé, 1959). Our test hypotheses are:

H_o : $\mu_i = \mu_k$ (average profitability/Sharpe's index of pair *i* isn't significant different from average profitability/Sharpe's index of pair *k*, i, k = PSI, ELECTRE and CAPM).

H_1 : $\mu_i \neq \mu_k$ (average profitability/Sharpe's index of pair *i* is significant different from average profitability/Sharpe's index of pair *k*, i, k = PSI, ELECTRE and CAPM).

c.1.) Follow-up - 1 y

To average profitability, for first follow-up period, table 43 and 44 show us that, with an error probability of 5%, we can conclude that average profitability in at least two ways of determine portfolio isn't significant different (*p-value* = 0,766 > α = 0,05, so we cannot reject H_0). To take conclusion about multiple comparisons, table 44 tells us that no way of determine average profitabilities is different (*p-value* for every comparison is bigger than α = 0,05) for a probability error of 5%, allowing us to answer to *T1* and *T3*.

Table nº 43 – Oneway - ANOVA results to average profitability (1st follow-up period)

Oneway – ANOVA					
PROFITABILITY					
	Sum of Squares	**df**	**Mean Square**	**F**	**Sig.**
Between Groups	17,570	2	8,785	0,267	0,766
Within Groups	8184,892	249	32,871		
Total	8202,461	251			

Source: SPSS output, September 2012.

Table nº 44 – ANOVA Multiple comparisons results to average profitability (1st follow-up period)

Post Hoc - Multiple Comparisons							
Dependent Variable: PROFITABILITY							
	(I) METHOD	**(J) METHOD**	**Mean Difference (I-J)**	**Std. Error**	**Sig.**	**95% Confidence Interval**	
						Lower Bound	**Upper Bound**
Tukey HSD	**PSI**	**ELECTRE**	-0,31537	0,88467	0,932	-2,4013	1,7705
		CAPM	-0,64671	0,88467	0,745	-2,7326	1,4392
	ELECTRE	**PSI**	0,31537	0,88467	0,932	-1,7705	2,4013
		CAPM	-0,33134	0,88467	0,926	-2,4172	1,7546
	CAPM	**PSI**	0,64671	0,88467	0,745	-1,4392	2,7326
		ELECTRE	0,33134	0,88467	0,926	-1,7546	2,4172
Scheffe	**PSI**	**ELECTRE**	-0,31537	0,88467	0,938	-2,4939	1,8632
		CAPM	-0,64671	0,88467	0,766	-2,8253	1,5318
	ELECTRE	**PSI**	0,31537	0,88467	0,938	-1,8632	2,4939
		CAPM	-0,33134	0,88467	0,932	-2,5099	1,8472
	CAPM	**PSI**	0,64671	0,88467	0,766	-1,5318	2,8253
		ELECTRE	0,33134	0,88467	0,932	-1,8472	2,5099

Source: SPSS output, September 2012.

To Sharpe's, index, for first follow-up period, table 45 show us that, with an error probability of 5%, we can conclude that average Sharpe's index in at least two ways of determine portfolio isn't significant different (*p-value* = 0,837 > α = 0,05, so we cannot reject H_0). To take conclusion about multiple comparisons, table 46 tells us that no way of determine average Sharpe's index is different (*p-value* for every comparison is bigger than α = 0,05) for a probability error of 5%, allowing us to answer to *T2* and *T4*.

Table nº 45 – Oneway - ANOVA results to Sharpe's Index (1st follow-up period)

ANOVA					
SHARPE					
	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	1123,067	2	561,534	0,178	0,837
Within Groups	787591,948	249	3163,020		
Total	788715,016	251			

Source: SPSS output, September 2012.

Table nº 46 – ANOVA Multiple comparisons results to Sharpe's Index (1st follow-up period)

Multiple Comparisons							
Dependent Variable:SHARPE							
	(I) METHOD	(J) METHOD	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Tukey HSD	PSI	ELECTRE	3,71360	8,67813	0,904	-16,7478	24,1750
		CAPM	4,97315	8,67813	0,835	-15,4883	25,4346
	ELECTRE	PSI	-3,71360	8,67813	0,904	-24,1750	16,7478
		CAPM	1,25955	8,67813	0,988	-19,2019	21,7210
	CAPM	PSI	-4,97315	8,67813	0,835	-25,4346	15,4883
		ELECTRE	-1,25955	8,67813	0,988	-21,7210	19,2019
Scheffe	PSI	ELECTRE	3,71360	8,67813	0,913	-17,6567	25,0839
		CAPM	4,97315	8,67813	0,849	-16,3971	26,3434
	ELECTRE	PSI	-3,71360	8,67813	0,913	-25,0839	17,6567
		CAPM	1,25955	8,67813	0,990	-20,1108	22,6298
	CAPM	PSI	-4,97315	8,67813	0,849	-26,3434	16,3971
		ELECTRE	-1,25955	8,67813	0,990	-22,6298	20,1108

Source: SPSS output, September 2012.

c.2.) Follow-up - 2 y

Regarding average profitability, for second follow-up period, results expressed by table 47 are very similar to the one taken to first follow-up period: with an error probability of 5%, we can conclude that average profitability in at least two ways of determine portfolio isn't significant different (*p-value* = 0,842 > α = 0,05, so we cannot reject H_0). To take conclusions about multiple comparisons, table 48 tells us that no way of determining average profitabilities is different (*p-value* for every comparison is bigger than α = 0,05) for a probability error of 5%, answering to *T1* and *T3*.

Table nº 47 – Oneway - ANOVA results to average profitability (2nd follow-up period)

Oneway – ANOVA					
PROFITABILITY					
	Sum of Squares	Df	Mean Square	F	Sig.
Between Groups	12,084	2	6,042	0,172	0,842
Within Groups	12944,677	369	35,080		
Total	12956,761	371			

Source: SPSS output, September 2012.

Table nº 48 – ANOVA Multiple comparisons results to average profitability (2nd follow-up period)

Oneway – Multiple Comparisons							
Dependent Variable: **PROFITABILITY**							
	(I) METHOD	**(J) METHOD**	**Mean Difference (I-J)**	**Std. Error**	**Sig.**	**95% Confidence Interval**	
						Lower Bound	**Upper Bound**
Tukey HSD	**PSI**	**ELECTRE**	-0,43244	0,81317	0,856	-2,3460	1,4811
		CAPM	-0,42968	0,81317	0,857	-2,3432	1,4839
	ELECTRE	**PSI**	0,43244	0,81317	0,856	-1,4811	2,3460
		CAPM	0,00275	0,69802	1,000	-1,6398	1,6453
	CAPM	**PSI**	0,42968	0,81317	0,857	-1,4839	2,3432
		ELECTRE	-0,00275	0,69802	1,000	-1,6453	1,6398
Scheffe	**PSI**	**ELECTRE**	-0,43244	0,81317	0,868	-2,4310	1,5661
		CAPM	-0,42968	0,81317	0,870	-2,4282	1,5688
	ELECTRE	**PSI**	0,43244	0,81317	0,868	-1,5661	2,4310
		CAPM	0,00275	0,69802	1,000	-1,7128	1,7183
	CAPM	**PSI**	0,42968	0,81317	0,870	-1,5688	2,4282
		ELECTRE	-0,00275	0,69802	1,000	-1,7183	1,7128

Source: SPSS output, September 2012.

Relative to average Sharpe's index (see tables 49 and 50), with an error probability of 5%, we can conclude that at least two ways of determine portfolio isn't significant different (*p-value* = 0,749 > α = 0,05, so we cannot reject H_0). To take conclusion about multiple comparisons, table 50 tells us that no way of calculating average Sharpe's index is different (*p-value* for every comparison is bigger than α = 0,05) for a probability error of 5%, answering to *T2* and *T4*.

Table nº 49 – Oneway - ANOVA results to Sharpe's Index (2nd follow-up period)

ANOVA					
SHARPE					
	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	2157,847	2	1078,923	0,289	0,749
Within Groups	1378291,287	369	3735,207		
Total	1380449,133	371			

Source: SPSS output, September 2012.

Table nº 50 – ANOVA Multiple comparisons results to Sharpe's Index (2nd follow-up period)

Multiple Comparisons							
Dependent Variable:SHARPE							
	(I) METHOD	(J) METHOD	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Tukey HSD	**PSI**	**ELECTRE**	5,51406	8,39081	0,788	-14,2313	25,2594
		CAPM	5,96705	8,39081	0,757	-13,7783	25,7124
	ELECTRE	**PSI**	-5,51406	8,39081	0,788	-25,2594	14,2313
		CAPM	,45299	7,20263	0,998	-16,4963	17,4023
	CAPM	**PSI**	-5,96705	8,39081	0,757	-25,7124	13,7783
		ELECTRE	-,45299	7,20263	0,998	-17,4023	16,4963
Scheffe	**PSI**	**ELECTRE**	5,51406	8,39081	0,806	-15,1082	26,1363
		CAPM	5,96705	8,39081	0,777	-14,6552	26,5893
	ELECTRE	**PSI**	-5,51406	8,39081	0,806	-26,1363	15,1082
		CAPM	,45299	7,20263	0,998	-17,2490	18,1550
	CAPM	**PSI**	-5,96705	8,39081	0,777	-26,5893	14,6552
		ELECTRE	-,45299	7,20263	0,998	-18,1550	17,2490

Source: SPSS output, September 2012.

c.3.) Follow-up - 3 y

Table nº 51 – Oneway - ANOVA results to average profitability (3rd follow-up period)

Oneway – ANOVA					
PROFITABILITY					
	Sum of Squares	**Df**	**Mean Square**	**F**	**Sig.**
Between Groups	20,849	2	10,425	0,279	0,757
Within Groups	16497,681	441	37,410		
Total	16518,531	443			

Source: SPSS output, September 2012.

Table nº 52 – ANOVA Multiple comparisons results to average profitability (3rd follow-up period)

Oneway – Multiple Comparisons							
Dependent Variable: **PROFITABILITY**							
	(I) METHOD	**(J) METHOD**	**Mean Difference (I-J)**	**Std. Error**	**Sig.**	**95% Confidence Interval**	
						Lower Bound	**Upper Bound**
Tukey HSD	**PSI**	**ELECTRE**	-,59627	0,80820	0,741	-2,4969	1,3043
		CAPM	-,33910	0,80820	0,908	-2,2397	1,5615
	ELECTRE	**PSI**	,59627	0,80820	0,741	-1,3043	2,4969
		CAPM	,25717	0,64472	0,916	-1,2590	1,7733
	CAPM	**PSI**	,33910	0,80820	0,908	-1,5615	2,2397
		ELECTRE	-,25717	0,64472	0,916	-1,7733	1,2590

Scheffe	**PSI**	**ELECTRE**	-,59627	0,80820	0,762	-2,5813	1,3887
		CAPM	-,33910	0,80820	0,916	-2,3241	1,6459
	ELECTRE	**PSI**	,59627	0,80820	0,762	-1,3887	2,5813
		CAPM	,25717	0,64472	0,924	-1,3263	1,8407
	CAPM	**PSI**	,33910	0,80820	0,916	-1,6459	2,3241
		ELECTRE	-,25717	0,64472	0,924	-1,8407	1,3263

Source: SPSS output, September 2012.

Concerning average profitability, for third follow-up period, results obtained previously are confirmed. From table 51, we can see that with an error probability of 5%, we can conclude that average profitability in at least two ways of calculating portfolio isn't significant different (*p-value* = 0,757 > α = 0,05, so we cannot reject H_0). To take conclusion about multiple comparisons, table 52 tells us that no way of determining average profitabilities is different (*p-value* for every comparison is bigger than α = 0,05) for a probability error of 5%, giving answer to *T1* and *T3*.

Table nº 53 – Oneway - ANOVA results to Sharpe's Index (3rd follow-up period)

ANOVA					
SHARPE					
	Sum of Squares	**df**	**Mean Square**	**F**	**Sig.**
Between Groups	5020,266	2	2510,133	0,628	0,534
Within Groups	1761344,595	441	3993,979		
Total	1766364,861	443			

Source: SPSS output, September 2012.

Table nº 54 – ANOVA Multiple comparisons results to Sharpe's Index (3rd follow-up period)

Multiple Comparisons							
Dependent Variable:SHARPE							
	(I) METHOD	**(J) METHOD**	**Mean Difference (I-J)**	**Std. Error**	**Sig.**	**95% Confidence Interval**	
						Lower Bound	**Upper Bound**
Tukey HSD	**PSI**	**ELECTRE**	1,36495	8,35082	0,985	-18,2733	21,0032
		CAPM	7,70630	8,35082	0,626	-11,9319	27,3445
	ELECTRE	**PSI**	-1,36495	8,35082	0,985	-21,0032	18,2733
		CAPM	6,34136	6,66165	0,608	-9,3245	22,0072
	CAPM	**PSI**	-7,70630	8,35082	0,626	-27,3445	11,9319
		ELECTRE	-6,34136	6,66165	0,608	-22,0072	9,3245
Scheffe	**PSI**	**ELECTRE**	1,36495	8,35082	0,987	-19,1454	21,8753
		CAPM	7,70630	8,35082	0,654	-12,8040	28,2166
	ELECTRE	**PSI**	-1,36495	8,35082	0,987	-21,8753	19,1454
		CAPM	6,34136	6,66165	0,636	-10,0202	22,7029
	CAPM	**PSI**	-7,70630	8,35082	0,654	-28,2166	12,8040
		ELECTRE	-6,34136	6,66165	0,636	-22,7029	10,0202

Source: SPSS output, September 2012.

Relative to average Sharpe's index, for third follow-up period, results obtained previously are confirmed. From table nº 53, we can see that with an error probability of 5%, we can conclude that average Sharpe's index in at least two ways of calculating portfolio isn't significant different (*p-value* = 0,534 > α = 0,05, so we cannot reject H_0). To take conclusion about multiple comparisons, table 54 tells us that no way of determining average Sharpe's index is different (*p-value* for every comparison is bigger than α = 0,05) for a probability error of 5%, giving answer to *T2* and *T4*.

All results obtained, by multiple comparations, lead us to conclude that in every follow-up we cannot infer that one way of calculating portfolio's average profitability and portfolio's Sharpe's index is better than the other, statistically. Although average profitability and Sharpe's means expressed in table 28 are greater when using ELECTRE method in a 3 years holding, probably due to sample limitations (few observations) statistical results are not able to confirm this conclusions.

4.4.2.1.2. Nonparametric tests

Nonparametric tests are an alternative to parametric tests, because they do no require the normality of the variable and the variance homogeneity: nonparametric tests are distribution-free tests. One of the alternatives is to conduct a Wilcoxon-Mann-Whitney test, (Mann and Whitney, 1947), witch is the nonparametric test analog to *t*-student for independent samples. According to Maroco (2007), this test has a 95,5% efficiency of *t*-student test. So, having calculated average profitability and Sharpe's index, what we want to see is if distribution of one way of calculating input is high than distribution obtained with other way of calculating the same input, with an error probability of 10%. Therefore, our hypotheses are:

H_0: $F(X_i) \geq F(X_j)$ ($F(X_j$ and $F(X_j))$ are *i* function distribution, with i, j = PSI, ELECTRE and CAPM).

H_1: $F(X_i) < F(X_j)$ (i, j = PSI, ELECTRE and CAPM)

We also conducted Kruskal-Wallis tests (Kruskal and Wallis, 1952), witch is similar to ANOVA one-way test. Our hypotheses are,

H_0: $F(X_i) = F(X_j) = F(X_k)$ (distributions of dependent variable are identical in k populations).

H_1: $F(X_i) \neq F(X_j)$ (exists at least one population where dependent variable distribution is different from one of the distributions from others populations).

a) Follow-up -1 Y

Table nº 55 – Kruskal-Wallis Test to average profitability (1 y holding)

Test Statistics[a,b,c]	
PROFITABILITY	
Chi-Square	0,231
Df	2
Asymp. Sig.	0,891
a. Kruskal Wallis Test	
b. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Table nº 56 – Kruskal-Wallis Test to Sharpe's Index (1 y holding)

Test Statistics[a,b,c]	
SHARPE	
Chi-Square	0,665
Df	2
Asymp. Sig.	0,717
a. Kruskal Wallis Test	
b. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Analyzing results obtained to average profitability (table 55), and based on *p-value* = 0,891 > α = 0,10, we cannot reject H_0, meaning that no way of calculating portfolio's average profitability is greater than the other way. Now, analyzing results obtained to average Sharpe's index (table 56), and based on *p-value* = 0,717 > α = 0,10, we cannot reject H_0, meaning that no way of calculating portfolio's average Sharpe's index is greater than the other way.

a.1.) PSI vs ELECTRE

Table nº 57 - Mann-Whitney Test to average profitability (Ranks for 1 y holding)

Ranks				
METHOD		N	Mean Rank	Sum of Ranks
PROFITABILITY	PSI	84	84,96	7137,00
	ELECTRE	84	84,04	7059,00
	Total	168		

Source: SPSS output, September 2012.

Table nº 58 - Mann-Whitney Test to average profitability (Test statistics for 1 y holding)

Test Statistics[a]	
	PROFITABILITY
Mann-Whitney U	3489,000
Wilcoxon W	7059,000
Z	-0,124
Asymp. Sig. (2-tailed)	0,902
Exact Sig. (2-tailed)	0,903
Exact Sig. (1-tailed)	0,452
Point Probability	0,001
a. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Results allow us to concluded that (see tables 57 and 58), seeing *p-value* = 0,452 > α =0,10, we cannot reject H_0. So, average profitability of PSI is greater than average profitability obtained with ELECTRE, being these conclusions consistent with observed data: mean rank is higher to PSI (84,96) than to ELECTRE (84,04), answering to *T1*.

Table nº 59 - Mann-Whitney Test to Sharpe's Index (Ranks for 1 y holding)

Ranks				
METHOD		N	Mean Rank	Sum of Ranks
SHARPE	PSI	84	88,04	7395,00
	ELECTRE	84	80,96	6801,00
	Total	168		

Source: SPSS output, September 2012.

Table nº 60 - Mann-Whitney Test to Sharpe's Index (Test statistics for 1 y holding)

Test Statistics[a]	
	SHARPE
Mann-Whitney U	3,231E3
Wilcoxon W	6,801E3
Z	-0,942
Asymp. Sig. (2-tailed)	0,346
Exact Sig. (2-tailed)	0,348
Exact Sig. (1-tailed)	0,174
Point Probability	0,001
a. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Results allow us to concluded that (tables 59 and 60), seeing *p-value* = 0,174 > α =0,10, we cannot reject H_0. So, average Sharpe's index of PSI is greater than average Sharpe's index obtained with ELECTRE, being these conclusions consistent with observed data: mean rank is higher to PSI (88,04) than to ELECTRE (80,96), answering to *T2*.

a.2.) PSI vs CAPM

Table nº 61 - Mann-Whitney Test to average profitability (Ranks for 1 y holding)

Ranks				
METHOD		**N**	**Mean Rank**	**Sum of Ranks**
PROFITABILITY	**PSI**	84	83,11	6981,00
	CAPM	84	85,89	7215,00
	Total	168		

Source: SPSS output, September 2012.

Table nº 62 - Mann-Whitney Test to average profitability (Test statistics for 1 y holding)

Test Statistics[a]	
	PROFITABILITY
Mann-Whitney U	3411,000
Wilcoxon W	6981,000
Z	-0,371
Asymp. Sig. (2-tailed)	0,711
Exact Sig. (2-tailed)	0,712
Exact Sig. (1-tailed)	0,356
Point Probability	0,001
a. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Results allow us to concluded that (see tables 61 and 62), seeing *p-value* = 0,356 > α =0,10, we cannot reject H_0. So, average profitability of PSI is greater than average profitability obtained with CAPM, being these conclusions not consistent with observed data: mean rank is higher to CAPM (85,89) than to PSI (83,11).

Table nº 63 - Mann-Whitney Test to Sharpe's Index (Ranks for 1 y holding)

Ranks				
METHOD		N	Mean Rank	Sum of Ranks
SHARPE	PSI	84	85,87	7213,00
	CAPM	84	83,13	6983,00
	Total	168		

Source: SPSS output, September 2012.

Table nº 64 - Mann-Whitney Test to Sharpe's Index (Test statistics for 1 y holding)

Test Statistics[a]	
	SHARPE
Mann-Whitney U	3,413E3
Wilcoxon W	6,983E3
Z	-0,365
Asymp. Sig. (2-tailed)	0,715
Exact Sig. (2-tailed)	0,717
Exact Sig. (1-tailed)	0,359
Point Probability	0,001
a. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Results allow us to concluded that (see tables 63 and 64), seeing *p-value* = 0,359 > α =0,10, we cannot reject H_0. So, average Sharpe's index of PSI is greater than average Sharpe's index obtained with CAPM, being these conclusions consistent with observed data: mean rank is higher to PSI (85,87) than to CAPM (83,13).

a.3.) ELECTRE vs CAPM

Table nº 65 - Mann-Whitney Test to average profitability (Ranks for 1 y holding)

Ranks				
METHOD		**N**	**Mean Rank**	**Sum of Ranks**
PROFITABILITY	**ELECTRE**	84	82,85	6959,00
	CAPM	84	86,15	7237,00
	Total	168		

Source: SPSS output, September 2012.

Table nº 66 - Mann-Whitney Test to average profitability (Test statistics for 1 y holding)

Test Statistics[a]	
	PROFITABILITY
Mann-Whitney U	3389,000
Wilcoxon W	6959,000
Z	-0,441
Asymp. Sig. (2-tailed)	0,659
Exact Sig. (2-tailed)	0,661
Exact Sig. (1-tailed)	0,331
Point Probability	0,001
a. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Results allow us to concluded that (see tables 65 and 66), seeing *p-value* = 0,331 > α =0,10, we cannot reject H_0. So, average profitability of ELECTRE is greater than average profitability obtained with CAPM, being these conclusions not consistent with observed data: mean rank is higher to CAPM (86,15) than to ELECTRE (82,85), answering to *T3*.

Table nº 67 - Mann-Whitney Test to Sharpe's Index (Ranks for 1 y holding)

Ranks				
METHOD		N	Mean Rank	Sum of Ranks
SHARPE	ELECTRE	84	83,86	7044,00
	CAPM	84	85,14	7152,00
	Total	168		

Source: SPSS output, September 2012.

Table nº 68 - Mann-Whitney Test to Sharpe's Index (Test statistics for 1 y holding)

Test Statistics[a]	
	SHARPE
Mann-Whitney U	3,474E3
Wilcoxon W	7,044E3
Z	-0,171
Asymp. Sig. (2-tailed)	0,864
Exact Sig. (2-tailed)	0,866
Exact Sig. (1-tailed)	0,433
Point Probability	0,001
a. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Results allow us to concluded that (tables 67 and 68), seeing *p-value* = 0,433 > α =0,10, we cannot reject H_0. So, average Sharpe's index of ELECTRE is greater than average Sharpe's index obtained with CAPM, being these conclusions not consistent with observed data: mean rank is higher to CAPM (85,14) than to ELECTRE (83,86), answering to *T4*.

b) Follow-up - 2 years

Table nº 69 – Kruskal-Wallis Test to average profitability (2 y holding)

Test Statistics[a,b,c]	
PROFITABILITY	
Chi-Square	0,086
Df	2
Asymp. Sig.	0,958
a. Kruskal Wallis Test	
b. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Table nº 70 – Kruskal-Wallis Test to Sharpe's Index (2 y holding)

Test Statistics[a,b,c]	
SHARPE	
Chi-Square	,802
Df	2
Asymp. Sig.	,670
a. Kruskal Wallis Test	
b. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Analyzing results obtained to average profitability (table 69), and based on *p-value* = 0,958 > α = 0,10, we cannot reject H_0, meaning that no way of calculating portfolio's average profitability is greater than the other way.

Analyzing results obtained to average Sharpe's index (table 70), and based on *p-value* = 0,670 > α = 0,10, we cannot reject H_0, meaning that no way of calculating portfolio's average Sharpe's index is greater than the other way.

b.1.) PSI VS ELECTRE

Table nº 71 - Mann-Whitney Test to average profitability (Ranks for 2 y holding)

Ranks				
METHOD		N	Mean Rank	Sum of Ranks
PROFITABILITY	PSI	84	112,99	9491,00
	ELECTRE	144	115,38	16615,00
	Total	228		

Source: SPSS output, September 2012.

Table nº 72 - Mann-Whitney Test to average profitability (Test statistics for 2 y holding)

Test Statistics[a]	
	PROFITABILITY
Mann-Whitney U	5921,000
Wilcoxon W	9491,000
Z	-0,264
Asymp. Sig. (2-tailed)	0,792
Exact Sig. (2-tailed)	0,792
Exact Sig. (1-tailed)	0,396
Point Probability	0,000
a. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Results allow us to conclude that (see tables 71 and 72), seeing *p-value* = 0,396 > α =0,10, we cannot reject H_0. So, average profitability of PSI is greater than average profitability obtained with ELECTRE, being these conclusions not consistent with observed data: mean rank is higher to ELECTRE (115,38) than to PSI (112,99), answering to *T1*.

Table nº 73 - Mann-Whitney Test to Sharpe's Index (Ranks for 2 y holding)

Ranks				
METHOD		N	Mean Rank	Sum of Ranks
SHARPE	PSI	84	118,50	9954,00
	ELECTRE	144	112,17	16152,00
	Total	228		

Source: SPSS output, September 2012.

Table nº 74 - Mann-Whitney Test to Sharpe's Index (Test statistics for 2 y holding)

Test Statistics[a]	
	SHARPE
Mann-Whitney U	5,712E3
Wilcoxon W	1,615E4
Z	-0,699
Asymp. Sig. (2-tailed)	0,484
Exact Sig. (2-tailed)	0,486
Exact Sig. (1-tailed)	0,243
Point Probability	0,001
a. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Results allow us to concluded that (tables 73 and 74), seeing *p-value* = 0,243 > α =0,10, we cannot reject H_0. So, average Sharpe's index of PSI is greater than average Sharpe's index obtained with ELECTRE, being these conclusions not consistent with observed data: mean rank is higher to ELECTRE (115,38) than to PSI (112,99), answering to *T2*.

b.2.) PSI VS CAPM

Table nº 75 - Mann-Whitney Test to average profitability (Ranks for 2 y holding)

Ranks				
METHOD		**N**	**Mean Rank**	**Sum of Ranks**
PROFITABILITY	**PSI**	84	112,99	9491,00
	CAPM	144	115,38	16615,00
	Total	228		

Source: SPSS output, September 2012.

Table nº 76 - Mann-Whitney Test to average profitability (Test statistics for 2 y holding)

Test Statistics[a]	
	PROFITABILITY
Mann-Whitney U	5921,000
Wilcoxon W	9491,000
Z	-0,264
Asymp. Sig. (2-tailed)	0,792
Exact Sig. (2-tailed)	0,793
Exact Sig. (1-tailed)	0,396
Point Probability	0,001
a. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Results allow us to concluded that (tables 75 and 76), seeing *p-value* = 0,396 > α =0,10, we cannot reject H_0. So, average profitability of PSI is greater than average profitability obtained with CAPM, being these conclusions not consistent with observed data: mean rank is higher to CAPM (115,38) than to PSI (112,99).

Table nº 77 - Mann-Whitney Test to Sharpe's Index (Ranks for 2 y holding)

Ranks				
METHOD		N	Mean Rank	Sum of Ranks
SHARPE	PSI	84	119,63	10049,00
	CAPM	144	111,51	16057,00
	Total	228		

Source: SPSS output, September 2012.

Table nº 78 - Mann-Whitney Test to Sharpe's Index (Test statistics for 2 y holding)

Test Statistics[a]	
	SHARPE
Mann-Whitney U	5,617E3
Wilcoxon W	1,606E4
Z	-0,897
Asymp. Sig. (2-tailed)	0,370
Exact Sig. (2-tailed)	0,371
Exact Sig. (1-tailed)	0,185
Point Probability	0,001
a. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Results allow us to concluded that (see tables 77 and 78), seeing *p-value* = 0,185 > α =0,10, we cannot reject H_0. So, average Sharpe's index of PSI is greater than average Sharpe's index obtained with CAPM, being these conclusions consistent with observed data: mean rank is higher to PSI (119,63) than to CAPM (111,51).

b.3.) ELECTRE VS CAPM

Table nº 79 - Mann-Whitney Test to average profitability (Ranks for 2 y holding)

Ranks				
METHOD		**N**	**Mean Rank**	**Sum of Ranks**
PROFITABILITY	**ELECTRE**	144	144,61	20824,00
	CAPM	144	144,39	20792,00
	Total	288		

Source: SPSS output, September 2012.

Table nº 80 - Mann-Whitney Test to average profitability (Test statistics for 2 y holding)

Test Statistics[a]	
PROFITABILITY	
Mann-Whitney U	10352,000
Wilcoxon W	20792,000
Z	-0,023
Asymp. Sig. (2-tailed)	0,982
Exact Sig. (2-tailed)	0,982
Exact Sig. (1-tailed)	0,491
Point Probability	0,000
a. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Results allow us to concluded that (tables 79 and 80), seeing *p-value* = 0,491 > α =0,10, we cannot reject H_0. So, average profitability of ELECTRE is greater than average profitability obtained with CAPM, being these conclusions consistent with observed data: mean rank is higher to ELECTRE (144,61) than to CAPM (144,39), answering to *T3*.

Table nº 81 - Mann-Whitney Test to Sharpe's Index (Ranks for 2 y holding)

Ranks				
METHOD		N	Mean Rank	Sum of Ranks
SHARPE	ELECTRE	144	145,05	20887,00
	CAPM	144	143,95	20729,00
	Total	288		

Source: SPSS output, September 2012.

Table nº 82 - Mann-Whitney Test to Sharpe's Index (Test statistics for 2 y holding)

Test Statistics[a]	
	SHARPE
Mann-Whitney U	1,029E4
Wilcoxon W	2,073E4
Z	-0,112
Asymp. Sig. (2-tailed)	0,911
Exact Sig. (2-tailed)	0,912
Exact Sig. (1-tailed)	0,456
Point Probability	0,001
a. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Results allow us to conclude that (tables 81 and 82), seeing *p-value* = 0,456 > α =0,10, we cannot reject H_0. So, average Sharpe's index of ELECTRE is greater than average Sharpe's index obtained with CAPM, being these conclusions consistent with observed data: mean rank is higher to ELECTRE (145,05) than to CAPM (143,95), answering to *T4*.

c) Follow-up - 3 years

Table nº 83 – Kruskal-Wallis Test to average profitability (3 y holding)

Test Statistics[a,b,c]	
PROFITABILITY	
Chi-Square	0,369
Df	2
Asymp. Sig.	0,831
a. Kruskal Wallis Test b. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Table nº 84 – Kruskal-Wallis Test to Sharpe's Index (3 y holding)

Test Statistics[a,b,c]	
SHARPE	
Chi-Square	1,762
Df	2
Asymp. Sig.	0,414
a. Kruskal Wallis Test b. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Regarding average profitability (table 83), based on *p-value* = 0,831 > α = 0,10, we cannot reject H_0, meaning that no way of calculating portfolio's average profitability is greater than the other way.

Concerning Sharpe's index (table 84), and based on *p-value* = 0,414 > α = 0,10, we cannot reject H_0, meaning that no way of calculating portfolio's average Sharpe's index is greater than the other way.

c.1.) PSI VS ELECTRE

Table nº 85 - Mann-Whitney Test to average profitability (Ranks for 3 y holding)

Ranks				
METHOD		**N**	**Mean Rank**	**Sum of Ranks**
PROFITABILITY	**PSI**	84	128,86	10824,00
	ELECTRE	180	134,20	24156,00
	Total	264		

Source: SPSS output, September 2012.

Table nº 86 - Mann-Whitney Test to average profitability (Test statistics for 3 y holding)

Test Statistics[a]	
	PROFITABILITY
Mann-Whitney U	7254,000
Wilcoxon W	10824,000
Z	-0,530
Asymp. Sig. (2-tailed)	0,596
Exact Sig. (2-tailed)	0,598
Exact Sig. (1-tailed)	0,299
Point Probability	0,001
a. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Results allow us to concluded that (tables 85 and 86), seeing *p-value* = 0,299 > α =0,10, we cannot reject H_0. So, average profitability of PSI is greater than average profitability obtained with ELECTRE, being these conclusions not consistent with observed data: mean rank is higher to ELECTRE (134,20) than to PSI (128,86), answering to *T1*.

Table nº 87 - Mann-Whitney Test to Sharpe's Index (Ranks for 3 y holding)

Ranks				
METHOD		N	Mean Rank	Sum of Ranks
SHARPE	PSI	84	133,99	11255,00
	ELECTRE	180	131,81	23725,00
	Total	264		

Source: SPSS output, September 2012.

Table nº 88 - Mann-Whitney Test to Sharpe's Index (Test statistics for 3 y holding)

Test Statistics[a]	
	SHARPE
Mann-Whitney U	7,435E3
Wilcoxon W	2,372E4
Z	-0,216
Asymp. Sig. (2-tailed)	0,829
Exact Sig. (2-tailed)	0,830
Exact Sig. (1-tailed)	0,415
Point Probability	0,001
a. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Results allow us to concluded that (tables 87 and 88), seeing *p-value* = 0,415 > α =0,10, we cannot reject H_0. So, average Sharpe's index of PSI is greater than average Sharpe's index obtained with ELECTRE, being these conclusions consistent with observed data: mean rank is higher to PSI (133,99) than to ELECTRE (131,81), answering to *T2*.

c.2.) PSI VS CAPM

Table nº 89 - Mann-Whitney Test to average profitability (Ranks for 3 y holding)

Ranks				
METHOD		**N**	**Mean Rank**	**Sum of Ranks**
PROFITABILITY	**PSI**	84	131,39	11037,00
	CAPM	180	133,02	23943,00
	Total	264		

Source: SPSS output, September 2012.

Table nº 90 - Mann-Whitney Test to average profitability (Test statistics for 3 y holding)

Test Statistics[a]	
PROFITABILITY	
Mann-Whitney U	7467,000
Wilcoxon W	11037,000
Z	-0,161
Asymp. Sig. (2-tailed)	0,872
Exact Sig. (2-tailed)	0,873
Exact Sig. (1-tailed)	0,437
Point Probability	0,001
a. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Results allow us to concluded that (tables 89 and 90), seeing *p-value* = 0,437 > α =0,10, we cannot reject H_0. So, average profitability of PSI is greater than average profitability obtained with CAPM, being these conclusions not consistent with observed data: mean rank is higher to CAPM (133,02) than to PSI (131,39).

Table nº 91 - Mann-Whitney Test to Sharpe's Index (Ranks for 3 y holding)

Ranks				
METHOD		N	Mean Rank	Sum of Ranks
SHARPE	PSI	84	141,79	11910,00
	CAPM	180	128,17	23070,00
	Total	264		

Source: SPSS output, September 2012.

Table nº 92 - Mann-Whitney Test to Sharpe's Index (Test statistics for 3 y holding)

Test Statistics[a]	
	SHARPE
Mann-Whitney U	6,780E3
Wilcoxon W	2,307E4
Z	-1,350
Asymp. Sig. (2-tailed)	0,177
Exact Sig. (2-tailed)	0,178
Exact Sig. (1-tailed)	0,089
Point Probability	0,000
a. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Results allow us to concluded that (tables nºs 91 and 92), seeing *p-value* = 0,089 < α =0,10, we can reject H_0. So, average Sharpe's index of PSI isn't greater than average Sharpe's index obtained with CAPM, being these conclusions not consistent with observed data: mean rank is higher to PSI (141,79) than to CAPM (128,17). To notice that, if α = 0,05 (*p-value* = 0,089 > α), we could not reject H_0, meaning average Sharpe's index of PSI would be greater than average Sharpe's index obtained with CAPM, being consistent with mean rank observed.

c.3.) ELECTRE VS CAPM

Table nº 93 - Mann-Whitney Test to average profitability (Ranks for 3 y holding)

Ranks				
METHOD		**N**	**Mean Rank**	**Sum of Ranks**
PROFITABILITY	**ELECTRE**	180	183,13	32964,00
	CAPM	180	177,87	32016,00
	Total	360		

Source: SPSS output, September 2012.

Table nº 94 - Mann-Whitney Test to average profitability (Test statistics for 3 y holding)

Test Statistics[a]	
	PROFITABILITY
Mann-Whitney U	15726,000
Wilcoxon W	32016,000
Z	-0,480
Asymp. Sig. (2-tailed)	0,631
Exact Sig. (2-tailed)	0,632
Exact Sig. (1-tailed)	0,316
Point Probability	0,000
a. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Results allow us to concluded that (tables nºs 93 and 94), seeing *p-value* = 0,316 > α =0,10, we cannot reject H_0. So, average profitability of ELECTRE is greater than average profitability obtained with CAPM, being these conclusions consistent with observed data: mean rank is higher to ELECTRE (183,13) than to CAPM (177,87), answering to *T3*.

Table nº 95 - Mann-Whitney Test to Sharpe's Index (Ranks for 3 y holding)

Ranks				
METHOD		N	Mean Rank	Sum of Ranks
SHARPE	ELECTRE	180	185,55	33399,00
	CAPM	180	175,45	31581,00
	Total	360		

Source: SPSS output, September 2012.

Table nº 96 - Mann-Whitney Test to Sharpe's Index (Test statistics for 3 y holding)

Test Statistics[a]	
	SHARPE
Mann-Whitney U	1,529E4
Wilcoxon W	3,158E4
Z	-0,921
Asymp. Sig. (2-tailed)	0,357
Exact Sig. (2-tailed)	0,358
Exact Sig. (1-tailed)	0,179
Point Probability	0,000
a. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Results allow us to concluded that (table 95 and 96), seeing *p-value* = 0,179 > α =0,10, we cannot reject H_0. So, average Sharpe's index of ELECTRE is greater than average Sharpe's index obtained with CAPM, being these conclusions consistent with observed data: mean rank is higher to ELECTRE (183,13) than to CAPM (177,87), answering to *T4*.

All results obtained tell us that, in absolute terms, in a 3 years holding period, ELECTRE achieved high average profitability and average Sharpe's index than the market (PSI). Between CAPM and ELECTRE, ELECTRE behaved better than CAPM in three years holding period, meaning that in a long term period, portfolios formed with shares selected using ELECTRE III assumptions had better performance than portfolios formed under CAPM assumptions, in absolute terms, also achieving greater profitability by unit of risk. Despite these conclusions, statistically some means could be differentiated, based on Mann-Whitney test, for instance, average profitability and Sharpe's index to 1 and 2 years holding where PSI behaved better than ELECTRE, Sharpe's Index to 1, 2 and 3 years holding, where PSI behaved better than CAPM, average profitability and average Sharpe's index to 2 and 3 years holding where ELECTRE behaved better than CAPM.

4.4.2.2. Statistics results obtained with follow-up periods - two samples

From charts nº 27 and nº 28, we clearly see that there is a big difference in average profitabilities and Sharpe's index between 2005-2007, where all average profitabilities and Sharpe's index are positive, and 2008-2011, were all average profitabilities and Sharpe's index are negative (being very negative in 2008), except for 2009. The reason to this behavior lies in sub prime crises, which culminated in Portuguese stock market in 2008.

According to the annual reports of the Bank of Portugal, in *2005*, the Portuguese stock market valuations continued to be significant, reflecting the overall positive evolution of traded companies. Both number of assets transacted and market capitalization showed positive growth over the previous year (around respectively 11 and 9 percent) being the average turnover ratio (defined as the ratio between the value of shares traded and value of shares quoted) remained practically unchanged, slightly above 50 percent. In *2006*, the Portuguese stock market recorded an increase particularly significant, with the PSI-Geral to increase about 33 percent. The variation of the Portuguese index, the largest since 1997, projected it to a new record (high record) at the end of the year. The trend observed was associated, on one hand, to the general improvement of traded companies' results and, on the other hand, the takeover over Portugal Telecom (PT) and over Portuguese Investment Bank (BPI), which were announced in the first quarter of the year. Market dynamics was visible not only in the evolution of prices but also in the significant increase in transactions, achieving a remarkable average turnover ratio. In *2007*, particularly in the second half, the turbulence that emerged in international financial markets, triggered by the sub prime crisis in the United States, came to significantly alter the conditions in the financial markets of the most advanced economies. The Portuguese stock market recorded an increase of 18.3 percent. This market has, however, a different behavior throughout the years. Indeed, in line with the trend observed since 2003, the PSI-Geral clearly evolved positively during the first seven months of the year, registering an increase of 24 percent. Later, in the context of observed turbulence in international financial markets, the Portuguese index declined by about 5 percent by the end of the year. This development was part of the observed trends in the advanced economies equity markets, being particularly noticeable in the prices of financial companies. Additionally, the evolution of the Portuguese index was conditioned in the last months of 2007 (and early 2008) due to the proposal of a merger agreement between BPI and Millennium BCP. Face to 2006, there was a significant increase in the value of transactions, particularly involving shares of EDP, BCP and Portugal Telecom, which contributed to raised, in such a remarkable way, the average turnover ratio.

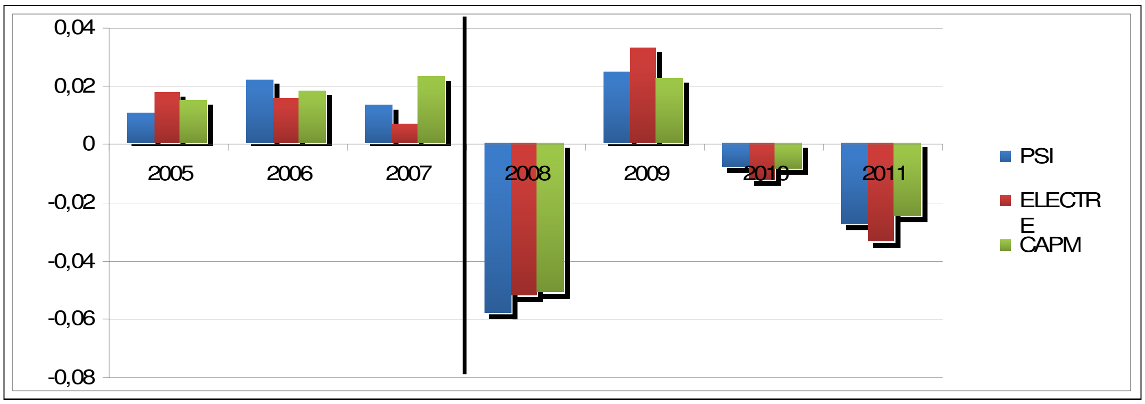
The year *2008* was marked by a sharp reduction in major stock indices in Europe and North America: the prices of the Standard & Poor's 500 fell 38 percent during 2008, being at the end of the year in the same level of mid-1997. The Eurostoxx 50 and the Portuguese PSI - 20 index fell, respectively, 44 and 51 percent, and returned in late 2008 to 2003 levels. The losses in the stock indices occurred throughout the year, but had special relevance during the month of October. The evolution of financial institutions' shares traded on the Portuguese stock market was less favorable than that of other titles. For example, the weight of these institutions in total capitalization decreased to 18.4% in December 2008 face to 26.5% in December 2007. This pattern has been found, in general, in Europe and the United States stock indexes. The fall in stock indices continued until early March 2009. Thereafter, there was a recovery between 15% and 26% over the level at the beginning of March, which stood at the end of April. The average turnover ratio, which has been increasing since 2003, rose slightly compared to 2007, suggesting maintenance, in terms of volume, of activity level in the Portuguese stock market. The negative news ensued throughout the year. It should be noted, by its symbolic nature, the collapse of British bank Northern Rock in February, due to a mass influx to their desks by the depositors. In March, the U.S. investment bank Bear Stearns was bought in disrepair by another American bank with Government support. Later, in September, it was announced that U.S. investment bank Lehman Brothers would start the procedures of filing for bankruptcy. This event has highlighted the extreme interdependence between institutions, massificating instability to an increasing number of financial institutions, other financial market segments and other countries. The effects of the crisis were not limited to the financial sector. In a context where we've recorded a fall of shares in nonfinancial firms, although less pronounced than financial entities, these developments have led to a global crisis of confidence and the general perception of a high risk of the counterparty. This helped to see an abrupt deterioration of the outlook on economic activity in developed economies, which quickly generalized to the most economies of emerging countries and the quotation of assets in all sectors. In late September the market saw the collapse of Fortis bank, with Belgian, Dutch and Luxembourg capital, which had to be partially nationalized by those Governments. During the first week of October, and despite news realizing approving a rescue plan for the financial sector of the United States in the amount of 700 billion dollars, continued to raise developments very negative for the markets. We saw the collapse of the Icelandic krona, with international agents skeptical about the ability of Iceland to make the payments on its sovereign debt. As on other occasions during this crisis, the events in Iceland had major repercussions on other economies. These developments illustrate the global nature of this crisis, where news from certain markets decisively influenced the performance of other markets. Between 1 and 10 October the S & P index depreciated by 22.7 percent, while the Eurostoxx 50 fell 21.1 percent and PSI-20 fell 4.21 percent. In *2009*, the PSI-Geral depreciated in the first two months of the year, following the very negative trend observed in 2008. Thus, since the beginning of 2009 until March 2010, the PSI-Geral appreciated by 33.9 percent, with the Dow Jones Euro Stoxx increased 24.4 percent in the same period. However, the values of the same ratios at the end of March 2010 were still, respectively, 36.0 percent and 33.7 percent below the values observed in July 2007. The year *2010* was particularly marked by the sovereign debt crisis in Europe, which made back the major stock indexes worldwide. During the last quarter of the year happened a strong recovery in some global indexes, such as the U.S. (S & P500) and German (DAX) rebounded 11% and 14%, respectively. But, the Portuguese market has been severely penalized, dropping the PSI-Geral about 10% in 2010. In *2011* remained a scenario of financial instability, particularly in the European Union, being reflected in the evolution of equity markets. Taking the variation of PSI-Geral as rough indicator of the profitability of Portuguese equity market, this indicator was -27.6% in 2011, meaning that in the last four years, only in 2009 the equity market provided a positive return for investors. The value of transactions in shares of companies traded in the PSI20, as happened in 2008 and 2009, recorded once again a significant decrease (-30.9%), reflecting the difficulties faced by companies.

Table nº 97 – Average profitability obtained, by year, in follow-up periods

	Jan05-Dec05	Jan06-Dec06	Jan07-Dec07	Jan08-Dec08	Jan09-Dec09	Jan10-Dec10	Jan11-Dec11
PSI	1,06%	2,18%	1,30%	-5,83%	2,45%	-0,78%	-2,78%
ELECTRE	1,78%	1,56%	0,65%	-5,21%	3,32%	-1,21%	-3,35%
CAPM	1,48%	1,80%	2,29%	-5,11%	2,27%	-0,85%	-2,51%

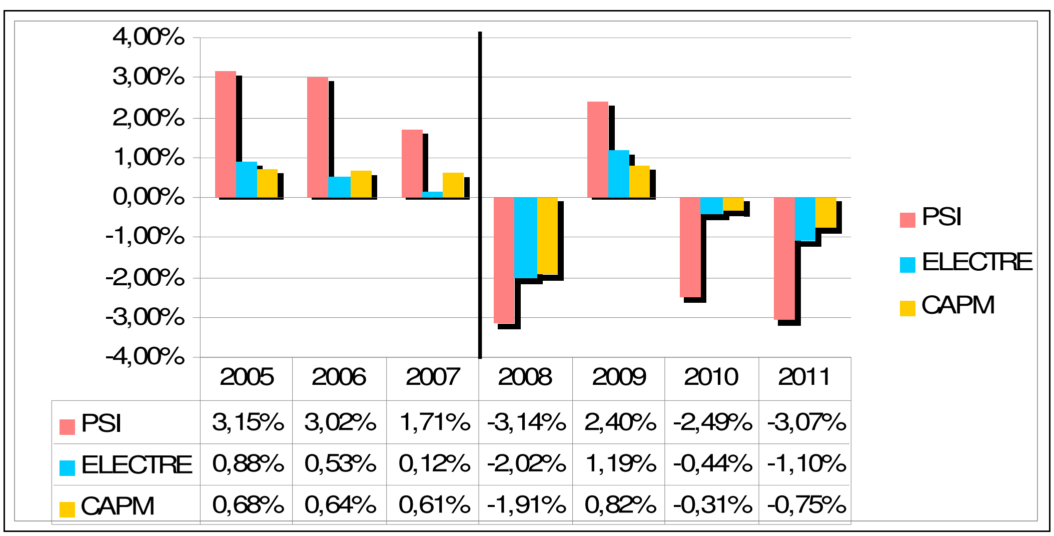
Source: Own elaboration, September 2012.

Chart nº 27 – Average profitability obtained, by year, in follow-up periods



Source: Own elaboration, September 2012.

Chart nº 28 – Sharpe's Index obtained, by year, in follow-up periods



Source: Own elaboration, September 2012.

With all this, we subdivided the all period into two sub periods. One sub period concerning the period before sub prime crises (2005-2007), and a second sub period after sub prime crises (2008-2011), and compare if there is significant differences between both means (average profitabilities and Sharpe's index).

Table nº 98 - 1st follow-up sub period

Portfolio nº	Follow-up period		
	1y	2y	3y
P1	Jan05-Dez05	Jan05-Dez06	Jan05-Dez07
P2	Jan06-Dez06	Jan06-Dez07	Jan06-Dez08
P3	Jan07-Dez07	Jan07-Dez08	Jan07-Dez09

Source: Own elaboration, September 2012.

Table nº 99 – 2nd follow-up sub period

Portfolio nº	Follow-up period		
	1y	2y	3y
P4	Jan08-Dez08	Jan08-Dez09	Jan08-Dez10
P5	Jan09-Dez09	Jan09-Dez10	Jan09-Dez11
P6	Jan10-Dez10	Jan10-Dez11	
P7	Jan11-Dez11		

Source: Own elaboration, September 2012.

4.4.2.2.1. Parametric tests

As mentioned previously, to test normality, the most used test is the Kolmogorov-Smirnov test[30], or alternatively, Shapiro-Wilk test (Shapiro and Wilk, 1965). To homogeneity in variance, usually Levene test (Levene, 1969) is the most used.

We start by testing if PSI-20 TR, ELECTRE and CAPM follows, or not, a normal distribution. Then, *t*-student test for individual method and One-way ANOVA test were conducted.

[30] Usually referred as K-S test.

4.4.2.2.1.1. 1st follow-up sub period

a) Normally test

a.1.) Follow-up, 1 year

Table nº 100 – Tests of normality to average profitability for 1st follow-up period (1st sub period)

Tests of Normality							
METHOD_GROUP		Kolmogorov-Smirnov[a]			Shapiro-Wilk		
		Statistic	df	Sig.	Statistic	df	Sig.
PROFITABILITY	PSI	0,088	36	0,200*	0,972	36	0,497
	ELECTRE	0,083	36	0,200*	0,966	36	0,331
	CAPM	0,086	36	0,200*	0,970	36	0,419

a. Lilliefors Significance Correction

*. This is a lower bound of the true significance.

Source: SPSS output, September 2012.

Table nº 101 – Tests of homogeneity to average profitability for 1st follow-up period (1st sub period)

Test of Homogeneity of Variance				
PROFITABILITY	Levene Statistic	df1	df2	Sig.
Based on Mean	0,064	2	105	0,938
Based on Median	0,077	2	105	0,926
Based on Median and with adjusted df	0,077	2	103,550	0,926
Based on trimmed mean	0,067	2	105	0,936

Source: SPSS output, September 2012.

From tables nºs 100 and 101, we can conclude that, for average profitability, PSI, ELECTRE and CAPM have a normal distribution, according to K-S test, with Lilliefors correction, all with *p-value* > α = 0,05. Concerning homogeneity of variance, all have homogenous variances, seeing all *p-value* > α = 0,05.

Table nº 102 – Tests of normality to Sharpe's Index for 1st follow-up period (1st sub period)

Tests of Normality							
METHOD		Kolmogorov-Smirnov[a]			Shapiro-Wilk		
		Statistic	df	Sig.	Statistic	df	Sig.
SHARPE	PSI	0,156	36	0,026	0,964	36	0,285
	ELECTRE	0,096	36	0,200*	0,981	36	0,789
	CAPM	0,101	36	0,200*	0,975	36	0,561

a. Lilliefors Significance Correction

*. This is a lower bound of the true significance.

Source: SPSS output, September 2012.

Table nº 103 – Tests of homogeneity to Sharpe's Index for 1st follow-up period (1st sub period)

Test of Homogeneity of Variance				
SHARPE	Levene Statistic	df1	df2	Sig.
Based on Mean	7,760	2	105	0,001
Based on Median	7,904	2	105	0,001
Based on Median and with adjusted df	7,904	2	91,828	0,001
Based on trimmed mean	7,817	2	105	0,001

Source: SPSS output, September 2012.

From tables nºs 102 and 103, we can conclude that, for average Sharpe's index, ELECTRE and CAPM have a normal distribution, according to K-S test, with Lilliefors correction, all with *p-value* > α = 0,05. Concerning homogeneity of variance, no way have homogenous variances, seeing all *p-value* < α = 0,05.

a.2.) Follow-up, 2 years

Table nº 104 – Tests of normality to average profitability for 2nd follow-up period (1st sub period)

Tests of Normality							
METHOD_GROUP		Kolmogorov-Smirnov[a]			Shapiro-Wilk		
		Statistic	df	Sig.	Statistic	df	Sig.
PROFITABILITY	PSI	0,158	48	0,004	0,844	48	0,000
	ELECTRE	0,124	72	0,008	0,892	72	0,000
	CAPM	0,105	72	0,048	0,904	72	0,000
a. Lilliefors Significance Correction							

Source: SPSS output, September 2012.

Table nº 105 – Tests of homogeneity to average profitability for 2nd follow-up period (1st sub period)

Test of Homogeneity of Variance				
PROFITABILITY	Levene Statistic	df1	df2	Sig.
Based on Mean	0,185	2	189	0,831
Based on Median	0,113	2	189	0,893
Based on Median and with adjusted df	0,113	2	183,801	0,893
Based on trimmed mean	0,132	2	189	0,877

Source: SPSS output, September 2012.

Looking for estimation results for second follow-up period, no way of calculating average profitability has a normal distribution, according to K-S test, with Lilliefors correction, with *p-value* < α = 0,05. Concerning homogeneity of variance, all have homogenous variances, seeing all *p-value* > α = 0,05.

Table nº 106 – Tests of normality to Sharpe's Index for 2nd follow-up period (1st sub period)

Tests of Normality							
METHOD		Kolmogorov-Smirnov[a]			Shapiro-Wilk		
		Statistic	df	Sig.	Statistic	df	Sig.
SHARPE	PSI	0,130	48	0,042	0,966	48	0,175
	ELECTRE	0,069	72	0,200*	0,992	72	0,914
	CAPM	0,065	720	,200*	0,988	72	0,742

a. Lilliefors Significance Correction

*. This is a lower bound of the true significance.

Source: SPSS output, September 2012.

Table nº 107 – Tests of homogeneity to Sharpe's Index for 2nd follow-up period (1st sub period)

Test of Homogeneity of Variance				
SHARPE	Levene Statistic	df1	df2	Sig.
Based on Mean	10,161	2	189	0,000
Based on Median	9,852	2	189	0,000
Based on Median and with adjusted df	9,852	2	158,841	0,000
Based on trimmed mean	10,117	2	189	0,000

Source: SPSS output, September 2012.

Looking for estimation results for second follow-up period, ELECTRE and CAPM have a normal distribution, according to K-S test, with Lilliefors correction, with *p-value* > α = 0,05. Concerning homogeneity of variance, no way have homogenous variances, seeing all *p-value* < α = 0,05.

a.3.) Follow-up, 3 years

Table nº 108 – Tests of normality to average profitability for 3rd follow-up period (1st sub period)

Tests of Normality							
METHOD_GROUP		**Kolmogorov-Smirnov[a]**			**Shapiro-Wilk**		
		Statistic	**Df**	**Sig.**	**Statistic**	**df**	**Sig.**
PROFITABILITY	**PSI**	0,160	60	0,001	0,869	60	0,000
	ELECTRE	0,085	108	0,052	0,924	108	0,000
	CAPM	0,087	108	0,045	0,932	108	0,000
a. Lilliefors Significance Correction							

Source: SPSS output, September 2012.

Table nº 109 – Tests of homogeneity to average profitability for 3rd follow-up period (1st sub period)

Test of Homogeneity of Variance				
PROFITABILITY	**Levene Statistic**	**Df1**	**df2**	**Sig.**
Based on Mean	0,421	2	273	0,657
Based on Median	0,353	2	273	0,703
Based on Median and with adjusted df	0,353	2	269,908	0,703
Based on trimmed mean	0,393	2	273	0,675

Source: SPSS output, September 2012.

Results estimation for third follow-up period revels that only ELECTRE has a normal distribution to average profitability, according to K-S test, with Lilliefors correction, seeing *p-value* > α = 0,05. Concerning homogeneity of variance, all have homogenous variances, seeing all *p-value* > α = 0,05.

Table nº 110 – Tests of normality to Sharpe's Index for 3rd follow-up period (1st sub period)

Tests of Normality							
METHOD		Kolmogorov-Smirnov[a]			Shapiro-Wilk		
		Statistic	df	Sig.	Statistic	df	Sig.
SHARPE	PSI	0,128	60	0,016	0,966	60	0,094
	ELECTRE	0,062	108	0,200*	0,992	108	0,799
	CAPM	0,074	108	0,175	0,989	108	0,545

a. Lilliefors Significance Correction

*. This is a lower bound of the true significance.

Source: SPSS output, September 2012.

Table nº 111 – Tests of homogeneity to Sharpe's Index for 3rd follow-up period (1st sub period)

Test of Homogeneity of Variance				
SHARPE	Levene Statistic	df1	df2	Sig.
Based on Mean	16,251	2	273	0,000
Based on Median	15,787	2	273	0,000
Based on Median and with adjusted df	15,787	2	230,980	0,000
Based on trimmed mean	16,264	2	273	0,000

Source: SPSS output, September 2012.

Results estimation for third follow-up period revels that only ELECTRE and CAPM have a normal distribution to average Sharpe's index, according to K-S test, with Lilliefors correction, seeing *p-value* > $\alpha = 0{,}05$. Concerning homogeneity of variance, no way have homogenous variances, seeing all *p-value* < $\alpha = 0{,}05$.

b) *t-student* for individual method

This procedure allow us to test if the average profitability and average Sharpe's index for PSI, ELECTRE and CAPM differ from zero, at 95% confidence level, for each follow-up/holding period. Our test hypotheses are:

H_o : μ = 0% (average profitability/Sharpe's index of *i* isn't significant different from zero, with i = PSI, ELECTRE and CAPM).

H_1 : μ ≠, < , > 0% (average profitability/Sharpe's index of *i* is significant different from zero, with i = PSI, ELECTRE and CAPM).

Table nº 112 – *t*-student results to average profitability (1st sub period)

PROFITABILITY	t*			Sig. (2-tailed)*		
	1 y	2 y	3 y	1 y	2 y	3 y
PSI	2,543	-0,187	0,486	,059	0,852	0,629
ELECTRE	2,619	0,429	0,885	,013	0,669	0,378
CAPM	3,268	1,130	0,923	,002	0,262	0,358
*Test Value = 0% with Confidence interval = 95%.						

Source: Own elaboration based on SPSS outputs, 2012.

Table nº 113 – *t*-student results to Sharpe's Index (1st sub period)

SHARPE'S INDEX	t*			Sig. (2-tailed)*		
	1 y	2 y	3 y	1 y	2 y	3 y
PSI	2,546	1,532	2,081	0,015	0,132	0,042
ELECTRE	2,863	1,984	1,787	0,007	0,051	0,077
CAPM	2,556	1,100	0,916	0,015	0,275	0,362
*Test Value = 0% with Confidence interval = 95%.						

Source: Own elaboration based on SPSS outputs, 2012.

b.1.) Follow-up period – 1 year

From results obtained to average profitability, we can conclude that to first follow-up period, with a 95% confidence interval, for ELECTRE *p-value* =0,013 < α = 0,05, meaning that we can reject H_o in favor of H_1, meaning our average profitability is significant different from 0% with an error probability of 5%. For CAPM, similar results were obtained seeing *p-value* = 0,002 < α = 0,05, meaning that we can reject H_o. So, with an error probability of 5%, CAPM average profitability is significantly different from 0%. Finally to PSI, *p-value* =0,059 > α = 0,05, meaning that we cannot reject H_o. So, with an error probability of 5%, PSI average profitability isn't significantly different from 0% despite, to an error probability of 10%, we can reject H_0 meaning average profitability is significantly different from 0% (*p-value* = 0,059 < α = 0,10). All these results are confirmed by *t* value: for ELECTRE 2,619 > 1,96, for CAPM 3,268 > 1,96 and for PSI 2,543 > 1,96. Generally, we conclude that every way of calculating average profitability is significantly different from 0%, with α = 0,10.

Regarding results obtained to average Sharpe's index, we can conclude that to first follow-up period, with a 95% confidence interval, for ELECTRE *p-value* =0,007 < α = 0,05, meaning that we can reject H_o in favor of H_1, meaning our average Sharpe's index is significant different from 0% with an error probability of 5%. For CAPM, similar results were obtained seeing *p-value* = 0,015 < α = 0,05, meaning that we can reject H_o. So, with an error probability of 5%, CAPM average Sharpe's index is significantly different from 0%. Finally to PSI, *p-value* = 0,015 < α = 0,05, meaning that we can reject H_o. So, with an error probability of 5%, every way of calculating average Sharpe's index is significantly different from 0%. All these results are confirmed by *t* value: for ELECTRE 2,863 > 1,96, for CAPM 2,556 > 1,96 and for PSI 2,546 > 1,96. Generally, we conclude that every way of calculating average Sharpe's index is significantly different from 0%.

b.2.) Follow-up period – 2 years

Relative to results obtained to average profitability to second follow-up period, with an 95% confidence interval, for ELECTRE *p-value* =0,669 > α = 0,05, meaning that we cannot reject H_o. So, with an error probability of 5%, ELECTRE average profitability isn't significantly different from 0%. For CAPM, similar results were obtained, CAPM *p-value* = 0,262 > α = 0,05, meaning that we cannot reject H_o. So, with an error probability of 5%, CAPM average profitability isn't significantly different from 0%. Finally to PSI, *p-value*

=0,852 > α = 0,05, meaning that we cannot reject H_o. So, with an error probability of 5%, PSI average profitability isn't significantly different from 0%. All these results are confirmed by *t* value: for ELECTRE 0,429 < 1,96, for CAPM 1,13 < 1,96 and for PSI -1,96 < -0,187. Generally, we conclude that every way of calculating average profitability isn't significantly different from 0%.

Relative to results obtained to Sharpe's index to second follow-up period, with a 95% confidence interval, for ELECTRE *p-value* =0,051 > α = 0,05, meaning that we cannot reject H_o. So, with an error probability of 5%, ELECTRE average Sharpe's index isn't significantly different from 0% despite, to a error probability of 10%, we can reject H_0 meaning average Sharpe's index is significantly different from 0% (*p-value* = 0,051 < α = 0,10). For CAPM, similar results were obtained, CAPM *p-value* = 0,275 > α = 0,05, meaning that we cannot reject H_o. So, with an error probability of 5%, CAPM average Sharpe's index isn't significantly different from 0%. Finally to PSI, *p-value* = 1,320 > α = 0,05, meaning that we cannot reject H_o. So, with an error probability of 5%, no way of determining average Sharpe's index is significantly different from 0%. All these results are confirmed by *t* value: for ELECTRE 1,96 < 1,984, for CAPM 1,100 < 1,96 and for PSI 1,532 < 1,96. Generally, we conclude that every way of calculating average Sharpe's index isn't significantly different from 0%, with α = 0,05.

b.3.) Follow-up period - 3 years

Referring to results obtained to average profitability to third follow-up period, with a 95% confidence interval, for ELECTRE *p-value* =0,378 > α = 0,05, meaning that we cannot reject H_o. So, with an error probability of 5%, ELECTRE average profitability isn't significantly different from 0%. For CAPM, similar results were obtained, CAPM *p-value* = 0,358 > α = 0,05, meaning that we cannot reject H_o. So, with an error probability of 5%, CAPM average profitability isn't significantly different from 0%. Finally to PSI, *p-value* = 0,629 > α = 0,05, meaning that we cannot reject H_o. So, with an error probability of 5%, PSI average profitability isn't significantly different from 0%. All these results are confirmed by *t* value: for ELECTRE 0,885 < 1,96, for CAPM 0,923 < 1,96 and for PSI 0,486 < 1,96. Generally, we conclude that every way of calculating average profitability isn't significantly different from 0%.

Concerning results obtained to average Sharpe's index to third follow-up period, with a 95% confidence interval, for ELECTRE *p-value* =0,077 > α = 0,05, meaning that we cannot reject H_o. So, with an error probability of 5%, ELECTRE average Sharpe's index isn't significantly different from 0% despite, to a error probability of 10%, we can reject H_0 meaning average Sharpe's index is significantly different from 0% (*p-value* = 0,077 < α = 0,10). For CAPM, similar results were obtained, CAPM *p-value* = 0,362 > α = 0,05, meaning that we cannot reject H_o. So, with an error probability of 5%, CAPM average Sharpe's index isn't significantly different from 0%. Finally to PSI, *p-value* = 0,042 < α = 0,05, meaning that we can reject H_o. So, with an error probability of 5%, PSI average Sharpe's index is significantly different from 0%. All these results are confirmed by *t* value: for ELECTRE 1,787 < 1,96, for CAPM 0,916 < 1,96 and for PSI 2,081 > 1,96. Generally, we conclude that every way of calculating average Sharpe's index isn't significantly different from 0%, except for market.

c) One-way ANOVA

One-way ANOVA test in a way to determine if exist differences among the average profitabilities and Sharpe's index, and which means differ. Conducting a one-way ANOVA post hoc test, range tests and pair wise multiple comparisons can determine which means differ. On one hand, range tests identify homogeneous subsets of means that are not different from each other. On the other hand, pair wise multiple comparisons test the difference between each pair of means and yield a matrix where asterisks indicate significantly different group means at an alpha level of 0,05. So, estimation was based on Tukey's test (Tukey, 1953), and Scheffé's test (Sheffé, 1959). Our test hypotheses are:

H_o : $\mu_i = \mu_k$ (average profitability/Sharpe's index of pair *i* isn't significant different from average profitability/Sharpe's index of pair *k*, i, k = PSI, ELECTRE and CAPM).

H_1 : $\mu_i \neq \mu_k$ (average profitability/Sharpe's index of pair *i* is significant different from average profitability/Sharpe's index of pair *k*, i, k = PSI, ELECTRE and CAPM).

c.1.) Follow-up, 1 year

For first follow-up period, results obtained show us that, with an error probability of 5%, we can conclude that average profitability in at least two ways of calculating portfolio isn't significant different (*p-value* = 0,838 > α = 0,05, so we cannot reject H_0). To take conclusion about multiple comparisons, table 115 tells us that no way of determining average profitabilities is different (*p-value* for every comparison is bigger than α = 0,05) for a probability error of 5%, answering to *T1* and *T3*.

Table nº 114 – ANOVA results to average profitability (1st follow-up period, 1st sub period)

ANOVA					
PROFITABILITY	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	0,001	2	0,000	0,177	0,838
Within Groups	0,162	105	0,002		
Total	0,163	107			

Source: SPSS output, September 2012.

Table nº 115 – ANOVA Multiple comparisons results to average profitability (1st follow-up period, 1st sub period)

Multiple Comparisons							
Dependent Variable:PROFITABILITY							
	(I) METHOD_GROUP	(J) METHOD_GROUP	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Tukey HSD	PSI	ELECTRE	-0,0022417	0,0092603	0,968	-0,024257	0,019774
		CAPM	-0,0054750	0,0092603	0,825	-0,027490	0,016540
	ELECTRE	PSI	0,0022417	0,0092603	0,968	-0,019774	0,024257
		CAPM	-0,0032333	0,0092603	0,935	-0,025249	0,018782
	CAPM	PSI	0,0054750	0,0092603	0,825	-0,016540	0,027490
		ELECTRE	0,0032333	0,0092603	0,935	-0,018782	0,025249
Scheffe	PSI	ELECTRE	-0,0022417	0,0092603	0,971	-0,025236	0,020752
		CAPM	-0,0054750	0,0092603	0,840	-0,028469	0,017519

	ELECTRE	**PSI**	0,0022417	0,0092603	0,971	-0,020752	0,025236
		CAPM	-0,0032333	0,0092603	0,941	-0,026227	0,019761
	CAPM	**PSI**	0,0054750	0,0092603	0,840	-0,017519	0,028469
		ELECTRE	0,0032333	0,0092603	0,941	-0,019761	0,026227

Source: SPSS output, September 2012.

For first follow-up period, results obtained show us that, with an error probability of 5%, we can conclude that average Sharpe's index in at least two ways of calculating portfolio isn't significant different (*p-value* = 0,386 > α = 0,05, so we cannot reject H_0). To take conclusion about multiple comparisons, table 117 tells us that no way of calculating average Sharpe's index is different (*p-value* for every comparison is bigger than α = 0,05) for a probability error of 5%, answering to *T2* and *T4*.

Table nº 116 – ANOVA results to Sharpe's Index (1st follow-up period, 1st sub period)

ANOVA					
SHARPE	**Sum of Squares**	**df**	**Mean Square**	**F**	**Sig.**
Between Groups	3416,497	2	1708,249	0,961	0,386
Within Groups	186665,593	105	1777,768		
Total	190082,091	107			

Source: SPSS output, September 2012.

Table nº 117 – ANOVA Multiple comparisons results to Sharpe's Index (1st follow-up period, 1st sub period)

Multiple Comparisons							
Dependent Variable:SHARPE							
	(I) METHOD	**(J) METHOD**	**Mean Difference (I-J)**	**Std. Error**	**Sig.**	**95% Confidence Interval**	
						Lower Bound	**Upper Bound**
Tukey HSD	**PSI**	**ELECTRE**	-13,76175	9,93805	0,353	-37,3885	9,8650
		CAPM	-7,44189	9,93805	0,735	-31,0687	16,1849
	ELECTRE	**PSI**	13,76175	9,93805	0,353	-9,8650	37,3885
		CAPM	6,31986	9,93805	0,801	-17,3069	29,9466
	CAPM	**PSI**	7,44189	9,93805	0,735	-16,1849	31,0687
		ELECTRE	-6,31986	9,93805	0,801	-29,9466	17,3069
Scheffe	**PSI**	**ELECTRE**	-13,76175	9,93805	0,387	-38,4388	10,9153
		CAPM	-7,44189	9,93805	0,756	-32,1189	17,2351
	ELECTRE	**PSI**	13,76175	9,93805	0,387	-10,9153	38,4388
		CAPM	6,31986	9,93805	0,817	-18,3572	30,9969
	CAPM	**PSI**	7,44189	9,93805	0,756	-17,2351	32,1189
		ELECTRE	-6,31986	9,93805	0,817	-30,9969	18,3572

Source: SPSS output, September 2012.

c.2.) Follow-up, 2 years

For second follow-up period, results expressed by table 118 are very similar to the one taken to first follow-up period: with an error probability of 5%, we can conclude that average profitability in at least two ways of determining portfolio isn't significant different (*p-value* = 0,654 > α = 0,05, so we cannot reject H_0). To take conclusion about multiple comparisons, table 119 tells us that no way of calculating average profitabilities is different (*p-value* for every comparison is bigger than α = 0,05) for a probability error of 5%, answering to *T1* and *T3*.

Table nº 118 – ANOVA results to average profitability (2nd follow-up period, 1st sub period)

ANOVA					
PROFITABILITY	**Sum of Squares**	**Df**	**Mean Square**	**F**	**Sig.**
Between Groups	0,003	2	0,001	0,426	0,654
Within Groups	0,624	189	0,003		
Total	0,627	191			

Source: SPSS output, September 2012.

Table nº 119 – ANOVA Multiple comparisons results to average profitability (2nd follow-up period, 1st sub period)

Multiple Comparisons							
Dependent Variable:PROFITABILITY							
	(I) METHOD_ GROUP	**(J) METHOD _GROUP**	**Mean Difference (I-J)**	**Std. Error**	**Sig.**	**95% Confidence Interval**	
						Lower Bound	**Upper Bound**
Tukey HSD	**PSI**	ELECTRE	-0,0088722	0,0107066	0,686	-0,034165	0,016420
		CAPM	-0,0088014	0,0107066	0,690	-0,034094	0,016491
	ELECTRE	PSI	0,0088722	0,0107066	0,686	-0,016420	0,034165
		CAPM	0,0000708	0,0095762	1,000	-0,022551	0,022693
	CAPM	PSI	0,0088014	0,0107066	0,690	-0,016491	0,034094
		ELECTRE	-0,0000708	0,0095762	1,000	-0,022693	0,022551
Scheffe	**PSI**	ELECTRE	-0,0088722	0,0107066	0,710	-0,035288	0,017544
		CAPM	-0,0088014	0,0107066	0,714	-0,035217	0,017615
	ELECTRE	PSI	0,0088722	0,0107066	0,710	-0,017544	0,035288
		CAPM	0,0000708	0,0095762	1,000	-0,023556	0,023698
	CAPM	PSI	0,0088014	0,0107066	0,714	-0,017615	0,035217
		ELECTRE	-0,0000708	0,0095762	1,000	-0,023698	0,023556

Source: SPSS output, September 2012.

For second follow-up period, statistical results are very similar to the one taken to first follow-up period: with an error probability of 5%, we can conclude that average Sharpe's index in at least two ways of determining portfolio isn't significant different (*p-value* = 0,644

> α = 0,05, so we cannot reject H_0). To take conclusion about multiple comparisons, table 121 tells us that no way of calculating average Sharpe's index is different (*p-value* for every comparison is bigger than α = 0,05) for a probability error of 5%, answering to *T2* and *T4*.

Table nº 120 – Oneway - ANOVA results to Sharpe's Index (2nd follow-up period, 1st sub period)

ANOVA					
SHARPE	**Sum of Squares**	**df**	**Mean Square**	**F**	**Sig.**
Between Groups	1994,317	2	997,158	0,441	0,644
Within Groups	427149,282	189	2260,049		
Total	429143,598	191			

Source: SPSS output, September 2012.

Table nº 121 – ANOVA Multiple comparisons results to Sharpe's Index (2nd follow-up period, 1st sub period)

Multiple Comparisons							
Dependent Variable:SHARPE							
						95% Confidence Interval	
	(I) METHOD	**(J) METHOD**	**Mean Difference (I-J)**	**Std. Error**	**Sig.**	**Lower Bound**	**Upper Bound**
Tukey HSD	**PSI**	**ELECTRE**	-6,87206	8,85855	0,718	-27,7988	14,0547
		CAPM	-,36903	8,85855	0,999	-21,2958	20,5578
	ELECTRE	**PSI**	6,87206	8,85855	0,718	-14,0547	27,7988
		CAPM	6,50303	7,92333	0,691	-12,2145	25,2205
	CAPM	**PSI**	,36903	8,85855	0,999	-20,5578	21,2958
		ELECTRE	-6,50303	7,92333	0,691	-25,2205	12,2145
Scheffe	**PSI**	**ELECTRE**	-6,87206	8,85855	0,741	-28,7285	14,9844
		CAPM	-,36903	8,85855	0,999	-22,2255	21,4874
	ELECTRE	**PSI**	6,87206	8,85855	0,741	-14,9844	28,7285
		CAPM	6,50303	7,92333	0,714	-13,0460	26,0521
	CAPM	**PSI**	,36903	8,85855	0,999	-21,4874	22,2255

Multiple Comparisons							
Dependent Variable:SHARPE							
	(I) METHOD	**(J) METHOD**	**Mean Difference (I-J)**	**Std. Error**	**Sig.**	**95% Confidence Interval**	
						Lower Bound	**Upper Bound**
Tukey HSD	**PSI**	**ELECTRE**	-6,87206	8,85855	0,718	-27,7988	14,0547
		CAPM	-,36903	8,85855	0,999	-21,2958	20,5578
	ELECTRE	**PSI**	6,87206	8,85855	0,718	-14,0547	27,7988
		CAPM	6,50303	7,92333	0,691	-12,2145	25,2205
	CAPM	**PSI**	,36903	8,85855	0,999	-20,5578	21,2958
		ELECTRE	-6,50303	7,92333	0,691	-25,2205	12,2145
	PSI	**ELECTRE**	-6,87206	8,85855	0,741	-28,7285	14,9844
		CAPM	-,36903	8,85855	0,999	-22,2255	21,4874
	ELECTRE	**PSI**	6,87206	8,85855	0,741	-14,9844	28,7285
		CAPM	6,50303	7,92333	0,714	-13,0460	26,0521
		PSI	,36903	8,85855	0,999	-21,4874	22,2255
		ELECTRE	-6,50303	7,92333	0,714	-26,0521	13,0460

Source: SPSS output, September 2012.

c.3.) Follow-up, 3 years

For third follow-up period, results obtained previously are confirmed. From table 122, we can see that with an error probability of 5%, we can conclude that average profitability in at least two ways of determining portfolio isn't significant different (*p-value* = 0,985 > α = 0,05, so we cannot reject H_0. To take conclusion about multiple comparisons, results tells us that no way of calculating average profitabilities is different (*p-value* for every comparison is bigger than α = 0,05) for a probability error of 5%, giving answer to *T1* and *T3*.

Table nº 122 – ANOVA results to average profitability (3rd follow-up period, 1st sub period)

ANOVA					
PROFITABILITY	**Sum of Squares**	**df**	**Mean Square**	**F**	**Sig.**
Between Groups	0,000	2	0,000	0,015	0,985
Within Groups	1,008	273	0,004		
Total	1,008	275			

Source: SPSS output, September 2012.

Table nº 123 – ANOVA Multiple comparisons results to average profitability (3rd follow-up period, 1st sub period)

Multiple Comparisons							
Dependent Variable:PROFITABILITY							
	(I) METHOD_GROUP	**(J) METHOD_GROUP**	**Mean Difference (I-J)**	**Std. Error**	**Sig.**	**95% Confidence Interval**	
						Lower Bound	**Upper Bound**
Tukey HSD	**PSI**	**ELECTRE**	-0,0016350	0,0097855	0,985	-0,024695	0,021425
		CAPM	-0,0013730	0,0097855	0,989	-0,024433	0,021687
	ELECTRE	**PSI**	0,0016350	0,0097855	0,985	-0,021425	0,024695
		CAPM	0,0002620	0,0082703	0,999	-0,019227	0,019752
	CAPM	**PSI**	0,0013730	0,0097855	0,989	-0,021687	0,024433
		ELECTRE	-0,0002620	0,0082703	0,999	-0,019752	0,019227
Scheffe	**PSI**	**ELECTRE**	-0,0016350	0,0097855	0,986	-0,025720	0,022450
		CAPM	-0,0013730	0,0097855	0,990	-0,025458	0,022712
	ELECTRE	**PSI**	0,0016350	0,0097855	0,986	-0,022450	0,025720
		CAPM	0,0002620	0,0082703	0,999	-0,020093	0,020617
	CAPM	**PSI**	0,0013730	0,0097855	0,990	-0,022712	0,025458
		ELECTRE	-0,0002620	0,0082703	0,999	-0,020617	0,020093

Source: SPSS output, September 2012.

For third follow-up period, results obtained previously are confirmed. From table 124, we can see that with an error probability of 5%, we can conclude that average Sharpe's index in at least two ways of determining portfolio isn't significant different (*p-value* = 0,730 > α = 0,05, so we cannot reject H_0). To take conclusion about multiple comparisons, results tells us that no way of determining average Sharpe's index is different (*p-value* for every comparison is bigger than α = 0,05) for a probability error of 5%, giving answer to *T2* and *T4*.

Table nº 124 –ANOVA results to Sharpe's Index (3rd follow-up period, 1st sub period)

ANOVA					
SHARPE	**Sum of Squares**	**df**	**Mean Square**	**F**	**Sig.**
Between Groups	1833,420	2	916,710	0,314	0,730
Within Groups	795850,149	273	2915,202		
Total	797683,569	275			

Source: SPSS output, September 2012.

Table nº 125 – ANOVA Multiple comparisons results to Sharpe's Index (3rd follow-up period, 1st sub period)

Multiple Comparisons							
Dependent Variable:SHARPE							
	(I) METHOD	**(J) METHOD**	**Mean Difference (I-J)**	**Std. Error**	**Sig.**	**95% Confidence Interval**	
						Lower Bound	**Upper Bound**
Tukey HSD	**PSI**	**ELECTRE**	-3,44939	8,69364	0,917	-23,9365	17,0378
		CAPM	2,35508	8,69364	0,960	-18,1321	22,8422
	ELECTRE	**PSI**	3,44939	8,69364	0,917	-17,0378	23,9365
		CAPM	5,80447	7,34746	0,709	-11,5103	23,1193
	CAPM	**PSI**	-2,35508	8,69364	0,960	-22,8422	18,1321
		ELECTRE	-5,80447	7,34746	0,709	-23,1193	11,5103
Scheffe	**PSI**	**ELECTRE**	-3,44939	8,69364	0,924	-24,8465	17,9477
		CAPM	2,35508	8,69364	0,964	-19,0420	23,7522
	ELECTRE	**PSI**	3,44939	8,69364	0,924	-17,9477	24,8465
		CAPM	5,80447	7,34746	0,732	-12,2794	23,8883
	CAPM	**PSI**	-2,35508	8,69364	0,964	-23,7522	19,0420
		ELECTRE	-5,80447	7,34746	0,732	-23,8883	12,2794

Source: SPSS output, September 2012.

All results obtained, by multiple comparations, lead us to conclude that in every follow-up we cannot infer that one way of calculating portfolio's average profitability and portfolio's average Sharpe's index is better than the other statistically. Although average profitability and Sharpe's index means are greater when using ELECTRE method in a 3 years holding, probably due to sample limitations (few observations) statistical results are not able to confirm this conclusions.

4.4.2.2.1.2. 2nd follow-up sub period

a) Normally test

a.1.) Follow-up, 1 year

Table nº 126 – Tests of normality to average profitability for 1st follow-up period (2nd sub period)

Tests of Normality							
METHOD_GROUP		Kolmogorov-Smirnov[a]			Shapiro-Wilk		
		Statistic	df	Sig.	Statistic	df	Sig.
PROFITABILITY	PSI	0,110	48	0,200*	0,929	48	0,006
	ELECTRE	0,100	48	0,200*	0,970	48	0,257
	CAPM	0,090	48	0,200*	0,972	48	0,308
a. Lilliefors Significance Correction							
*. This is a lower bound of the true significance.							

Source: SPSS output, September 2012.

Table nº 127 – Tests of homogeneity to average profitability for 1st follow-up period (2nd sub period)

Test of Homogeneity of Variance				
PROFITABILITY	**Levene Statistic**	**df1**	**df2**	**Sig.**
Based on Mean	0,042	2	141	0,959
Based on Median	0,014	2	141	0,986
Based on Median and with adjusted df	0,014	2	140,067	0,986
Based on trimmed mean	0,026	2	141	0,974

Source: SPSS output, September 2012.

From tables nºs 126 and 127, we can conclude that every ways of determine average profitability follows a normal distribution, according to K-S test, with Lilliefors correction, both with *p-value* > α = 0,05. Concerning homogeneity of variance, all have homogenous variances, seeing all *p-value* > α = 0,05.

Table nº 128 – Tests of normality to Sharpe's Index for 1st follow-up period (2nd sub period)

Tests of Normality							
METHOD		**Kolmogorov-Smirnov**[a]			**Shapiro-Wilk**		
		Statistic	**df**	**Sig.**	**Statistic**	**df**	**Sig.**
SHARPE	**PSI**	0,098	48	0,200*	0,956	48	0,067
	ELECTRE	0,123	48	0,065	0,952	48	0,049
	CAPM	0,075	48	0,200*	0,983	48	0,723
a. Lilliefors Significance Correction							
*. This is a lower bound of the true significance.							

Source: SPSS output, September 2012.

Table nº 129 – Tests of homogeneity to Sharpe's Index for 1st follow-up period (2nd sub period)

Test of Homogeneity of Variance				
SHARPE	**Levene Statistic**	**df1**	**df2**	**Sig.**
Based on Mean	17,807	2	141	0,000
Based on Median	15,977	2	141	0,000
Based on Median and with adjusted df	15,977	2	102,367	0,000
Based on trimmed mean	17,622	2	141	0,000

Source: SPSS output, September 2012.

From tables above, we can conclude that every ways of determine average Sharpe's index follows a normal distribution, according to K-S test, with Lilliefors correction, both with *p-value* > α = 0,05. Concerning homogeneity of variance, no way have homogenous variances, seeing all *p-value* < α = 0,05.

a.2.) Follow-up, 2 years

Table nº 130 – Tests of normality to average profitability for 2nd follow-up period (2nd sub period)

Tests of Normality							
METHOD_GROUP		**Kolmogorov-Smirnov[a]**			**Shapiro-Wilk**		
		Statistic	**df**	**Sig.**	**Statistic**	**df**	**Sig.**
PROFITABILITY	**PSI**	0,110	48	0,200*	0,929	48	0,006
	ELECTRE	0,078	72	0,200*	0,979	72	0,274
	CAPM	0,094	72	0,194	0,977	72	0,209

a. Lilliefors Significance Correction

*. This is a lower bound of the true significance.

Source: SPSS output, September 2012.

Table nº 131 – Tests of homogeneity to average profitability for 2nd follow-up period (2nd sub period)

Test of Homogeneity of Variance				
PROFITABILITY	**Levene Statistic**	**df1**	**df2**	**Sig.**
Based on Mean	,164	2	189	,849
Based on Median	,115	2	189	,892
Based on Median and with adjusted df	,115	2	185,489	,892
Based on trimmed mean	,134	2	189	,875

Source: SPSS output, September 2012.

Looking for estimation results for second follow-up period (tables 130 and 131), also every ways of calculating average profitability have a normal distribution, according to K-S test, with Lilliefors correction, with *p-value* > α = 0,05. Concerning homogeneity of variance, all have homogenous variances, seeing all *p-value* > α = 0,05.

Table nº 132 – Tests of normality to Sharpe's Index for 2nd follow-up period (2nd sub period)

Tests of Normality							
METHOD		**Kolmogorov-Smirnov[a]**			**Shapiro-Wilk**		
		Statistic	**df**	**Sig.**	**Statistic**	**df**	**Sig.**
SHARPE	**PSI**	0,130	48	0,042	0,966	48	0,175
	ELECTRE	0,104	72	0,052	0,972	72	0,103
	CAPM	0,078	72	0,200*	0,977	72	0,205
a. Lilliefors Significance Correction							
*. This is a lower bound of the true significance.							

Source: SPSS output, September 2012.

Table nº 133 – Tests of homogeneity to Sharpe's Index for 2nd follow-up period (2nd sub period)

Test of Homogeneity of Variance				
SHARPE	**Levene Statistic**	**df1**	**df2**	**Sig.**
Based on Mean	18,427	2	189	,000
Based on Median	17,117	2	189	,000
Based on Median and with adjusted df	17,117	2	151,122	,000
Based on trimmed mean	18,206	2	189	,000

Source: SPSS output, September 2012.

Looking for estimation results for second follow-up period, only ELECTRE and CAPM have a normal distribution, according to K-S test, with Lilliefors correction, with *p-value* > α = 0,05. Concerning homogeneity of variance, no way have homogenous variances, seeing all *p-value* < α = 0,05.

a.3.) Follow-up, 3 years

Table nº 134 – Tests of normality to average profitability for 3rd follow-up period (2nd sub period)

Tests of Normality							
METHOD		**Kolmogorov-Smirnov[a]**			**Shapiro-Wilk**		
		Statistic	**df**	**Sig.**	**Statistic**	**df**	**Sig.**
PROFITABILITY	**PSI**	0,110	48	0,200*	0,929	48	0,006
	ELECTRE	0,089	72	0,200*	0,978	72	0,236
	CAPM	0,070	72	0,200*	0,980	72	0,302
a. Lilliefors Significance Correction							
*. This is a lower bound of the true significance.							

Source: SPSS output, September 2012.

Table nº 135 – Tests of homogeneity to average profitability for 3rd follow-up period (2nd sub period)

Test of Homogeneity of Variance				
PROFITABILITY	**Levene Statistic**	**df1**	**df2**	**Sig.**
Based on Mean	0,121	2	189	0,886
Based on Median	0,051	2	189	0,950
Based on Median and with adjusted df	0,051	2	183,986	0,950
Based on trimmed mean	0,082	2	189	0,921

Source: SPSS output, September 2012.

Results estimation for third follow-up period (tables nºs 134 and 135) revels that every way of calculating average profitability has a normal distribution, according to K-S test, with Lilliefors correction, because all *p-value* > α = 0,05. Concerning homogeneity of variance, all have homogenous variances, seeing all *p-value* > α = 0,05.

Table nº 136 – Tests of normality to Sharpe's Index for 3rd follow-up period (2nd sub period)

Tests of Normality							
METHOD		**Kolmogorov-Smirnov[a]**			**Shapiro-Wilk**		
		Statistic	**df**	**Sig.**	**Statistic**	**df**	**Sig.**
SHARPE	**PSI**	0,098	48	0,200*	0,956	48	0,067
	ELECTRE	0,085	72	0,200*	0,979	72	0,288
	CAPM	0,100	72	0,070	0,964	72	0,039
a. Lilliefors Significance Correction							
*. This is a lower bound of the true significance.							

Source: SPSS output, September 2012.

Table nº 137 – Tests of homogeneity to Sharpe's Index for 3rd follow-up period (2nd sub period)

Test of Homogeneity of Variance				
SHARPE	**Levene Statistic**	**df1**	**df2**	**Sig.**
Based on Mean	18,636	2	189	0,000
Based on Median	16,215	2	189	0,000
Based on Median and with adjusted df	16,215	2	147,985	0,000
Based on trimmed mean	18,259	2	189	0,000

Source: SPSS output, September 2012.

Results estimation for third follow-up period revels that every way of calculating average Sharpe's index has a normal distribution, according to K-S test, with Lilliefors correction, because all *p-value* > $\alpha = 0{,}05$. Concerning homogeneity of variance, no way have homogenous variances, seeing all *p-value* < $\alpha = 0{,}05$.

b) *t-student* for individual method

The *t*-student test if whether the mean of a single variable differs from a specified constant, assuming that the data are normally distributed. However, this test is fairly robust to departures from normality. So, we want to test if the average profitability and Sharpe's index for PSI, ELECTRE and CAPM differ from zero, at 95% confidence level, for each follow-up/holding period. Our test hypotheses are:

H_o : $\mu = 0\%$ (average profitability/Sharpe's index of *i* isn't significant different from zero, with i = PSI, ELECTRE and CAPM).

H_1 : $\mu \neq, <, > 0\%$ (average profitability/Sharpe's index of *i* is significant different from zero, with i = PSI, ELECTRE and CAPM).

Table nº 138 – *t*-student results for individual method to average profitability (2nd sub period)

PROFITABILITY	t*			Sig. (2-tailed)*		
	1 y	2 y	3 y	1 y	2 y	3 y
PSI	-1,609	-1,609	-1,609	0,114	0,114	0,114
ELECTRE	-0,926	-0,380	-0,393	0,359	0,705	0,695
CAPM	-0,896	-0,369	-0,615	0,375	0,713	0,540
*Test Value = 0% with Confidence interval = 95%.						

Source: Own elaboration based on SPSS outputs, 2012.

Table nº 139 – *t*-student results for individual method to Sharpe's Index (2nd sub period)

SHARPE'S INDEX	t*			Sig. (2-tailed)*		
	1 y	2 y	3 y	1 y	2 y	3 y
PSI	-0,791	1,532	-0,791	0,433	0,132	0,433
ELECTRE	-2,301	-1,977	-1,404	0,026	0,052	0,165
CAPM	-1,484	-1,351	-1,953	0,145	0,181	0,055
*Test Value = 0% with Confidence interval = 95%.						

Source: Own elaboration based on SPSS outputs, 2012.

b.1.) Follow-up period – 1 year

Concerning average profitability (table nº 138), we can conclude that for first follow-up period, with a 95% confidence interval, for ELECTRE *p-value* =0,359 > α = 0,05, meaning that we cannot reject H_o. So, with an error probability of 5%, ELECTRE average profitability isn't significantly different from 0%. For CAPM, similar results were obtained, CAPM *p-value* = 0,375 > α = 0,05, meaning that we cannot reject H_o. So, with an error probability of 5%, CAPM average profitability isn't significantly different from 0%. Finally to PSI, *p-value*

=0,114 > α = 0,05, meaning that we cannot reject H_o. So, with an error probability of 5%, PSI average profitability isn't significantly different from 0%. All these results are confirmed by *t* value: for ELECTRE -1,96 < -0,926, for CAPM -1,96 < -0,896 and for PSI -1,96 < -1,609. Generally, we conclude that every way of calculating average profitability isn't significantly different from 0%.

Regarding average Sharpe's index (table nº 139), we can conclude that for first follow-up period, with a 95% confidence interval, for ELECTRE *p-value* =0,026 < α = 0,05, meaning that we can reject H_o. So, with an error probability of 5%, ELECTRE average Sharpe's index is significantly different from 0%. For CAPM, *p-value* = 0,145 > α = 0,05, meaning that we cannot reject H_o. So, with an error probability of 5%, CAPM average Sharpe's index isn't significantly different from 0%. Finally to PSI, *p-value* =0,433 > α = 0,05, meaning that we cannot reject H_o. So, with an error probability of 5%, average Sharpe's index calculated to PSI is significantly different from 0%. All these results are confirmed by *t* value: for ELECTRE -2,301 < -1,96, for CAPM -1,96 < -1,484 and for PSI -1,96 < -0,791. Generally, we conclude that every way of calculating average Sharpe's index isn't significantly different from 0%, except for ELECTRE.

b.2.) Follow-up period – 2 years

Now, looking to results obtained to average profitability to second follow-up period (see table nº 138), with a 95% confidence interval, for ELECTRE *p-value* =0,705 > α = 0,05, meaning that we cannot reject H_o. So, with an error probability of 5%, ELECTRE average profitability isn't significantly different from 0%. For CAPM, similar results were obtained, CAPM *p-value* = 0,713 > α = 0,05, meaning that we cannot reject H_o. So, with an error probability of 5%, CAPM average profitability isn't significantly different from 0%. Finally to PSI, *p-value* =0,114 > α = 0,05, meaning that we cannot reject H_o. So, with an error probability of 5%, PSI average profitability isn't significantly different from 0%. All these results are confirmed by *t* value: for ELECTRE -1,96 < -0,380, for CAPM -1,96 < -0,369 and for PSI -1,96 < -1,609. Generally, we conclude that every way of determining average profitability isn't significantly different from 0%.

Relative to Sharpe's index (see table nº 139), results obtained to second follow-up period, with a 95% confidence interval, for ELECTRE *p-value* =0,052 > α = 0,05, meaning that we cannot reject H_o. So, with an error probability of 5%, ELECTRE average Sharpe's index isn't significantly different from 0% despite, to a error probability of 10%, we can reject H_0 meaning average Sharpe's index is significantly different from 0% (*p-value* = 0,052 < α = 0,10). For CAPM, similar results were obtained, CAPM *p-value* = 0,181 > α = 0,05, meaning that we cannot reject H_o. So, with an error probability of 5%, CAPM average Sharpe's index isn't significantly different from 0%. Finally to PSI, *p-value* =0,132 > α = 0,05, meaning that we cannot reject H_o. So, with an error probability of 5%, PSI average Sharpe's index isn't significantly different from 0%. All these results are confirmed by *t* value: for ELECTRE -1,977 < -1,96, for CAPM -1,96 < -1,351 and for PSI 1,532 < 1,96. Generally, we conclude that every way of determining average Sharpe's index isn't significantly different from 0%, except for ELECTRE with α = 0,10.

b.3.) Follow-up period - 3 years

Relatively to average profitability (see table nº 138), to third follow-up period, with a 95% confidence interval, for ELECTRE *p-value* =0,695 > α = 0,05, meaning that we cannot reject H_o. So, with an error probability of 5%, ELECTRE average profitability isn't significantly different from 0%. For CAPM, similar results were obtained, CAPM *p-value* = 0,540 > α = 0,05, meaning that we cannot reject H_o. So, with an error probability of 5%, CAPM average profitability isn't significantly different from 0%. Finally to PSI, *p-value* = 0,114 > α = 0,05, meaning that we cannot reject H_o. So, with an error probability of 5%, PSI average profitability isn't significantly different from 0%. All these results are confirmed by *t* value: for ELECTRE -1,96 < -0,393, for CAPM -1,96 < -0,615 and for PSI -1,96 < -1,609. Generally, we conclude that every way of determining average profitability isn't significantly different from 0%.

Concerning average Sharpe's index (see table nº 139), to third follow-up period, with a 95% confidence interval, for ELECTRE *p-value* = 0,165 > α = 0,05, meaning that we cannot reject H_o. So, with an error probability of 5%, ELECTRE average Sharpe's index isn't significantly different from 0%. For CAPM, similar results were obtained, CAPM *p-value* = 0,055 > α = 0,05, meaning that we cannot reject H_o despite, to a error probability of 10%, we can reject H_0 meaning average Sharpe's index is significantly different from 0% (*p-value* = 0,055 < α = 0,10). So, with an error probability of 5%, CAPM average Sharpe's index isn't significantly different from 0%. Finally to PSI, *p-value* = 0,433 > α = 0,05, meaning that we cannot reject H_o. So, with an error probability of 5%, PSI average Sharpe's index isn't significantly different from 0%. All these results are confirmed by *t* value: for ELECTRE

-1,96 < -1,404, for CAPM -1,96 < -1,953 and for PSI -1,96 < -0,791. Generally, we conclude that every way of determining average Sharpe's index isn't significantly different from 0%.

c) ANOVA

In order to obtain more robust results than the ones previous obtained we must conduct a One-way ANOVA. This test allows us to determine if exists differences among the average profitabilities and Sharpe's index, and which means differ. In order to do so, PSI, ELECTRE and CAPM should come from populations with equal variances (see Levene's homogeneity-of-variance test). Conducting a one-way ANOVA post hoc test, range tests and pair wise multiple comparisons can determine which means differ. On one hand, range tests identify homogeneous subsets of means that are not different from each other. On the other hand, pair wise multiple comparisons test the difference between each pair of means and yield a matrix where asterisks indicate significantly different group means at an alpha level of 0,05. So, estimation was based on Tukey's test (Tukey, 1953), and Scheffé's test (Sheffé, 1959). Our test hypotheses are:

H_o : $\mu_i = \mu_k$ (average profitability/Sharpe's index of pair *i* isn't significant different from average profitability/Sharpe's index of pair *k*, i, k = PSI, ELECTRE and CAPM).

H_1 : $\mu_i \neq \mu_k$ (average profitability/Sharpe's index of pair *i* is significant different from average profitability/Sharpe's index of pair *k*, i, k = PSI, ELECTRE and CAPM).

c.1.) Follow-up, 1 year

For first follow-up period, table 140 show us that, with an error probability of 5%, we can conclude that average profitability in at least two ways of determine portfolio isn't significant different (*p-value* = 0,834 > α = 0,05, so we cannot reject H_0). To take conclusion about multiple comparisons, table 141 tells us that no way of calculating average profitabilities is different (*p-value* for every comparison is bigger than α = 0,05) for a probability error of 5%, answering to *T1* and *T3*.

Table nº 140 – ANOVA results to average profitability (1st follow-up period, 2nd sub period)

ANOVA					
PROFITABILITY	**Sum of Squares**	**df**	**Mean Square**	**F**	**Sig.**
Between Groups	0,002	2	0,001	0,182	0,834
Within Groups	0,602	141	0,004		
Total	0,604	143			

Source: SPSS output, September 2012.

Table nº 141 – ANOVA Multiple comparisons results to average profitability (1st follow-up period, 2nd sub period)

Multiple Comparisons							
Dependent Variable:PROFITABILITY							
	(I) METHOD _GROUP	**(J) METHOD _GROUP**	**Mean Difference (I-J)**	**Std. Error**	**Sig.**	**95% Confidence Interval**	
						Lower Bound	**Upper Bound**
Tukey HSD	**PSI**	**ELECTRE**	-0,0066917	0,0133393	0,871	-,038289	,024906
		CAPM	-0,0072188	0,0133393	0,851	-,038816	,024378
	ELECTRE	**PSI**	0,0066917	0,0133393	0,871	-,024906	,038289
		CAPM	-0,0005271	0,0133393	0,999	-,032124	,031070
	CAPM	**PSI**	0,0072188	0,0133393	0,851	-,024378	,038816
		ELECTRE	0,0005271	0,0133393	0,999	-,031070	,032124
Scheffe	**PSI**	**ELECTRE**	-0,0066917	0,0133393	0,882	-,039693	,026310
		CAPM	-0,0072188	0,0133393	0,864	-,040220	,025783
	ELECTRE	**PSI**	0,0066917	0,0133393	0,882	-,026310	,039693
		CAPM	-0,0005271	0,0133393	0,999	-,033528	,032474
	CAPM	**PSI**	0,0072188	0,0133393	0,864	-,025783	,040220
		ELECTRE	0,0005271	0,0133393	0,999	-,032474	,033528

Source: SPSS output, September 2012.

For first follow-up period, table nº 142 show us that, with an error probability of 5%, we can conclude that average Sharpe's index in at least two ways of determine portfolio isn't significant different (*p-value* = 0,351 > α = 0,05, so we cannot reject H_0). To take conclusion about multiple comparisons, table 143 tells us that no way of calculating average Sharpe's index is different (*p-value* for every comparison is bigger than α = 0,05) for a probability error of 5%, answering to *T2* and *T4*.

Table nº 142 – ANOVA results to Sharpe's Index (1st follow-up period, 2nd sub period)

ANOVA					
SHARPE	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	7894,323	2	3947,161	1,054	0,351
Within Groups	527933,529	141	3744,209		
Total	535827,852	143			

Source: SPSS output, September 2012.

Table nº 143 – ANOVA Multiple comparisons results to Sharpe's Index (1st follow-up period, 2nd sub period)

Multiple Comparisons							
Dependent Variable:SHARPE							
						95% Confidence Interval	
	(I) METHOD	(J) METHOD	Mean Difference (I-J)	Std. Error	Sig.	Lower Bound	Upper Bound
Tukey HSD	PSI	ELECTRE	16,82021	12,49035	0,372	-12,7660	46,4064
		CAPM	14,28446	12,49035	0,489	-15,3017	43,8706
	ELECTRE	PSI	-16,82021	12,49035	0,372	-46,4064	12,7660
		CAPM	-2,53575	12,49035	0,978	-32,1219	27,0504
	CAPM	PSI	-14,28446	12,49035	0,489	-43,8706	15,3017
		ELECTRE	2,53575	12,49035	0,978	-27,0504	32,1219
Scheffe	PSI	ELECTRE	16,82021	12,49035	0,406	-14,0807	47,7211
		CAPM	14,28446	12,49035	0,522	-16,6164	45,1853
	ELECTRE	PSI	-16,82021	12,49035	0,406	-47,7211	14,0807
		CAPM	-2,53575	12,49035	0,980	-33,4366	28,3651
	CAPM	PSI	-14,28446	12,49035	0,522	-45,1853	16,6164
		ELECTRE	2,53575	12,49035	0,980	-28,3651	33,4366

Source: SPSS output, September 2012.

c.2.) Follow-up, 2 years

For second follow-up period, results expressed by table 144 are very similar to the one taken to first follow-up period: with an error probability of 5%, we can conclude that average profitability in at least two ways of calculating portfolio isn't significant different (*p-value* = 0,486 > α = 0,05, so we cannot reject H_0). To take conclusion about multiple comparisons, table 145 tells us that no way of determining average profitabilities is different (*p-value* for every comparison is bigger than α = 0,05) for a probability error of 5%, giving answer to *T1* and *T3*.

Table nº 144 – Oneway - ANOVA results to average profitability (2nd follow-up period, 2nd sub period)

ANOVA					
PROFITABILITY	**Sum of Squares**	**Df**	**Mean Square**	**F**	**Sig.**
Between Groups	0,006	2	0,003	0,724	0,486
Within Groups	0,760	189	0,004		
Total	0,766	191			

Source: SPSS output, September 2012.

Table nº 145 – ANOVA Multiple comparisons results to average profitability (2nd follow-up period, 2nd sub period)

Multiple Comparisons							
Dependent Variable:PROFITABILITY							
	(I) METHOD _GROUP	**(J) METHOD _GROUP**	**Mean Difference (I-J)**	**Std. Error**	**Sig.**	**95% Confidence Interval Lower Bound**	**95% Confidence Interval Upper Bound**
Tukey HSD	**PSI**	**ELECTRE**	-0,0126111	0,0118148	0,535	-0,040522	0,015299
		CAPM	-0,0128208	0,0118148	0,524	-0,040731	0,015090
	ELECTRE	**PSI**	0,0126111	0,0118148	0,535	-0,015299	0,040522
		CAPM	-0,0002097	0,0105675	1,000	-0,025174	0,024754
	CAPM	**PSI**	0,0128208	0,0118148	0,524	-0,015090	0,040731

		ELECTRE	0,0002097	0,0105675	1,000	-0,024754	0,025174
Scheffe	**PSI**	**ELECTRE**	-0,0126111	0,0118148	0,567	-0,041762	0,016539
		CAPM	-0,0128208	0,0118148	0,556	-0,041971	0,016330
	ELECTRE	**PSI**	0,0126111	0,0118148	0,567	-0,016539	0,041762
		CAPM	-0,0002097	0,0105675	1,000	-0,026283	0,025863
	CAPM	**PSI**	0,0128208	0,0118148	0,556	-0,016330	0,041971
		ELECTRE	0,0002097	0,0105675	1,000	-0,025863	0,026283

Source: SPSS output, September 2012.

For second follow-up period, results expressed by table 146 are very similar to the one taken to first follow-up period: with an error probability of 5%, we can conclude that average Sharpe's index in at least two ways of calculating portfolio isn't significant different (*p-value* = 0,162 > α = 0,05, so we cannot reject H_0). To take conclusion about multiple comparisons, table 147 tells us that no way of determining average Sharpe's index is different (*p-value* for every comparison is bigger than α = 0,05) for a probability error of 5%, giving answer to *T2* and *T4*.

Table nº 146 – Oneway - ANOVA results to Sharpe's Index (2nd follow-up period, 2nd sub period)

ANOVA					
SHARPE					
	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	17619,982	2	8809,991	1,837	0,162
Within Groups	906395,068	189	4795,741		
Total	924015,050	191			

Source: SPSS output, September 2012.

Table nº 147 – ANOVA Multiple comparisons results to Sharpe's Index (2nd follow-up period, 2nd sub period)

Multiple Comparisons							
Dependent Variable:SHARPE							
	(I) METHOD	(J) METHOD	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Tukey HSD	PSI	ELECTRE	24,20218	12,90422	0,149	-6,2818	54,6861
		CAPM	18,60518	12,90422	0,322	-11,8788	49,0891
	ELECTRE	PSI	-24,20218	12,90422	0,149	-54,6861	6,2818
		CAPM	-5,59700	11,54188	0,879	-32,8627	21,6687
	CAPM	PSI	-18,60518	12,90422	0,322	-49,0891	11,8788
		ELECTRE	5,59700	11,54188	0,879	-21,6687	32,8627
Scheffe	PSI	ELECTRE	24,20218	12,90422	0,175	-7,6361	56,0404
		CAPM	18,60518	12,90422	0,356	-13,2331	50,4434
	ELECTRE	PSI	-24,20218	12,90422	0,175	-56,0404	7,6361
		CAPM	-5,59700	11,54188	0,889	-34,0740	22,8800
	CAPM	PSI	-18,60518	12,90422	0,356	-50,4434	13,2331
		ELECTRE	5,59700	11,54188	0,889	-22,8800	34,0740

Source: SPSS output, September 2012.

c.3.) Follow-up, 3 years

Table nº 148 – ANOVA results to average profitability (3rd follow-up period, 2nd sub period)

ANOVA					
PROFITABILITY	Sum of Squares	Df	Mean Square	F	Sig.
Between Groups	0,005	2	0,003	0,658	0,519
Within Groups	0,738	189	0,004		
Total	0,743	191			

Source: SPSS output, September 2012.

Table nº 149 – ANOVA Multiple comparisons results to average profitability (3rd follow-up period, 2nd sub period)

Multiple Comparisons							
Dependent Variable:PROFITABILITY							
	(I) METHOD	**(J) METHOD**	**Mean Difference (I-J)**	**Std. Error**	**Sig.**	**95% Confidence Interval**	
						Lower Bound	**Upper Bound**
Tukey HSD	**PSI**	**ELECTRE**	-0,01262	0,01164	0,526	-0,0401	0,0149
		CAPM	-0,01107	0,01164	0,609	-0,0386	0,0164
	ELECTRE	**PSI**	0,01262	0,01164	0,526	-0,0149	0,0401
		CAPM	0,00154	0,01042	0,988	-0,0231	0,0261
	CAPM	**PSI**	0,01107	0,01164	0,609	-0,0164	0,0386
		ELECTRE	-0,00154	0,01042	0,988	-0,0261	0,0231
Scheffe	**PSI**	**ELECTRE**	-0,01262	0,01164	0,557	-0,0413	0,0161
		CAPM	-0,01107	0,01164	0,637	-0,0398	0,0177
	ELECTRE	**PSI**	0,01262	0,01164	0,557	-0,0161	0,0413
		CAPM	0,00154	0,01042	0,989	-0,0242	0,0272
	CAPM	**PSI**	0,01107	0,01164	0,637	-0,0177	0,0398
		ELECTRE	-0,00154	0,01042	0,989	-0,0272	0,0242

Source: SPSS output, September 2012.

For third follow-up period, results obtained previously are confirmed. From table 148, we can see that with an error probability of 5%, we can conclude that average profitability in at least two ways of determining portfolio isn't significant different (*p-value* = 0,519 > α = 0,05, so we cannot reject H_0). To take conclusion about multiple comparisons, table 149 tells us that no way of determining average profitabilities is different (*p-value* for every comparison is bigger than α = 0,05) for a probability error of 5%, answering to *T1* and *T3*.

Table nº 150 – Oneway - ANOVA results to Sharpe's Index (3rd follow-up period, 2nd sub period)

ANOVA					
SHARPE	Sum of Squares	df	Mean Square	F	Sig.
Between Groups	7491,108	2	3745,554	0,760	0,469
Within Groups	931973,847	189	4931,079		
Total	939464,955	191			

Source: SPSS output, September 2012.

Table nº 151 – ANOVA Multiple comparisons results to Sharpe's Index period, 2nd sub period)

Multiple Comparisons							
Dependent Variable:SHARPE							
	(I) METHOD	(J) METHOD	Mean Difference (I-J)	Std. Error	Sig.	95% Confidence Interval	
						Lower Bound	Upper Bound
Tukey HSD	PSI	ELECTRE	8,95700	13,08503	0,773	-21,9541	39,8681
		CAPM	16,10375	13,08503	0,437	-14,8074	47,0149
	ELECTRE	PSI	-8,95700	13,08503	0,773	-39,8681	21,9541
		CAPM	7,14675	11,70361	0,814	-20,5010	34,7945
	CAPM	PSI	-16,10375	13,08503	0,437	-47,0149	14,8074
		ELECTRE	-7,14675	11,70361	0,814	-34,7945	20,5010
Scheffe	PSI	ELECTRE	8,95700	13,08503	0,791	-23,3274	41,2414
		CAPM	16,10375	13,08503	0,470	-16,1806	48,3881
	ELECTRE	PSI	-8,95700	13,08503	0,791	-41,2414	23,3274
		CAPM	7,14675	11,70361	0,830	-21,7293	36,0228
	CAPM	PSI	-16,10375	13,08503	0,470	-48,3881	16,1806
		ELECTRE	-7,14675	11,70361	0,830	-36,0228	21,7293

Source: SPSS output, September 2012.

For third follow-up period, results obtained previously are confirmed. From table 150, we can see that with an error probability of 5%, we can conclude that average Sharpe's index in at least two ways of determining portfolio isn't significant different (*p-value* = 0,469 > α = 0,05, so we cannot reject H_0). To take conclusion about multiple comparisons, table 151 tells us that no way of determining average Sharpe's index is different (*p-value* for every comparison is bigger than α = 0,05) for a probability error of 5%, answering to *T2* and *T4*.

All results obtained, by pair of group and by follow-up period, lead us to conclude that in every follow-up we cannot infer that one way of calculating portfolio's average profitability and portfolio's average Sharpe's index is better than the other, statistically. Although average profitability and average Sharpe's index are greater when using ELECTRE method in a 3 years holding, in absolute terms, probably due to sample limitations (few observations) statistical results are not able to confirm this conclusions.

4.4.2.2.1.3. Compare means of both subperiods (t-student test to compare two groups)

This *t*-student test procedure compares means for two groups of cases (1^{st} sub period versus 2^{nd} sub period), for each follow-up period, and give information if their mean are, or not, significant different, with a 95% confidence interval. Our test hypotheses are:

H_o : $\mu_i - \mu_k = 0$ (average profitability/Sharpe's index of $Group_i$ isn't significant different from average profitability/Sharpe's index of $Group_k$, with i, k = 1^{st} sub period and 2^{nd} sub period).

H_1 : $\mu_i - \mu_k \neq 0$ (average profitability/Sharpe's index of $Group_i$ isn't significant different from average profitability/Sharpe's index of $Group_k$, with i, k = 1^{st} sub period and 2^{nd} sub period).

a) Follow-up, 1 year

Table nº 152 – 1st and 2nd sub periods main statistics to average profitability (1st follow-up period)

Group Statistics					
SUBPERIOD		N	Mean	Std. Deviation	Std. Error Mean
PROFITABILITY	1ST FOLLOW-UP SUBPERIOD	108	0,010788	0,0525198	0,0050537
	2ND FOLLOW-UP SUBPERIOD	144	-0,010830	0,0649743	0,0054145

Source: SPSS output, September 2012.

Table nº 153 – 1st and 2nd sub periods comparisations results (1st follow-up period)

Independent Samples Test									
PROFITABILITY	Levene's Test for Equality of Variances		t-test for Equality of Means						
	F	Sig.	T	Df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference	
								Lower	Upper
Equal variances assumed	3,626	0,058	2,832	250	0,005	0,0216178	0,0076327	0,0065852	0,0366504
Equal variances not assumed			2,919	248,566	0,004	0,0216178	0,0074066	0,0070302	0,0362054

Source: SPSS output, September 2012.

Table nº 154 – 1st and 2nd sub periods main statistics to Sharpe's Index (1st follow-up period)

Group Statistics					
SUBPERIOD		N	Mean	Std. Deviation	Std. Error Mean
SHARPE	1SUBPERIOD	108	18,2429	42,14817	4,05571
	2SUBPERIOD	144	-13,6581	61,21314	5,10109

Source: SPSS output, September 2012.

Table nº 155 – 1st and 2nd sub periods comparisations results to Sharpe's Index (1st follow-up period)

Independent Samples Test									
SHARPE	Levene's Test for Equality of Variances		t-test for Equality of Means						
	F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference Lower	Upper
Equal variances assumed	10,104	0,002	4,651	250	0,000	31,90100	6,85928	18,39166	45,41035
Equal variances not assumed			4,895	248,320	0,000	31,90100	6,51690	19,06556	44,73644

Source: SPSS output, September 2012.

Results obtained to average profitability (tables nº 152 and 153) leads us to reject H_0, meaning that these two groups are statistically different seeing *p-value* = 0,005 < α = 0,05 (t (250) = 2,832; p = 0,005).

Results obtained average Sharpe's index (tables nº 154 and 155) leads us to reject H_0, meaning that these two groups are statistically different seeing *p-value* = 0,000 < α = 0,10 (t (250) = 4,651; p = 0,000).

b) Follow-up, 2 years

Table nº 156 – 1st and 2nd sub periods main statistics to average profitability (2nd follow-up period)

Group Statistics					
SUBPERIOD		N	Mean	Std. Deviation	Std. Error Mean
PROFITABILITY	**1ST FOLLOW-UP SUBPERIOD**	192	0,004948	0,0572844	0,0041341
	2ND FOLLOW-UP SUBPERIOD	192	-0,005930	0,0633134	0,0045693

Source: SPSS output, September 2012.

Table nº 157 – 1st and 2nd sub periods comparisations results to average profitability (2nd follow-up period)

Independent Samples Test									
PROFITABILITY	**Levene's Test for Equality of Variances**		**t-test for Equality of Means**						
								95% Confidence Interval of the Difference	
	F	**Sig.**	**t**	**df**	**Sig. (2-tailed)**	**Mean Difference**	**Std. Error Difference**	**Lower**	**Upper**
Equal variances assumed	2,052	0,153	1,765	382	0,078	0,0108781	0,0061619	-0,0012374	0,0229936
Equal variances not assumed			1,765	378,238	0,078	0,0108781	0,0061619	-0,0012378	0,0229940

Source: SPSS output, September 2012.

Table nº 158 – 1st and 2nd sub periods main statistics to Sharpe's Index (2nd follow-up period)

Group Statistics					
SUBPERIOD		N	Mean	Std. Deviation	Std. Error Mean
SHARPE	1SUBPERIOD	192	8,7758	47,40069	3,42085
	2SUBPERIOD	192	-9,9924	69,55412	5,01964

Source: SPSS output, September 2012.

Table nº 159 – 1st and 2nd sub periods comparisations results to Sharpe's Index (2nd follow-up period)

Independent Samples Test									
SHARPE	Levene's Test for Equality of Variances		t-test for Equality of Means						
	F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference Lower	Upper
Equal variances assumed	21,479	0,000	3,090	382	0,002	18,76817	6,07445	6,82462	30,71171
Equal variances not assumed			3,090	336,936	0,002	18,76817	6,07445	6,81954	30,71679

Source: SPSS output, September 2012.

Results obtained to average profitability for a two holding period (tables nº 156 and 157) do not allow us to reject H_0, meaning these two groups aren't statistically different seeing *p-value* = 0,078 > α = 0,050 (t (382) = 1,765; p = 0,078) despite, to a error probability of 10%, we can reject H_0 meaning average profitability for these two groups can be differentiated statistically (*p-value* = 0,078 < α = 0,10).

Results obtained to average Sharpe's index for a two holding period (tables nº 158 and 159) allow us to reject H_0, so these two groups are statistically different seeing *p-value* = 0,002 < α = 0,10 (t (382) = 3,090; p = 0,002).

c) Follow-up, 3 years

Table nº 160 – 1st and 2nd sub periods main statistics to average profitability (3rd follow-up period)

Group Statistics						
SUBPERIOD		N	Mean	Std. Deviation	Std. Error Mean
PROFITABILITY	1SUBPERIOD	277	0,0051	0,06048	0,00363
	2SUBPERIOD	192	-0,0066	0,06238	0,00450

Source: SPSS output, September 2012.

Table nº 161 – 1st and 2nd sub periods comparisations results to average profitability (3rd follow-up period)

Independent Samples Test									
PROFITABILITY	Levene's Test for Equality of Variances		t-test for Equality of Means						
								95% Confidence Interval of the Difference	
	F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	Lower	Upper
Equal variances assumed	0,389	0,533	2,028	467	0,043	0,01167	0,00575	0,00036	0,02298
Equal variances not assumed			2,017	402,698	0,044	0,01167	0,00579	0,00030	0,02304

Source: SPSS output, September 2012.

Table nº 162 – 1st and 2nd sub periods main statistics to Sharpe's Index (3rd follow-up period)

Group Statistics					
SUBPERIOD		N	Mean	Std. Deviation	Std. Error Mean
SHARPE	1SUBPERIOD	277	7,4550	53,93730	3,24078
	2SUBPERIOD	191	-12,4139	70,21460	5,08055

Source: SPSS output, September 2012.

Table nº 163 – 1st and 2nd sub periods comparisations results to Sharpe's Index (3rd follow-up period)

Independent Samples Test									
SHARPE	Levene's Test for Equality of Variances		t-test for Equality of Means						
	F	Sig.	t	df	Sig. (2-tailed)	Mean Difference	Std. Error Difference	95% Confidence Interval of the Difference Lower	Upper
Equal variances assumed	11,055	0,001	3,458	466	0,001	19,86893	5,74654	8,57659	31,16126
Equal variances not assumed			3,297	337,598	0,001	19,86893	6,02616	8,01537	31,72248

Source: SPSS output, September 2012.

Results obtained to average profitability for a three holding period (tables nº 160 and 161) allow us to reject H_0, meaning these two groups are statistically different seeing *p-value* = 0,043 < α = 0,10 (t (467) = 2,028; p = 0,043).

Results obtained to average Sharpe's index for a three holding period (tables nºs 162 and 163) allow us to reject H_0, meaning these two groups are statistically different seeing *p-value* = 0,001 < α = 0,10 (t (466) = 3,458; p = 0,001).

From all this, and for all holding period, results allow us to reject H_0, (except average profitability in a two years holding with $\alpha = 0{,}05$) meaning portfolio's average profitability and portfolio's average Sharpe's index for these two sub periods are statistically different.

4.4.2.2.2. Nonparametric tests

As already mentioned, nonparametric tests are an alternative to parametric tests, because they do no require the normality of the variable and the variance homogeneity: nonparametric tests are distribution-free tests. One of the alternatives is to conduct a Wilcoxon-Mann-Whitney test, (Mann & Whitney, 1947), witch is the nonparametric test analog to *t*-student for independent samples. According to Maroco (2007), this test has a 95,5% efficiency of *t*-student test. So, having calculated average profitability and Sharpe's index, what we want to test is if distribution of one way is high than distribution obtained with other way, with an error probability of 10%. Therefore, our test hypotheses are:

H_0: $F(X_i) \geq F(X_j)$ ($F(X_j$ and $F(X_j))$ are function distribution, with i, j = PSI, ELECTRE and CAPM).

H_1: $F(X_i) < F(X_j)$ (i, j = PSI, ELECTRE and CAPM)

We also conducted Kruskal-Wallis tests (Kruskal and Wallis, 1952), witch is similar to ANOVA one-way test. Our hypotheses are,

H_0: $F(X_i) = F(X_j) = F(X_k)$ (distributions of dependent variable are identical in k populations).

H_1: $F(X_i) \neq F(X_j)$ (exists at least one population where dependent variable distribution is different from one of the distributions from others populations).

4.4.2.2.2.1. 1st follow-up sub period

a) Follow-up, 1 year

Table nº 164 – Kruskal-Wallis Test to average profitability (1 y holding, 1st sub period)

Test Statistics[a,b]	
PROFITABILITY	
Chi-Square	0,323
Df	2
Asymp. Sig.	0,851
a. Kruskal Wallis Test	
b. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Table nº 165 – Kruskal-Wallis Test to Sharpe's Index (1 y holding, 1st sub period)

Test Statistics[a,b]	
SHARPE	
Chi-Square	1,582
Df	2
Asymp. Sig.	0,453
a. Kruskal Wallis Test	
b. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Analyzing results obtained to average profitability (table nº 164), and based on *p-value* = 0,851 > α = 0,10, we cannot reject H_0, meaning that no way of calculating portfolio's average profitability is greater than the other way.

Analyzing results obtained to Sharpe's index (table nº 165), and based on *p-value* = 0,453 > α = 0,10, we cannot reject H_0, meaning that no way of calculating portfolio's average Sharpe's index is greater than the other way.

a.1.) PSI vs ELECTRE

Table nº 166 - Mann-Whitney Test to average profitability (Ranks for 1 y holding, 1st sub period)

Ranks				
METHOD_GROUP		N	Mean Rank	Sum of Ranks
PROFITABILITY	PSI	36	35,78	1288,00
	ELECTRE	36	37,22	1340,00
	Total	72		

Source: SPSS output, September 2012.

Table nº 167 - Mann-Whitney Test to average profitability (Test statistics for 1 y holding, 1st sub period)

Test Statistics[a]	
	PROFITABILITY
Mann-Whitney U	622,000
Wilcoxon W	1288,000
Z	-0,293
Asymp. Sig. (2-tailed)	0,770
Exact Sig. (2-tailed)	0,773
Exact Sig. (1-tailed)	0,387
Point Probability	0,002
a. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Results allow us to conclude that (see tables 166 and 167), seeing *p-value* = 0,387 > α =0,10, we cannot reject H_0. So, average profitability of PSI is greater than average profitability obtained with ELECTRE, being these conclusions not consistent with observed data: mean rank is higher to ELECTRE (37,22) than to PSI (35,78), answering to *T1*.

Table nº 168 - Mann-Whitney Test to Sharpe's Index (Ranks for 1 y holding, 1st sub period)

Ranks				
METHOD		N	Mean Rank	Sum of Ranks
SHARPE	PSI	36	33,39	1202,00
	ELECTRE	36	39,61	1426,00
	Total	72		

Source: SPSS output, September 2012.

Table nº 169 - Mann-Whitney Test to Sharpe's Index (Test statistics for 1 y holding, 1st sub period)

Test Statistics[a]	
	SHARPE
Mann-Whitney U	536,000
Wilcoxon W	1,202E3
Z	-1,261
Asymp. Sig. (2-tailed)	0,207
Exact Sig. (2-tailed)	0,211
Exact Sig. (1-tailed)	0,105
Point Probability	0,002
a. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Results allow us to conclude that (tables nº 168 and 169), seeing *p-value* = 0,105 > α =0,10, we cannot reject H_0. So, average Sharpe's index of PSI is greater than average Sharpe's index obtained with ELECTRE, being these conclusions not consistent with observed data: mean rank is higher to ELECTRE (39,61) than to PSI (33,39), answering to *T2*.

a.2.) PSI vs CAPM

Table nº 170 - Mann-Whitney Test to average profitability (Ranks for 1 y holding, 1st sub period)

Ranks				
METHOD_GROUP		**N**	**Mean Rank**	**Sum of Ranks**
PROFITABILITY	**PSI**	36	35,08	1263,00
	CAPM	36	37,92	1365,00
	Total	72		

Source: SPSS output, September 2012.

Table nº 171 - Mann-Whitney Test to average profitability (Test statistics for 1 y holding, 1st sub period)

Test Statistics[a]	
	PROFITABILITY
Mann-Whitney U	597,000
Wilcoxon W	1263,000
Z	-0,574
Asymp. Sig. (2-tailed)	0,566
Exact Sig. (2-tailed)	0,572
Exact Sig. (1-tailed)	0,286
Point Probability	0,004
a. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Results allow us to concluded that (tables nºs 170 and 171), seeing *p-value* = 0,286 > α =0,10, we cannot reject H_0. So, average profitability of PSI is greater than average profitability obtained with CAPM, being these conclusions not consistent with observed data: mean rank is higher to CAPM (37,92) than to PSI (35,08).

Table nº 172 - Mann-Whitney Test to Sharpe's Index (Ranks for 1 y holding, 1st sub period)

Ranks				
METHOD		N	Mean Rank	Sum of Ranks
SHARPE	PSI	36	34,75	1251,00
	CAPM	36	38,25	1377,00
	Total	72		

Source: SPSS output, September 2012.

Table nº 173 - Mann-Whitney Test to Sharpe's Index (Test statistics for 1 y holding, 1st sub period)

Test Statistics[a]	
	SHARPE
Mann-Whitney U	585,000
Wilcoxon W	1,251E3
Z	-0,710
Asymp. Sig. (2-tailed)	0,478
Exact Sig. (2-tailed)	0,484
Exact Sig. (1-tailed)	0,242
Point Probability	0,003
a. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Results allow us to concluded that (tables nº 172 and 173), seeing *p-value* = 0,242 > α = 0,10, we cannot reject H_0. So, average Sharpe's index of PSI is greater than average Sharpe's index obtained with CAPM, being these conclusions not consistent with observed data: mean rank is higher to CAPM (38,25) than to PSI (34,75).

a.3.) ELECTRE vs CAPM

Table nº 174 - Mann-Whitney Test to average profitability (Ranks for 1 y holding, 1st sub period)

Ranks				
METHOD_GROUP		N	Mean Rank	Sum of Ranks
PROFITABILITY	ELECTRE	36	35,86	1291,00
	CAPM	36	37,14	1337,00
	Total	72		

Source: SPSS output, September 2012.

Table nº 175 - Mann-Whitney Test to average profitability (Test statistics for 1 y holding, 1st sub period)

Test Statistics[a]	
	PROFITABILITY
Mann-Whitney U	625,000
Wilcoxon W	1291,000
Z	-0,259
Asymp. Sig. (2-tailed)	0,796
Exact Sig. (2-tailed)	0,799
Exact Sig. (1-tailed)	0,399
Point Probability	0,002
a. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Results, expressed in tables 174 and 175 allow us to concluded that, seeing *p-value* = 0,399 > α =0,10, we cannot reject H_0. So, average profitability of ELECTRE is greater than average profitability obtained with CAPM, being these conclusions not consistent with observed data: mean rank is higher to CAPM (37,14) than to ELECTRE (35,86), answering to *T3*.

Table nº 176 - Mann-Whitney Test to Sharpe's Index (Ranks for 1 y holding, 1st sub period)

Ranks				
METHOD		N	Mean Rank	Sum of Ranks
SHARPE	ELECTRE	36	37,78	1360,00
	CAPM	36	35,22	1268,00
	Total	72		

Source: SPSS output, September 2012.

Table nº 177 - Mann-Whitney Test to Sharpe's Index (Test statistics for 1 y holding, 1st sub period)

Test Statistics[a]	
	SHARPE
Mann-Whitney U	602,000
Wilcoxon W	1,268E3
Z	-0,518
Asymp. Sig. (2-tailed)	0,604
Exact Sig. (2-tailed)	0,610
Exact Sig. (1-tailed)	0,305
Point Probability	0,004
a. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Results allow us to concluded that (tables nºs 176 and 177), seeing *p-value* = 0,305 > α =0,10, we cannot reject H_0. So, average Sharpe's index of ELECTRE is greater than average Sharpe's index obtained with CAPM, being these conclusions consistent with observed data: mean rank is higher to ELECTRE (37,78) than to CAPM (35,22), answering to *T4*.

b) Follow-up, 2 years

Table nº 178 – Kruskal-Wallis Test to average profitability (2 y holding, 1st sub period)

Test Statistics[a,b]	
PROFITABILITY	
Chi-Square	0,743
Df	2
Asymp. Sig.	0,690
a. Kruskal Wallis Test b. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Table nº 179 – Kruskal-Wallis Test to Sharpe's Index (2 y holding, 1st sub period)

Test Statistics[a,b]	
SHARPE	
Chi-Square	1,136
Df	2
Asymp. Sig.	0,567
a. Kruskal Wallis Test b. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Analyzing results obtained to average profitability (table nº 178), and based on *p-value* = 0,690 > α = 0,10, we cannot reject H_0, meaning that no way of determining portfolio's average profitability is greater than the other way.

Analyzing results obtained to average Sharpe's index (table nº 179), and based on *p-value* = 0,567 > α = 0,10, we cannot reject H_0, meaning that no way of determining portfolio's average Sharpe's index is greater than the other way.

b.1.) PSI vs ELECTRE

Table nº 180 - Mann-Whitney Test to average profitability (Ranks for 2 y holding, 1st sub period)

Ranks				
METHOD_GROUP		**N**	**Mean Rank**	**Sum of Ranks**
PROFITABILITY	**PSI**	48	57,26	2748,50
	ELECTRE	72	62,66	4511,50
	Total	120		

Source: SPSS output, September 2012.

Table nº 181 - Mann-Whitney Test to average profitability (Test statistics for 2 y holding, 1st sub period)

Test Statistics[a]	
	PROFITABILITY
Mann-Whitney U	1572,500
Wilcoxon W	2748,500
Z	-0,833
Asymp. Sig. (2-tailed)	0,405
Exact Sig. (2-tailed)	0,407
Exact Sig. (1-tailed)	0,204
Point Probability	0,001
a. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Results allow us to conclude that (expressed in tables 180 and 181), seeing *p-value* = 0,204 > α =0,10, we cannot reject H_0. So, average profitability of PSI is greater than average profitability obtained with ELECTRE, being these conclusions not consistent with observed data: mean rank is higher to ELECTRE (62,66) than to PSI (57,26), answering to *T1*.

Table nº 182 - Mann-Whitney Test to Sharpe's Index (Ranks for 2 y holding, 1st sub period)

Ranks				
METHOD		N	Mean Rank	Sum of Ranks
SHARPE	PSI	48	56,48	2711,00
	ELECTRE	72	63,18	4549,00
	Total	120		

Source: SPSS output, September 2012.

Table nº 183 - Mann-Whitney Test to Sharpe's Index (Test statistics for 2 y holding, 1st sub period)

Test Statistics[a]	
	SHARPE
Mann-Whitney U	1,535E3
Wilcoxon W	2,711E3
Z	-1,034
Asymp. Sig. (2-tailed)	0,301
Exact Sig. (2-tailed)	0,304
Exact Sig. (1-tailed)	0,152
Point Probability	0,001
a. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Results allow us to concluded that (see tables 182 and 183), seeing *p-value* = 0,152 > α =0,10, we cannot reject H_0. So, average Sharpe's index of PSI is greater than average Sharpe's index obtained with ELECTRE, being these conclusions not consistent with observed data: mean rank is higher to ELECTRE (63,18) than to PSI (56,48), answering to *T2*.

b.2.) PSI vs CAPM

Table nº 184 - Mann-Whitney Test to average profitability (Ranks for 2 y holding, 1st sub period)

Ranks				
METHOD_GROUP		**N**	**Mean Rank**	**Sum of Ranks**
PROFITABILITY	**PSI**	48	57,88	2778,00
	CAPM	72	62,25	4482,00
	Total	120		

Source: SPSS output, September 2012.

Table nº 185 - Mann-Whitney Test to average profitability (Test statistics for 2 y holding, 1st sub period)

Test Statistics[a]	
PROFITABILITY	
Mann-Whitney U	1602,000
Wilcoxon W	2778,000
Z	-0,675
Asymp. Sig. (2-tailed)	0,500
Exact Sig. (2-tailed)	0,502
Exact Sig. (1-tailed)	0,251
Point Probability	0,001
a. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Results allow us to concluded that (tables 184 and 185), seeing *p-value* = 0,251 > α =0,10, we cannot reject H_0. So, average profitability of PSI is greater than average profitability obtained with CAPM, being these conclusions not consistent with observed data: mean rank is higher to CAPM (62,25) than to PSI (57,88).

Table nº 186 - Mann-Whitney Test to Sharpe's Index (Ranks for 2 y holding, 1st sub period)

Ranks				
METHOD		N	Mean Rank	Sum of Ranks
SHARPE	PSI	48	60,08	2884,00
	CAPM	72	60,78	4376,00
	Total	120		

Source: SPSS output, September 2012.

Table nº 187 - Mann-Whitney Test to Sharpe's Index (Test statistics for 2 y holding, 1st sub period)

Test Statistics[a]	
	SHARPE
Mann-Whitney U	1,708E3
Wilcoxon W	2,884E3
Z	-0,107
Asymp. Sig. (2-tailed)	0,915
Exact Sig. (2-tailed)	0,917
Exact Sig. (1-tailed)	0,459
Point Probability	0,002
a. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Results allow us to concluded that (see tables nº 186 and 187), seeing *p-value* = 0,459 > α =0,10, we cannot reject H_0. So, average Sharpe's index of PSI is greater than average Sharpe's index obtained with CAPM, being these conclusions not consistent with observed data: mean rank is higher to CAPM (60,78) than to PSI (60,08).

b.3.) ELECTRE vs CAPM

Table nº 188 - Mann-Whitney Test to average profitability (Ranks for 2 y holding, 1st sub period)

Ranks				
METHOD_GROUP		N	Mean Rank	Sum of Ranks
PROFITABILITY	ELECTRE	72	73,10	5263,00
	CAPM	72	71,90	5177,00
	Total	144		

Source: SPSS output, September 2012.

Table nº 189 - Mann-Whitney Test to average profitability (Test statistics for 2 y holding, 1st sub period)

Test Statistics[a]	
	PROFITABILITY
Mann-Whitney U	2549,000
Wilcoxon W	5177,000
Z	-0,172
Asymp. Sig. (2-tailed)	0,864
Exact Sig. (2-tailed)	0,865
Exact Sig. (1-tailed)	0,432
Point Probability	0,001
a. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Results indicated in tables nºs 188 and 189 allow us to concluded that, seeing *p-value* = 0,432 > α =0,10, we cannot reject H_0. So, average profitability of ELECTRE is greater than average profitability obtained with CAPM, being these conclusions consistent with observed data: mean rank is higher to ELECTRE (73,10) than to CAPM (71,90), answering to *T2*.

Table nº 190 - Mann-Whitney Test to Sharpe's Index (Ranks for 2 y holding, 1st sub period)

Ranks				
METHOD		N	Mean Rank	Sum of Ranks
SHARPE	ELECTRE	72	75,25	5418,00
	CAPM	72	69,75	5022,00
	Total	144		

Source: SPSS output, September 2012.

Table nº 191 - Mann-Whitney Test to Sharpe's Index (Test statistics for 2 y holding, 1st sub period)

Test Statistics[a]	
	SHARPE
Mann-Whitney U	2,394E3
Wilcoxon W	5,022E3
Z	-0,791
Asymp. Sig. (2-tailed)	0,429
Exact Sig. (2-tailed)	0,431
Exact Sig. (1-tailed)	0,216
Point Probability	0,001
a. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Results allow us to conclude that (tables nºs 190 and 191), seeing *p-value* = 0,216 > α =0,10, we cannot reject H_0. So, average Sharpe's index of ELECTRE is greater than average Sharpe's index obtained with CAPM, being these conclusions consistent with observed data: mean rank is higher to ELECTRE (75,25) than to CAPM (69,75), answering to *T4*.

c) Follow-up, 3 years

Table nº 192 – Kruskal-Wallis Test to average profitability (3 y holding, 1st sub period)

Test Statistics[a,b]	
PROFITABILITY	
Chi-Square	0,743
Df	2
Asymp. Sig.	0,690
a. Kruskal Wallis Test b. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Table nº 193 – Kruskal-Wallis Test to Sharpe's Index (3 y holding, 1st sub period)

Test Statistics[a,b]	
SHARPE	
Chi-Square	0,698
Df	2
Asymp. Sig.	0,706
a. Kruskal Wallis Test b. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Analyzing results obtained to average profitability (table nº 192), and based on *p-value* = 0,690 > α = 0,10, we cannot reject H_0, meaning that no way of calculating portfolio's average profitability is greater than the other way.

Looking to results obtained to Sharpe's index (table nº 193), and based on *p-value* = 0,706 > α = 0,10, we cannot reject H_0, meaning that no way of calculating portfolio's average Sharpe's index is greater than the other way.

c.1.) PSI vs ELECTRE

Table nº 194 - Mann-Whitney Test to average profitability (Ranks for 3 y holding, 1st sub period)

Ranks				
METHOD		N	Mean Rank	Sum of Ranks
PROFITABILITY	PSI	60	83,61	5016,50
	ELECTRE	108	85,00	9179,50
	Total	168		

Source: SPSS output, September 2012.

Table nº 195 - Mann-Whitney Test to average profitability (Test statistics for 3 y holding, 1st sub period)

Test Statistics[a]	
	PROFITABILITY
Mann-Whitney U	3186,500
Wilcoxon W	5016,500
Z	-0,177
Asymp. Sig. (2-tailed)	0,859
Exact Sig. (2-tailed)	0,860
Exact Sig. (1-tailed)	0,430
Point Probability	0,001
a. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Results allow us to concluded that (see tables 194 and 195), seeing *p-value* = 0,430 > α =0,10, we cannot reject H_0. So, average profitability of PSI is greater than average profitability obtained with ELECTRE, being these conclusions not consistent with observed data: mean rank is higher to ELECTRE (85,00) than to PSI (83,61), answering to *T1*.

Table nº 196 - Mann-Whitney Test to Sharpe's Index (Ranks for 3 y holding, 1st sub period)

Ranks				
METHOD		**N**	**Mean Rank**	**Sum of Ranks**
SHARPE	**PSI**	60	81,73	4904,00
	ELECTRE	108	86,04	9292,00
	Total	168		

Source: SPSS output, September 2012.

Table nº 197 - Mann-Whitney Test to Sharpe's Index (Test statistics for 3 y holding, 1st sub period)

Test Statistics[a]	
	SHARPE
Mann-Whitney U	3,074E3
Wilcoxon W	4,904E3
Z	-0,550
Asymp. Sig. (2-tailed)	0,583
Exact Sig. (2-tailed)	0,585
Exact Sig. (1-tailed)	0,292
Point Probability	0,001
a. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Results allow us to concluded that (see tables 196 and 197), seeing *p-value* = 0,292 > α =0,10, we cannot reject H_0. So, average Sharpe's index of PSI is greater than average Sharpe's index obtained with ELECTRE, being these conclusions not consistent with observed data: mean rank is higher to ELECTRE (86,04) than to PSI (81,73), answering to *T2*.

c.2.) PSI vs CAPM

Table nº 198 - Mann-Whitney Test to average profitability (Ranks for 3 y holding, 1st sub period)

Ranks				
METHOD_GROUP		**N**	**Mean Rank**	**Sum of Ranks**
PROFITABILITY	**PSI**	60	84,92	5095,00
	CAPM	108	84,27	9101,00
	Total	168		

Source: SPSS output, September 2012.

Table nº 199 - Mann-Whitney Test to average profitability (Test statistics for 3 y holding, 1st sub period)

Test Statistics[a]	
	PROFITABILITY
Mann-Whitney U	3215,000
Wilcoxon W	9101,000
Z	-0,083
Asymp. Sig. (2-tailed)	0,934
Exact Sig. (2-tailed)	0,935
Exact Sig. (1-tailed)	0,467
Point Probability	0,001
a. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Results allow us to concluded that (tables 198 and 199), seeing *p-value* = 0,467 > α =0,10, we cannot reject H_0. So, average profitability of PSI is greater than average profitability obtained with CAPM, being these conclusions consistent with observed data: mean rank is higher to PSI (84,92) than to CAPM (84,27).

Table nº 200 - Mann-Whitney Test to Sharpe's Index (Ranks for 3 y holding, 1st sub period)

Ranks				
METHOD		N	Mean Rank	Sum of Ranks
SHARPE	PSI	60	86,60	5196,00
	CAPM	108	83,33	9000,00
	Total	168		

Source: SPSS output, September 2012.

Table nº 201 - Mann-Whitney Test to Sharpe's Index (Test statistics for 3 y holding, 1st sub period)

Test Statistics[a]	
	SHARPE
Mann-Whitney U	3,114E3
Wilcoxon W	9,000E3
Z	-0,417
Asymp. Sig. (2-tailed)	0,677
Exact Sig. (2-tailed)	0,679
Exact Sig. (1-tailed)	0,339
Point Probability	0,001
a. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Results allow us to concluded that (tables 200 and 201), seeing *p-value* = 0,339 > α =0,10, we cannot reject H_0. So, average Sharpe's index of PSI is greater than average Sharpe's index obtained with CAPM, being these conclusions consistent with observed data: mean rank is higher to PSI (86,60) than to CAPM (83,33).

c.3.) ELECTRE vs CAPM

Table nº 202 - Mann-Whitney Test to average profitability (Ranks for 3 y holding, 1st sub period)

Ranks				
METHOD_GROUP		N	Mean Rank	Sum of Ranks
PROFITABILITY	ELECTRE	108	109,45	11820,50
	CAPM	108	107,55	11615,50
	Total	216		

Source: SPSS output, September 2012.

Table nº 203 - Mann-Whitney Test to average profitability (Test statistics for 3 y holding, 1st sub period)

Test Statistics[a]	
	PROFITABILITY
Mann-Whitney U	5729,500
Wilcoxon W	11615,500
Z	-0,223
Asymp. Sig. (2-tailed)	0,823
Exact Sig. (2-tailed)	0,824
Exact Sig. (1-tailed)	0,412
Point Probability	0,000

a. Grouping Variable: METHOD

Source: SPSS output, September 2012.

Results allow us to concluded that (see tables 202 and 203), seeing *p-value* = 0,412 > α =0,10, we cannot reject H_0. So, average profitability of ELECTRE is greater than average profitability obtained with CAPM, being these conclusions consistent with observed data: mean rank is higher to ELECTRE (109,45) than to CAPM (107,55), answering to *T3*.

Table nº 204 - Mann-Whitney Test to Sharpe's Index (Ranks for 3 y holding, 1st sub period)

Ranks				
METHOD		N	Mean Rank	Sum of Ranks
SHARPE	ELECTRE	108	111,67	12060,00
	CAPM	108	105,33	11376,00
	Total	216		

Source: SPSS output, September 2012.

Table nº 205 - Mann-Whitney Test to Sharpe's Index (Test statistics for 3 y holding, 1st sub period)

Test Statistics[a]	
	SHARPE
Mann-Whitney U	5,490E3
Wilcoxon W	1,138E4
Z	-0,745
Asymp. Sig. (2-tailed)	0,456
Exact Sig. (2-tailed)	0,458
Exact Sig. (1-tailed)	0,229
Point Probability	0,001
a. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Results allow us to concluded that (tables 204 and 205), seeing *p-value* = 0,229 > α =0,10, we cannot reject H_0. So, average Sharpe's index of ELECTRE is greater than average Sharpe's index obtained with CAPM, being these conclusions consistent with observed data: mean rank is higher to ELECTRE (111,67) than to CAPM (105,33), answering to *T4*.

All results obtained tell us that, in absolute terms, in a 3 years holding period, ELECTRE achieved high average profitability and average Sharpe's index than the market (PSI). Between CAPM and ELECTRE, ELECTRE behaved better than CAPM in three years holding period, meaning that in a long term period, portfolios formed with shares selected using ELECTRE III assumptions had better performance than portfolios formed under CAPM assumptions, in absolute terms, also achieving greater profitability by unit of risk. Despite these conclusions, statistically some means could be differentiated, based on Mann-Whitney test, for instance, Sharpe's Index for 1 year holding where ELECTRE behaved better than CAPM, average profitability and average Sharpe's index for 2 and 3 years holding, where ELECTRE behaved better than CAPM, average profitability and Sharpe's index for 3 years holding, where PSI behaved better than CAPM.

4.4.2.2.2.2. 2nd follow-up sub period

a) Follow-up, 1 year

Table nº 206 – Kruskal-Wallis Test to average profitability (1 y holding, 2nd sub period)

Test Statistics[a,b]	
PROFITABILITY	
Chi-Square	0,019
Df	2
Asymp. Sig.	0,990
a. Kruskal Wallis Test	
b. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Table nº 207 – Kruskal-Wallis Test to Sharpe's Index (1 y holding, 2nd sub period)

Test Statistics[a,b]	
SHARPE	
Chi-Square	5,197
Df	2
Asymp. Sig.	0,074
a. Kruskal Wallis Test	
b. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Analyzing results obtained to average profitability (see table 206), and based on *p-value* = 0,990 > α = 0,10, we cannot reject H_0, meaning that no way of calculating portfolio's average profitability is greater than the other way.

Analyzing results obtained to Sharpe's index (see table 207), and based on *p-value* = 0,074 < α = 0,10, we can reject H_0, meaning that certain way of calculating portfolio's average Sharpe's index is greater than the other way. But to α = 0,05 (*p-value* = 0,074 > α = 0,05) we cannot reject H_0, meaning that no way of calculating portfolio's average Sharpe's index is greater than the other way.

a.1.) PSI vs ELECTRE

Table nº 208 - Mann-Whitney Test to average profitability (Ranks for 1 y holding, 2nd sub period)

Ranks				
METHOD_GROUP		**N**	**Mean Rank**	**Sum of Ranks**
PROFITABILITY	**PSI**	48	48,60	2333,00
	ELECTRE	48	48,40	2323,00
	Total	96		

Source: SPSS output, September 2012.

Table nº 209 - Mann-Whitney Test to average profitability (Test statistics for 1 y holding, 2nd sub period)

Test Statistics[a]	
PROFITABILITY	
Mann-Whitney U	1147,000
Wilcoxon W	2323,000
Z	-0,037
Asymp. Sig. (2-tailed)	0,971
Exact Sig. (2-tailed)	0,972
Exact Sig. (1-tailed)	0,486
Point Probability	0,001
a. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Results, expressed in tables 208 and 209 allow us to concluded that, seeing *p-value* = 0,486 > α =0,10, we cannot reject H_0. So, average profitability of PSI is greater than average profitability obtained with ELECTRE, being these conclusions consistent with observed data: mean rank is higher to PSI (48,60) than to ELECTRE (48,40), answering to *T1*.

Table nº 210 - Mann-Whitney Test to Sharpe's Index (Ranks for 1 y holding, 2nd sub period)

Ranks				
METHOD		**N**	**Mean Rank**	**Sum of Ranks**
SHARPE	**PSI**	48	55,65	2671,00
	ELECTRE	48	41,35	1985,00
	Total	96		

Source: SPSS output, September 2012.

Table nº 211 - Mann-Whitney Test to Sharpe's Index (Test statistics for 1 y holding, 2nd sub period)

Test Statistics[a]	
SHARPE	
Mann-Whitney U	809,000
Wilcoxon W	1,985E3
Z	-2,513
Asymp. Sig. (2-tailed)	0,012
Exact Sig. (2-tailed)	0,012
Exact Sig. (1-tailed)	0,006
Point Probability	0,000
a. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Results, expressed in tables 210 and 211 allow us to concluded that, seeing *p-value* = 0,006 < α =0,10, we can reject H_0. So, average Sharpes' index of PSI isn't greater than average Sharpe's index obtained with ELECTRE, being these conclusions not consistent with observed data: mean rank is higher to PSI (55,65) than to ELECTRE (41,35), answering to *T2*.

a.2.) PSI vs CAPM

Table nº 212 - Mann-Whitney Test to average profitability (Ranks for 1 y holding, 2nd sub period)

Ranks				
METHOD_GROUP		**N**	**Mean Rank**	**Sum of Ranks**
PROFITABILITY	**PSI**	48	47,97	2302,50
	CAPM	48	49,03	2353,50
	Total	96		

Source: SPSS output, September 2012.

Table nº 213 - Mann-Whitney Test to average profitability (Test statistics for 1 y holding, 2nd sub period)

Test Statistics[a]	
PROFITABILITY	
Mann-Whitney U	1126,500
Wilcoxon W	2302,500
Z	-0,187
Asymp. Sig. (2-tailed)	0,852
Exact Sig. (2-tailed)	0,854
Exact Sig. (1-tailed)	0,427
Point Probability	0,001
a. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Results, expressed in tables 212 and 213 allow us to concluded that, seeing *p-value* = 0,427 > α =0,10, we cannot reject H_0. So, average profitability of PSI is greater than average profitability obtained with CAPM, being these conclusions not consistent with observed data: mean rank is higher to CAPM (49,03) than to PSI (47,97).

Table nº 214 - Mann-Whitney Test to Sharpe's Index (Ranks for 1 y holding, 2nd sub period)

Ranks				
METHOD		**N**	**Mean Rank**	**Sum of Ranks**
SHARPE	**PSI**	48	52,29	2510,00
	CAPM	48	44,71	2146,00
	Total	96		

Source: SPSS output, September 2012.

Table nº 215 - Mann-Whitney Test to Sharpe's Index (Test statistics for 1 y holding, 2nd sub period)

Test Statistics[a]	
SHARPE	
Mann-Whitney U	970,000
Wilcoxon W	2,146E3
Z	-1,334
Asymp. Sig. (2-tailed)	0,182
Exact Sig. (2-tailed)	0,184
Exact Sig. (1-tailed)	0,092
Point Probability	0,001
a. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Results, expressed in tables 214 and 215 allow us to concluded that, seeing *p-value* = 0,092 < α =0,10, we can reject H_0. So, average profitability of PSI isn't greater than average profitability obtained with CAPM, being these conclusions not consistent with observed data: mean rank is higher to PSI (52,29) than to CAPM (44,71).

a.3.) ELECTRE vs CAPM

Table nº 216 - Mann-Whitney Test to average profitability (Ranks for 1 y holding, 2nd sub period)

Ranks				
METHOD_GROUP		**N**	**Mean Rank**	**Sum of Ranks**
PROFITABILITY	**ELECTRE**	48	48,35	2321,00
	CAPM	48	48,65	2335,00
	Total	96		

Source: SPSS output, September 2012.

Table nº 217 - Mann-Whitney Test to average profitability (Test statistics for 1 y holding, 2nd sub period)

Test Statistics[a]	
PROFITABILITY	
Mann-Whitney U	1145,000
Wilcoxon W	2321,000
Z	-0,051
Asymp. Sig. (2-tailed)	0,959
Exact Sig. (2-tailed)	0,962
Exact Sig. (1-tailed)	0,481
Point Probability	0,003
a. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Results, expressed in tables 216 and 217 allow us to concluded that, seeing *p-value* = 0,481 > α =0,10, we cannot reject H_0. So, average profitability of ELECTRE is greater than average profitability obtained with CAPM, being these conclusions not consistent with observed data: mean rank is higher to CAPM (48,65) than to ELECTRE (48,35), answering to *T3*.

Table nº 218 - Mann-Whitney Test to Sharpe's Index (Ranks for 1 y holding, 2nd sub period)

Ranks				
METHOD		**N**	**Mean Rank**	**Sum of Ranks**
SHARPE	**ELECTRE**	48	48,06	2307,00
	CAPM	48	48,94	2349,00
	Total	96		

Source: SPSS output, September 2012.

Table nº 219 - Mann-Whitney Test to Sharpe's Index (Test statistics for 1 y holding, 2nd sub period)

Test Statistics[a]	
SHARPE	
Mann-Whitney U	1,131E3
Wilcoxon W	2,307E3
Z	-0,154
Asymp. Sig. (2-tailed)	0,878
Exact Sig. (2-tailed)	0,881
Exact Sig. (1-tailed)	0,441
Point Probability	0,003
a. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Results, expressed in tables 218 and 219 allow us to concluded that, seeing *p-value* = 0,441 > α =0,10, we cannot reject H_0. So, average Sharpe's index of ELECTRE is greater than average Sharpe's index obtained with CAPM, being these conclusions not consistent with observed data: mean rank is higher to CAPM (48,94) than to ELECTRE (48,06), answering to *T4*.

b) Follow-up, 2 years

Table nº 220 – Kruskal-Wallis Test to average profitability (2 y holding, 2nd sub period)

Test Statistics[a,b]	
PROFITABILITY	
Chi-Square	0,408
Df	2
Asymp. Sig.	0,815
a. Kruskal Wallis Test	
b. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Table nº 221 – Kruskal-Wallis Test to Sharpe's Index (2 y holding, 2nd sub period)

Test Statistics[a,b]	
SHARPE	
Chi-Square	6,836
Df	2
Asymp. Sig.	0,033
a. Kruskal Wallis Test	
b. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Analyzing results to average profitability (table nº 220), and based on *p-value* = 0,815 > α = 0,10, we cannot reject H_0, meaning that no way of calculating portfolio's average profitability is greater than the other way.

Regarding results obtained to Sharpe's index (table nº 221), and based on *p-value* = 0,033 < α = 0,10, we can reject H_0, meaning that a way of calculating portfolio's average Sharpe's index is greater than the other way.

b.1.) PSI vs ELECTRE

Table nº 222 - Mann-Whitney Test to average profitability (Ranks for 2 y holding, 2nd sub period)

Ranks				
METHOD_GROUP		**N**	**Mean Rank**	**Sum of Ranks**
PROFITABILITY	**PSI**	48	58,38	2802,00
	ELECTRE	72	61,92	4458,00
	Total	120		

Source: SPSS output, September 2012.

Table nº 223 - Mann-Whitney Test to average profitability (Test statistics for 2 y holding, 2nd sub period)

Test Statistics[a]	
PROFITABILITY	
Mann-Whitney U	1626,000
Wilcoxon W	2802,000
Z	-0,546
Asymp. Sig. (2-tailed)	0,585
Exact Sig. (2-tailed)	0,587
Exact Sig. (1-tailed)	0,294
Point Probability	0,001
a. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Results, expressed in tables 222 and 223 allow us to concluded that, seeing *p-value* = 0,294 > α =0,10, we cannot reject H_0. So, average profitability of PSI is greater than average profitability obtained with ELECTRE, being these conclusions not consistent with observed data: mean rank is higher to ELECTRE (61,92) than to PSI (58,38), answering to *T1*.

Table nº 224 - Mann-Whitney Test to Sharpe's Index (Ranks for 2 y holding, 2nd sub period)

Ranks				
METHOD		**N**	**Mean Rank**	**Sum of Ranks**
SHARPE	**PSI**	48	70,62	3390,00
	ELECTRE	72	53,75	3870,00
	Total	120		

Source: SPSS output, September 2012.

Table nº 225 - Mann-Whitney Test to Sharpe's Index (Test statistics for 2 y holding, 2nd sub period)

Test Statistics[a]	
SHARPE	
Mann-Whitney U	1,242E3
Wilcoxon W	3,870E3
Z	-2,603
Asymp. Sig. (2-tailed)	0,009
Exact Sig. (2-tailed)	0,009
Exact Sig. (1-tailed)	0,004
Point Probability	0,000
a. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Results, expressed in tables 224 and 225 allow us to concluded that, seeing *p-value* = 0,004 < α =0,10, we can reject H_0. So, average Sharpe's index of PSI isn't greater than average Sharpe's index obtained with ELECTRE, being these conclusions not consistent with observed data: mean rank is higher to PSI (70,62) than to ELECTRE (53,75), answering to *T2*.

b.2.) PSI vs CAPM

Table nº 226 - Mann-Whitney Test to average profitability (Ranks for 2 y holding, 2nd sub period)

Ranks				
METHOD_GROUP		**N**	**Mean Rank**	**Sum of Ranks**
PROFITABILITY	**PSI**	48	58,19	2793,00
	CAPM	72	62,04	4467,00
	Total	120		

Source: SPSS output, September 2012.

Table nº 227 - Mann-Whitney Test to average profitability (Test statistics for 2 y holding, 2nd sub period)

Test Statistics[a]	
PROFITABILITY	
Mann-Whitney U	1617,000
Wilcoxon W	2793,000
Z	-0,595
Asymp. Sig. (2-tailed)	0,552
Exact Sig. (2-tailed)	0,554
Exact Sig. (1-tailed)	0,277
Point Probability	0,001
a. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Results, expressed in tables 226 and 227 allow us to concluded that, seeing *p-value* = 0,2776 > α =0,10, we cannot reject H_0. So, average profitability of PSI is greater than average profitability obtained with CAPM, being these conclusions not consistent with observed data: mean rank is higher to CAPM (62,04) than to PSI (58,19).

Table nº 228 - Mann-Whitney Test to Sharpe's Index (Ranks for 2 y holding, 2nd sub period)

Ranks				
METHOD		**N**	**Mean Rank**	**Sum of Ranks**
SHARPE	**PSI**	48	68,21	3274,00
	CAPM	72	55,36	3986,00
	Total	120		

Source: SPSS output, September 2012.

Table nº 229 - Mann-Whitney Test to Sharpe's Index (Test statistics for 2 y holding, 2nd sub period)

Test Statistics[a]	
SHARPE	
Mann-Whitney U	1,358E3
Wilcoxon W	3,986E3
Z	-1,982
Asymp. Sig. (2-tailed)	0,047
Exact Sig. (2-tailed)	0,047
Exact Sig. (1-tailed)	0,024
Point Probability	0,000
a. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Results, expressed in tables 228 and 229 allow us to concluded that, seeing *p-value* = 0,024 < α =0,10, we can reject H_0. So, average Sharpe's index of PSI isn't greater than average Sharpe's index obtained with CAPM, being these conclusions not consistent with observed data: mean rank is higher to PSI (68,21) than to CAPM (55,36).

b.3.) ELECTRE vs CAPM

Table nº 230 - Mann-Whitney Test to average profitability (Ranks for 2 y holding, 2nd sub period)

Ranks					
METHOD_GROUP		**N**	**Mean Rank**	**Sum of Ranks**
PROFITABILITY	**ELECTRE**	72	72,53	5222,00
	CAPM	72	72,47	5218,00
	Total	144		

Source: SPSS output, September 2012.

Table nº 231 - Mann-Whitney Test to average profitability (Test statistics for 2 y holding, 2nd sub period)

Test Statistics[a]	
PROFITABILITY	
Mann-Whitney U	2590,000
Wilcoxon W	5218,000
Z	-0,008
Asymp. Sig. (2-tailed)	0,994
Exact Sig. (2-tailed)	0,994
Exact Sig. (1-tailed)	0,497
Point Probability	0,001
a. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Results, expressed in tables 230 and 231 allow us to concluded that, seeing *p-value* = 0,497 > α =0,10, we cannot reject H_0. So, average profitability of ELECTRE is greater than average profitability obtained with CAPM, being these conclusions consistent with observed data: mean rank is higher to ELECTRE (72,53) than to CAPM (72,47), answering to *T2*.

Table nº 232 - Mann-Whitney Test to Sharpe's Index (Ranks for 2 y holding, 2nd sub period)

Ranks				
METHOD		**N**	**Mean Rank**	**Sum of Ranks**
SHARPE	**ELECTRE**	72	71,01	5113,00
	CAPM	72	73,99	5327,00
	Total	144		

Source: SPSS output, September 2012.

Table nº 233 - Mann-Whitney Test to Sharpe's Index (Test statistics for 2 y holding, 2nd sub period)

Test Statistics[a]	
SHARPE	
Mann-Whitney U	2,485E3
Wilcoxon W	5,113E3
Z	-0,428
Asymp. Sig. (2-tailed)	0,669
Exact Sig. (2-tailed)	0,671
Exact Sig. (1-tailed)	0,336
Point Probability	0,001
a. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Results, expressed in tables 232 and 233, allow us to conclude that, seeing *p-value* = 0,336 > α =0,10, we cannot reject H_0. So, average Sharpe's index of ELECTRE is greater than average Sharpe's index obtained with CAPM, being these conclusions not consistent with observed data: mean rank is higher to CAPM (73,99) than to ELECTRE (71,01), answering to *T4*.

c) Follow-up, 3 years

Table nº 234 – Kruskal-Wallis Test to average profitability (3 y holding, 2nd sub period)

Test Statistics[a,b]	
PROFITABILITY	
Chi-Square	0,384
Df	2
Asymp. Sig.	0,825
a. Kruskal Wallis Test b. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Table nº 235 – Kruskal-Wallis Test to Sharpe's Index (3 y holding, 2nd sub period)

Test Statistics[a,b]	
SHARPE	
Chi-Square	4,199
df	2
Asymp. Sig.	0,123
a. Kruskal Wallis Test	
b. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Analyzing results obtained to average profitability (table nº 234), and based on *p-value* = 0,825 > α = 0,10, we cannot reject H_0, meaning that no way of calculating portfolio's average profitability is greater than the other way.

Analyzing results obtained to Sharpe's index (table nº 235), and based on *p-value* = 0,123 > α = 0,10, we cannot reject H_0, meaning that no way of calculating portfolio's average Sharpe's index is greater than the other way.

c.1.) PSI vs ELECTRE

Table nº 236 - Mann-Whitney Test to average profitability (Ranks for 3 y holding, 2nd sub period)

Ranks				
METHOD		**N**	**Mean Rank**	**Sum of Ranks**
PROFITABILITY	**PSI**	48	58,14	2790,50
	ELECTRE	72	62,08	4469,50
	Total	120		

Source: SPSS output, September 2012.

Table nº 237 - Mann-Whitney Test to average profitability (Test statistics for 3 y holding, 2nd sub period)

Test Statistics[a]	
PROFITABILITY	
Mann-Whitney U	1614,500
Wilcoxon W	2790,500
Z	-0,608
Asymp. Sig. (2-tailed)	0,543
Exact Sig. (2-tailed)	0,545
Exact Sig. (1-tailed)	0,273
Point Probability	0,001
a. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Results, expressed in tables 236 and 237 allow us to concluded that, seeing *p-value* = 0,273 > α =0,10, we cannot reject H_0. So, average profitability of PSI is greater than average profitability obtained with ELECTRE, being these conclusions not consistent with observed data: mean rank is higher to ELECTRE (62,08) than to PSI (58,14), answering to *T1*.

Table nº 238 - Mann-Whitney Test to Sharpe's Index (Ranks for 3 y holding, 2nd sub period)

Ranks				
METHOD		**N**	**Mean Rank**	**Sum of Ranks**
SHARPE	**PSI**	48	65,23	3131,00
	ELECTRE	72	57,35	4129,00
	Total	120		

Source: SPSS output, September 2012.

Table nº 239 - Mann-Whitney Test to Sharpe's Index (Test statistics for 3 y holding, 2nd sub period)

Test Statistics[a]	
SHARPE	
Mann-Whitney U	1,501E3
Wilcoxon W	4,129E3
Z	-1,216
Asymp. Sig. (2-tailed)	0,224
Exact Sig. (2-tailed)	0,226
Exact Sig. (1-tailed)	0,113
Point Probability	0,001
a. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Results, expressed in tables 238 and 239 allow us to concluded that, seeing *p-value* = 0,113 > α =0,10, we cannot reject H_0. So, average Sharpe's index of PSI is greater than average Sharpe's index obtained with ELECTRE, being these conclusions consistent with observed data: mean rank is higher to PSI (65,230) than to ELECTRE (57,35), answering to *T2*.

c.2.) PSI vs CAPM

Table nº 240 - Mann-Whitney Test to average profitability (Ranks for 3 y holding, 2nd sub period)

Ranks				
METHOD		**N**	**Mean Rank**	**Sum of Ranks**
PROFITABILITY	**PSI**	48	58,98	2831,00
	CAPM	72	61,51	4429,00
	Total	120		

Source: SPSS output, September 2012.

Table nº 241 - Mann-Whitney Test to average profitability (Test statistics for 3 y holding, 2nd sub period)

Test Statistics[a]	
PROFITABILITY	
Mann-Whitney U	1655,000
Wilcoxon W	2831,000
Z	-0,391
Asymp. Sig. (2-tailed)	0,696
Exact Sig. (2-tailed)	0,698
Exact Sig. (1-tailed)	0,349
Point Probability	0,001
a. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Results, expressed in tables 240 and 241 allow us to concluded that, seeing *p-value* = 0,349 > α =0,10, we cannot reject H_0. So, average profitability of PSI is greater than average profitability obtained with CAPM, being these conclusions not consistent with observed data: mean rank is higher to CAPM (61,51) than to PSI (58,98).

Table nº 242 - Mann-Whitney Test to Sharpe's Index (Ranks for 3 y holding, 2nd sub period)

Ranks				
METHOD		**N**	**Mean Rank**	**Sum of Ranks**
SHARPE	**PSI**	48	68,85	3305,00
	CAPM	72	54,93	3955,00
	Total	120		

Source: SPSS output, September 2012.

Table nº 243 - Mann-Whitney Test to Sharpe's Index (Test statistics for 3 y holding, 2nd sub period)

Test Statistics[a]	
SHARPE	
Mann-Whitney U	1,327E3
Wilcoxon W	3,955E3
Z	-2,148
Asymp. Sig. (2-tailed)	0,032
Exact Sig. (2-tailed)	0,032
Exact Sig. (1-tailed)	0,016
Point Probability	0,000
a. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Results, expressed in tables 242 and 243 allow us to concluded that, seeing *p-value* = 0,016 < α =0,10, we can reject H_0. So, average Sharpe's index of PSI isn't greater than average Sharpe's index obtained with CAPM, being these conclusions not consistent with observed data: mean rank is higher to PSI (68,85) than to CAPM (54,93).

c.3.) ELECTRE vs CAPM

Table nº 244 - Mann-Whitney Test to average profitability (Ranks for 3 y holding, 2nd sub period)

Ranks					
METHOD_GROUP		**N**	**Mean Rank**	**Sum of Ranks**
PROFITABILITY	**ELECTRE**	72	73,45	5288,50
	CAPM	72	71,55	5151,50
	Total	144		

Source: SPSS output, September 2012.

Table nº 245 - Mann-Whitney Test to average profitability (Test statistics for 3 y holding, 2nd sub period)

Test Statistics[a]	
PROFITABILITY	
Mann-Whitney U	2523,500
Wilcoxon W	5151,500
Z	-0,274
Asymp. Sig. (2-tailed)	0,784
Exact Sig. (2-tailed)	0,786
Exact Sig. (1-tailed)	0,393
Point Probability	0,001
a. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Results, expressed in tables 244 and 245 allow us to concluded that, seeing *p-value* = 0,393 > α =0,10, we cannot reject H_0. So, average profitability of ELECTRE is greater than average profitability obtained with CAPM, being these conclusions consistent with observed data: mean rank is higher to ELECTRE (73,45) than to CAPM (71,55), answering to *T3*.

Table nº 246 - Mann-Whitney Test to Sharpe's Index (Ranks for 3 y holding, 2nd sub period)

Ranks				
METHOD		**N**	**Mean Rank**	**Sum of Ranks**
SHARPE	**ELECTRE**	72	75,03	5402,00
	CAPM	72	69,97	5038,00
	Total	144		

Source: SPSS output, September 2012.

Table nº 247 - Mann-Whitney Test to Sharpe's Index (Test statistics for 3 y holding, 2nd sub period)

Test Statistics[a]	
SHARPE	
Mann-Whitney U	2,410E3
Wilcoxon W	5,038E3
Z	-0,727
Asymp. Sig. (2-tailed)	0,467
Exact Sig. (2-tailed)	0,469
Exact Sig. (1-tailed)	0,235
Point Probability	0,001
a. Grouping Variable: METHOD	

Source: SPSS output, September 2012.

Results, expressed in tables 246 and 247 allow us to concluded that, seeing *p-value* = 0,235 > α =0,10, we cannot reject H_0. So, average Sharpe's index of ELECTRE is greater than average Sharpe's index obtained with CAPM, being these conclusions consistent with observed data: mean rank is higher to ELECTRE (75,03) than to CAPM (69,97), answering to *T4*.

All results obtained tell us that, in absolute terms, in a 3 years holding period, ELECTRE achieved high average profitability and average Sharpe's index than the market (PSI). Between CAPM and ELECTRE, ELECTRE behaved better than CAPM in three years holding period, meaning that in a long term period, portfolios formed with shares selected using ELECTRE III assumptions had better performance than portfolios formed under CAPM assumptions, in absolute terms, also achieving greater profitability by unit of risk. Despite these conclusions, statistically some means could be differentiated, based on Mann-Whitney test, for instance, average profitability for 1 year holding where PSI behaved better than ELECTRE, average profitability for 2 years holding where ELECTRE behaved better than CAPM, Sharpe's Index for 3 years holding where PSI behaved better than ELECTRE, average profitability and Sharpe's index for 3 years holding where ELECTRE behaved better than CAPM.

4.4.2.2.2.3. Compare means of both sub period (Kruskal-Wallis and Mann-Whitney tests)

a) Follow-up, 1 year

Table nº 248 - Kruskal-Wallis Test to average profitability (Ranks for 1 y holding, 1st and 2nd sub period)

Ranks			
SUBPERIOD		N	Mean Rank
PROFITABILITY	1ST FOLLOW-UP SUBPERIOD	108	144,73
	2ND FOLLOW-UP SUBPERIOD	144	112,83
	Total	252	

Source: SPSS output, September 2012.

Table nº 249 - Kruskal-Wallis Test to average profitability (Test statistics for 1 y holding, 1st and 2nd sub period)

Test Statistics[a,b]	
	PROFITABILITY
Chi-Square	11,818
Df	1
Asymp. Sig.	0,001

a. Kruskal Wallis Test

b. Grouping Variable: SUBPERIOD

Source: SPSS output, September 2012.

Table nº 250 - Kruskal-Wallis Test to Sharpe's Index (Ranks for 1 y holding, 1st and 2nd sub period)

Ranks			
SUBPERIOD		**N**	**Mean Rank**
SHARPE	**1SUBPERIOD**	108	152,05
	2SUBPERIOD	144	107,34
	Total	252	

Source: SPSS output, September 2012.

Table nº 251 - Kruskal-Wallis Test to Sharpe's Index (Test statistics for 1 y holding, 1st and 2nd sub period)

Test Statistics[a,b]	
SHARPE	
Chi-Square	23,215
df	1
Asymp. Sig.	0,000

a. Kruskal Wallis Test

b. Grouping Variable: SUBPERIOD

Source: SPSS output, September 2012.

Kruskal-Wallis results to average profitability (see tables 248 and 249) tells us that with a *p-value* = 0,001 < α = 0,10, we can reject H_0, meaning that a sub period has greater average profitability than the other sub period (see chart nº 27).

Kruskal-Wallis results to average Sharpe's index (see tables 250 and 251) tells us that with a *p-value* = 0,000 < α = 0,10, we can reject H_0, meaning that a sub period has greater average Sharpe's index than the other sub period (see chart nº 28).

Table nº 252 - Mann-Whitney Test to average profitability (Ranks for 1 y holding, 1st and 2nd sub period)

Ranks				
SUBPERIOD		N	Mean Rank	Sum of Ranks
PROFITABILITY	1ST FOLLOW-UP SUBPERIOD	108	144,73	15630,50
	2ND FOLLOW-UP SUBPERIOD	144	112,83	16247,50
	Total	252		

Source: SPSS output, September 2012.

Table nº 253 - Mann-Whitney Test to average profitability (Test statistics for 1 y holding, 1st and 2nd sub period)

Test Statistics[a]	
	PROFITABILITY
Mann-Whitney U	5807,500
Wilcoxon W	16247,500
Z	-3,438
Asymp. Sig. (2-tailed)	0,001

a. Grouping Variable: SUBPERIOD

Source: SPSS output, September 2012.

Results, expressed in tables 252 and 253 allow us to concluded that, seeing *p-value* = 0,001 < α =0,10, we can reject H_0. So, average profitability of 1st sub period is greater than average profitability obtained in 2nd sub period, being these conclusions consistent with observed data because mean rank is higher to 1st sub period (144,73) than to 2nd sub period (112,83).

Table nº 254 - Mann-Whitney Test to Sharpe's Index (Ranks for 1 y holding, 1st and 2nd sub period)

Ranks				
SUBPERIOD		N	Mean Rank	Sum of Ranks
SHARPE	1SUBPERIOD	108	152,05	16421,00
	2SUBPERIOD	144	107,34	15457,00
	Total	252		

Source: SPSS output, September 2012.

Table nº 255 - Mann-Whitney Test to Sharpe's Index (Test statistics for 1 y holding, 1st and 2nd sub period)

Test Statistics[a]	
	SHARPE
Mann-Whitney U	5,017E3
Wilcoxon W	1,546E4
Z	-4,818
Asymp. Sig. (2-tailed)	0,000
Exact Sig. (2-tailed)	0,000
Exact Sig. (1-tailed)	0,000
Point Probability	0,000
a. Grouping Variable: SUBPERIOD	

Source: SPSS output, September 2012.

Results, expressed in tables 254 and 255 allow us to concluded that, seeing *p-value* = 0,000 < α =0,10, we can reject H_0. So, average Sharpe's index of 1st sub period is greater than average Sharpe's index obtained in 2nd sub period, being these conclusions consistent with observed data because mean rank is higher to 1st sub period (152,05) than to 2nd sub period (107,34).

b) Follow-up, 2 years

Table nº 256 - Kruskal-Wallis Test to average profitability (Ranks for 2 y holding, 1st and 2nd sub period)

Ranks			
SUBPERIOD		**N**	**Mean Rank**
PROFITABILITY	**1ST FOLLOW-UP SUBPERIOD**	192	208,37
	2ND FOLLOW-UP SUBPERIOD	192	176,63
	Total	384	

Source: SPSS output, September 2012.

Table nº 257 - Kruskal-Wallis Test to average profitability (Test statistics for 2 y holding, 1st and 2nd sub period)

Test Statistics[a,b]	
PROFITABILITY	
Chi-Square	7,850
Df	1
Asymp. Sig.	0,005

a. Kruskal Wallis Test

b. Grouping Variable: SUBPERIOD

Source: SPSS output, September 2012.

Table nº 258 - Kruskal-Wallis Test to Sharpe's Index (Ranks for 2 y holding, 1st and 2nd sub period)

Ranks			
SUBPERIOD		N	Mean Rank
SHARPE	**1SUBPERIOD**	192	210,63
	2SUBPERIOD	192	174,37
	Total	384	

Source: SPSS output, September 2012.

Table nº 259 - Kruskal-Wallis Test to Sharpe's Index (Test statistics for 2 y holding, 1st and 2nd sub period)

Test Statistics[a,b,c]	
SHARPE	
Chi-Square	10,245
Df	1
Asymp. Sig.	,001
a. Kruskal Wallis Test	
b. Grouping Variable: SUBPERIOD	

Source: SPSS output, September 2012.

Kruskal-Wallis results to average profitability (see tables 256 and 257) tells us that with a *p-value* = 0,005 < α = 0,10, we can reject H_0, meaning that a sub period has greater average profitability than the other sub period (see chart nº 27).

Kruskal-Wallis results to average Sharpe's index (see tables 258 and 259) tells us that with a *p-value* = 0,001 < α = 0,10, we can reject H_0, meaning that a sub period has greater average Sharpe's index than the other sub period (see chart nº 28).

Table nº 260 - Mann-Whitney Test to average profitability (Ranks for 2 y holding, 1st and 2nd sub period)

Ranks				
SUBPERIOD		**N**	**Mean Rank**	**Sum of Ranks**
PROFITABILITY	**1ST FOLLOW-UP SUBPERIOD**	192	208,37	40007,00
	2ND FOLLOW-UP SUBPERIOD	192	176,63	33913,00
	Total	384		

Source: SPSS output, September 2012.

Table nº 261 - Mann-Whitney Test to average profitability (Test statistics for 2 y holding, 1st and 2nd sub period)

Test Statistics[a]	
PROFITABILITY	
Mann-Whitney U	15385,000
Wilcoxon W	33913,000
Z	-2,802
Asymp. Sig. (2-tailed)	0,005
a. Grouping Variable: SUBPERIOD	

Source: SPSS output, September 2012.

Results, expressed in tables 260 and 261 allow us to concluded that, seeing *p-value* = 0,005 < α =0,10, we can reject H_0. So, average profitability of 1st sub period is greater than average profitability obtained in 2nd sub period, being these conclusions consistent with observed data because mean rank is higher to 1st sub period (208,37) than to 2nd sub period (176,63).

Table nº 262 - Mann-Whitney Test to Sharpe's Index (Ranks for 2 y holding, 1st and 2nd sub period)

Ranks				
SUBPERIOD		**N**	**Mean Rank**	**Sum of Ranks**
SHARPE	**1SUBPERIOD**	192	210,63	40441,00
	2SUBPERIOD	192	174,37	33479,00
	Total	384		

Source: SPSS output, September 2012.

Table nº 263 - Mann-Whitney Test to Sharpe's Index (Test statistics for 2 y holding, 1st and 2nd sub period)

Test Statistics[a]	
SHARPE	
Mann-Whitney U	1,495E4
Wilcoxon W	3,348E4
Z	-3,201
Asymp. Sig. (2-tailed)	0,001
a. Grouping Variable: SUBPERIOD	

Source: SPSS output, September 2012.

Results, expressed in tables 262 and 263 allow us to concluded that, seeing *p-value* = 0,001 < α =0,10, we can reject H_0. So, average Sharpe's index of 1st sub period is greater than average Sharpe's index obtained in 2nd sub period, being these conclusions consistent with observed data because mean rank is higher to 1st sub period (210,63) than to 2nd sub period (174,37).

d) Follow-up, 3 years

Table nº 264 - Kruskal-Wallis Test to average profitability (Ranks for 3 y holding, 1st and 2nd sub period)

Ranks			
SUBPERIOD		N	Mean Rank
PROFITABILITY	**1ST FOLLOW-UP SUBPERIOD**	277	249,94
	2ND FOLLOW-UP SUBPERIOD	192	213,45
	Total	469	

Source: SPSS output, September 2012.

Table nº 265 - Kruskal-Wallis Test to average profitability (Test statistics for 3 y holding, 1st and 2nd sub period)

Test Statistics[a,b]	
PROFITABILITY	
Chi-Square	8,220
Df	1
Asymp. Sig.	0,004
a. Kruskal Wallis Test	
b. Grouping Variable: SUBPERIOD	

Source: SPSS output, September 2012.

Table nº 266 - Kruskal-Wallis Test to Sharpe's Index (Ranks for 3 y holding, 1st and 2nd sub period)

Ranks			
SUBPERIOD		**N**	**Mean Rank**
SHARPE	**1SUBPERIOD**	277	253,12
	2SUBPERIOD	191	207,50
	Total	468	

Source: SPSS output, September 2012.

Table nº 267 - Kruskal-Wallis Test to Sharpe's Index (Test statistics for 3 y holding, 1st and 2nd sub period)

Test Statistics[a,b]	
SHARPE	
Chi-Square	12,859
df	1
Asymp. Sig.	0,000
a. Kruskal Wallis Test	
b. Grouping Variable: SUBPERIOD	

Source: SPSS output, September 2012.

Kruskal-Wallis results to average profitability (see tables 264 and 265) tells us that with a *p-value* = 0,004 < α = 0,10, we can reject H_0, meaning that a sub period has greater average profitability than the other sub period (see chart nº 27).

Kruskal-Wallis results to average Sharpe's index (see tables 266 and 267) tells us that with a *p-value* = 0,000 < α = 0,10, we can reject H_0, meaning that a sub period has greater average Sharpe's index than the other sub period (see chart nº 28).

Table nº 268 - Mann-Whitney Test to average profitability (Ranks for 3 y holding, 1st and 2nd sub period)

Ranks				
SUBPERIOD		N	Mean Rank	Sum of Ranks
PROFITABILITY	1ST FOLLOW-UP SUBPERIOD	277	249,94	69233,00
	2ND FOLLOW-UP SUBPERIOD	192	213,45	40982,00
	Total	469		

Source: SPSS output, September 2012.

Table nº 269 - Mann-Whitney Test to average profitability (Test statistics for 3 y holding, 1st and 2nd sub period)

Test Statistics[a]	
	PROFITABILITY
Mann-Whitney U	22454,000
Wilcoxon W	40982,000
Z	-2,867
Asymp. Sig. (2-tailed)	0,004

a. Grouping Variable: SUBPERIOD

Source: SPSS output, September 2012.

Results, expressed in tables 268 and 269, allow us to conclude that, seeing *p-value* = 0,004 < α =0,10, we can reject H_0. So, average profitability of 1st sub period is greater than average profitability obtained in 2nd sub period, being these conclusions consistent with observed data because mean rank is higher to 1st sub period (249,94) than to 2nd sub period (213,45).

Table nº 270 - Mann-Whitney Test to Sharpe's Index (Ranks for 3 y holding, 1st and 2nd sub period)

Ranks				
SUBPERIOD		**N**	**Mean Rank**	**Sum of Ranks**
SHARPE	**1SUBPERIOD**	277	253,12	70113,00
	2SUBPERIOD	191	207,50	39633,00
	Total	468		

Source: SPSS output, September 2012.

Table nº 271 - Mann-Whitney Test to Sharpe's Index (Test statistics for 3 y holding, 1st and 2nd sub period)

Test Statistics[a]	
	SHARPE
Mann-Whitney U	2,130E4
Wilcoxon W	3,963E4
Z	-3,586
Asymp. Sig. (2-tailed)	0,000
a. Grouping Variable: SUBPERIOD	

Source: SPSS output, September 2012.

Results, expressed in tables 270 and 271 allow us to conclude that, seeing *p-value* = 0,000 < α =0,10, we can reject H_0. So, average Sharpe's index of 1st sub period is greater than average Sharpe's index obtained in 2nd sub period, being these conclusions consistent with observed data because mean rank is higher to 1st sub period (253,12) than to 2nd sub period (207,50).

4.5. Main conclusions from empirical work

From this empirical work, we can generically conclude that multicriteria approach, throw companies' financial performance (financial ratios), is a successful approach in portfolio management, where financial ratios and shares are equally weighed. In absolute terms, portfolios based on ELECTRE III assumptions achieved greater average profitabilities and high Sharpe's index than PSI-20 TR or CAPM. Despite this conclusion, statistically we cannot infer this same conclusion (means cannot be differentiated) to all follow-up periods.

Our analysis underwent monitoring historical and follow-up periods, conducting a descriptive analysis to each portfolio individually, and a statistical analysis to follow-up periods. In the specific case of statistical analysis, we conducted parametric (*t*-student and ANOVA one-way tests) and nonparametric tests (Kruskal-Wallis and Mann-Whitney tests) in order to compare profitability and Sharpe's index means, subdivided into two analysis: on one hand a unit sample analysis from 2005 to 2011; on the other hand a two sample analysis, one from 2005 to 2007, and another one from 2008 to 2011. This subdivision of follow-up period analysis was necessary because average profitability and Sharpe's index from 2005-2007 are positive to ELECTRE, CAPM and PSI, but, after this period, from 2008-2011, they are negative, except for 2009. Economically this situation can be explained based on subprime crises that erupted in middle of 2007 in American market.

Answering to our test empirical hypothesis, from *T1* to *T4*, and based on a descriptive analysis,

- Relatively to *T1* (average profitability of ELECTRE is greater than PSI?) we can conclude that in every historical period portfolio behaved better than the market (PSI-20 TR). In follow-up periods, generally ELECTRE also achieved greater profitabilities than PSI;
- Concerning *T2* (portfolios under ELECTRE assumptions have greater profitability by unit of risk than PSI?), we can conclude that portfolio had an expected differential return per unit of risk, measured by Sharpe's Index, more significant than market (less risk), in historical period and in almost every follow-up periods;

- As to *T3* (average profitability achieved with ELECTRE is greater than achieved with CAPM?), we can conclude that in average by follow-up, in a holding period of 3 years, ELECTRE behaved better than CAPM;

- As regards *T4* (portfolios under ELECTRE assumptions have high profitability by unit of risk than portfolios under CAPM?) we can conclude that generally, in average, portfolio had a Sharpe's Index more significant than CAPM in every holding periods (less risk);

Now looking to statistical results obtained with parametric and nonparametric tests, and answering to *T1* and *T2* (average profitability/ Sharpe's index obtained with ELECTRE is greater than PSI?), and to *T3* and *T4* (average profitability/Sharpe's index obtained with ELECTRE is greater than CAPM?), results do not corroborated all conclusions taken in descriptive analysis. Let's analyze tables below.

Table nº 272 – Main statistical results obtained with Unit Sample

Sample tested	**Test Conducted**		***p-value* Obtained**	**Conclusions**			
				If p-value=0,10		**If p-value=0,05**	
Unit Sample							
Mann-Whitney (Nonparametric)	Average profitability and Average Sharpe's index	PSI vs. ELECTRE (1y)	0,452 0,174	$> \alpha = 0,10$ (we cannot reject H0)	MR_{PSI} > $MR_{ELECTRE}$ (Statistically means can be differentiated)	$> \alpha = 0,05$ (we cannot reject H0)	MR_{PSI} > $MR_{ELECTRE}$ (Statistically means can be differentiated)
	Average Sharpe's index	PSI vs. CAPM (1, 2y)	0,359 0,185	$> \alpha = 0,10$ (we cannot reject H0)	MR_{PSI} > MR_{CAPM} (Statistically means can be differentiated)	$> \alpha = 0,05$ (we cannot reject H0)	MR_{PSI} > MR_{CAPM} (Statistically means can be differentiated)

	Average profitability and Average Sharpe's index	ELECTRE vs. CAPM (2, 3 y)	0,491 0,456 0,316 0,179	> α = 0,10 (we cannot reject H0)	$MR_{ELECTRE}$ > MR_{CAPM} (Statistically means can be differentiated)	> α = 0,05 (we cannot reject H0)	$MR_{ELECTRE}$ > MR_{CAPM} (Statistically means can be differentiated)
	Average Sharpe's index	PSI vs. ELECTRE (3y)	0,415	> α = 0,10 (we cannot reject H0)	MR_{PSI} > $MR_{ELECTRE}$ (Statistically means can be differentiated)	> α = 0,05 (we cannot reject H0)	MR_{PSI} > $MR_{ELECTRE}$ (Statistically means can be differentiated)
	Average Sharpe's index	PSI vs. CAPM (3Y)	0,089	< α = 0,10 (we can reject H0)	MR_{PSI} > MR_{CAPM} (Statistically means cannot be differentiated)	> α = 0,05 (we cannot reject H0)	MR_{PSI} > MR_{CAPM} (Statistically means can be differentiated)

Source: Own elaboration, September 2012.

So, considering statistical results to a unit sample,

- Parametric test (ANOVA test) indicates that all results obtained, by multiple comparations, lead us to conclude that in every follow-up we cannot infer that one way of calculating portfolio's average profitability - *T1* and *T3* - and portfolio's Sharpe's index - *T2* and *T4* - is better than the other statistically (see tables from 43 to 54). Although average profitability and Sharpe's means expressed in descriptive analysis are greater when using ELECTRE method in a 3 years holding, probably due to sample limitations (few observations) statistical results are not able to confirm this conclusions;

- Nonparametric test (Kruskal-Wallis and Mann-Withney tests) indicates that, generally, we cannot reject H_0, meaning that no way of calculating portfolio's average profitability - *T1* and *T3* - and portfolio's Sharpe's index - *T2* and *T4* - is greater than the other way (see tables nºs 55 to 96). We may point out sample limitation as being a possible cause to these results. But, Mann-Whitney test allow us to differentiated statistically certain means, for instance, PSI vs. ELECTRE (1 year) for average profitability and average Sharpe's index (see tables 57 to 60 that answer to *T1* and *T2*), PSI vs. CAPM (1, 2 years) for average Sharpe's index (see tables 63, 64, 77 and 78), ELECTRE vs. CAPM (2, 3 years) for average profitability and average Sharpe's index (see tables 79 to 82 and 93 to 96, answering to *T4*), PSI vs. ELECTRE (3 years) for average Sharpe's index (see tables 87 and 88, answering to *T2*), and finally PSI vs. CAPM (3 years) for average Sharpe's index (see tables 91 and 92) but only to a error probability of 5%.

Now looking to statistic results obtained with two samples, some results corroborated descriptive analysis.

Table nº 273 – Main statistical results obtained with Two Samples (1st sub period)

Sample tested	**Test Conducted**		***p-value* Obtained**	**Conclusions**			
				If p-value=0,10		**If p-value=0,05**	
Two Samples (1st sub period)							
t-student (parametric)	Average profitability	PSI (1y)	0,059	< α = 0,10 (we can reject H0)	Statistically means are different from 0%	> α = 0,05 (we cannot reject H0)	The means are not statistically different from 0%
		ELECTRE (1y)	0,013	< α = 0,10 (we can reject H0)	Statistically means are different from 0%	< α = 0,05 (we can reject H0)	Statistically means are different from 0%
		CAPM (1y)	0,002	< α = 0,10 (we can reject H0)	Statistically means are different from 0%	< α = 0,05 (we can reject H0)	Statistically means are different from 0%
	Average Sharpe's	PSI (1, 3 y)	0,015 0,042	< α = 0,10 (we can	Statistically means are	< α = 0,05 (we can	Statistically means are

	index			reject H0)	different from 0%	reject H0)	different from 0%
		ELECTRE (1y)	0,007	< α = 0,10 (we can reject H0)	Statistically means are different from 0%	< α = 0,05 (we can reject H0)	Statistically means are different from 0%
		ELECTRE (2, 3 y)	0,051 0,077	< α = 0,10 (we can reject H0)	Statistically means are different from 0%	> α = 0,05 (we cannot reject H0)	The means are not statistically different from 0%
		CAPM (1y)	0,015	< α = 0,10 (we can reject H0)	Statistically means are different from 0%	< α = 0,05 (we can reject H0)	Statistically means are different from 0%
Mann-Whitney (Nonparametric)	Average Sharpe's index	ELECTRE vs. CAPM (1, 2, 3 y)	0,305 0,216 0,229	> α = 0,10 (we cannot reject H0)	$MR_{ELECTRE} > MR_{CAPM}$ (Statistically means can be differentiated)	> α = 0,05 (we cannot reject H0)	$MR_{ELECTRE} > MR_{CAPM}$ (Statistically means can be differentiated)
	Average profitability	ELECTRE vs. CAPM (2, 3 y)	0,432 0,412	> α = 0,10 (we cannot reject H0)	$MR_{ELECTRE} > MR_{CAPM}$ (Statistically means can be differentiated)	> α = 0,05 (we cannot reject H0)	$MR_{ELECTRE} > MR_{CAPM}$ (Statistically means can be differentiated)
	Average profitability and Average Sharpe's index	PSI vs. CAPM (3 y)	0,467 0,339	> α = 0,10 (we cannot reject H0)	$MR_{PSI} > MR_{CAPM}$ (Statistically means can be differentiated)	> α = 0,05 (we cannot reject H0)	$MR_{PSI} > MR_{CAPM}$ (Statistically means can be differentiated)

Source: Own elaboration, September 2012.

Concerning 1st sub period statistic results, expressed in table above:

- Parametric test (t-student) indicates that to a error probability of 5% and 10%, all way's of calculating portfolio's average profitability in a 1 year holding are different from 0%, except to PSI, when considering a error of 5% (table nº 112). To portfolio's average Sharpe's index, *t-student* results to 1 year holding tells that every mean are statistically different from 0%, with an error probability of 10%. In a 2 and 3 years holding, only ELECTRE is statistically different from 0% with an error probability of 10% (table nº 113).

  ANOVA test show us that all results obtained, by multiple comparations, lead us to conclude that in every follow-up we cannot infer that one way of calculating portfolio's average profitability - *T1* and *T3* - and portfolio's average Sharpe's index - *T2* and *T4* - is better than the other statistically (see tables from 114 to 125). Although, in absolute terms, average profitability and average Sharpe's index means are greater when using ELECTRE method in a 3 years holding, probably due to sample limitations (few observations) statistic results are not able to confirm these conclusions.

- Nonparametric test (Kruskal-Wallis and Mann-Withney tests) generally failed to demonstrate that, statistically, ELECTRE achieved high portfolio's average profitability and portfolio's average Sharpe's index than the market (PSI). They also generally failed to demonstrate that between CAPM and ELECTRE, ELECTRE achieved high portfolio's average profitability and portfolio's average profitability by unit of risk than CAPM. But, there are particular cases where Mann-Withney test differentiated means, in particular, ELECTRE vs. CAPM (1, 2, 3 years) for average Share's index (see tables 176, 177, 190, 191, 204 and 205, answering to *T4*), ELECTRE vs. CAPM (2, 3 years) for average profitability (see tables 188, 189, 202 and 203, answering to *T3*), and finally PSI vs. CAPM (3 years) for average profitability and average Sharpe's index (see tables 198 to 201).

Table nº 274 – Main statistical results obtained to Two Samples (2nd sub period)

Sample tested	Test Conducted		*p-value* Obtained	Conclusions			
				If p-value=0,10		If p-value=0,05	
Two Samples (2nd sub period)							
t-student (parametric)	Average Sharpe's Index	ELECTRE (1y)	0,026	< α = 0,10 (we can reject H0)	Statistically means are different from 0%	< α = 0,05 (we can reject H0)	Statistically means are different from 0%
		ELECTRE (2y)	0,052	< α = 0,10 (we can reject H0)	Statistically means are different from 0%	> α = 0,05 (we cannot reject H0)	The means are not statistically different from 0%
		CAPM (3y)	0,055	< α = 0,10 (we can reject H0)	Statistically means are different from 0%	> α = 0,05 (we cannot reject H0)	The means are not statistically different from 0%
Kruskal-Wallis (nonparametric)	Average Sharpe's Index	1 y	0,074	< α = 0,10 (we can reject H0)	Statistically means can be differentiated	> α = 0,05 (we can reject H0)	Statistically means cannot be differentiated
		2 y	0,033	< α = 0,10 (we can reject H0)	Statistically means can be differentiated	< α = 0,05 (we can reject H0)	Statistically means can be differentiated
Mann-Whitney (nonparametric)	Average profitability	PSI vs. ELECTRE (1 y)	0,486	> α = 0,10 (we cannot reject H0)	$MR_{PSI} > MR_{ELECTRE}$ (Statistically means can be differentiated)	> α = 0,05 (we cannot reject H0)	$MR_{PSI} > MR_{ELECTRE}$ (Statistically means can be differentiated)
		ELECTRE vs. CAPM (2, 3 y)	0,497	> α = 0,10 (we cannot reject H0)	$MR_{ELECTRE} > MR_{CAPM}$ (Statistically means can be	> α = 0,05 (we cannot reject H0)	$MR_{ELECTRE} > MR_{CAPM}$ (Statistically means can be

					differentiated)		differentiated)
	Average Sharpe's index	PSI vs. ELECTRE (3 y)	0,113	> α = 0,10 (we cannot reject H0)	MR_{PSI} > $MR_{ELECTRE}$ (Statistically means can be differentiated)	> α = 0,05 (we cannot reject H0)	MR_{PSI} > $MR_{ELECTRE}$ (Statistically means can be differentiated)
	Average Sharpe's index	ELECTRE vs. CAPM (3 y)	0,235	> α = 0,10 (we cannot reject H0)	$MR_{ELECTRE}$ > MR_{CAPM} (Statistically means can be differentiated)	> α = 0,05 (we cannot reject H0)	$MR_{ELECTRE}$ > MR_{CAPM} (Statistically means can be differentiated)

Source: Own elaboration, September 2012.

Regarding statistic results obtained to 2nd sub period,

- Parametric test (t-student) indicates that to an error probability of 5% and 10%, to average Sharpe's Index, only ELECTRE to a 1 year holding is statistically significant from 0%. All the others are statistically significant from 0%, but only to a error probability of 10%. To a 2 years holding, Sharpe's Index means calculated with ELECTRE are significantly different from 0% to an error probability of 10% (table nº 139).

  ANOVA test, by pair of group and by follow-up period, lead us to conclude that in every follow-up we cannot infer that one way of calculating portfolio's average profitability - *T1* and *T3* - and portfolio's average Sharpe's index - *T2* and *T4* - is better than the other, statistically (see tables from 140 to 151). Although average profitability and average Sharpe's index are greater when using ELECTRE method in a 3 years holding, in absolute terms, probably due to sample limitations (few observations) statistical results are not able to confirm these conclusions;

- Nonparametric test (Kruskal-Wallis and Mann-Withney tests) generally failed to prove that CAPM and ELECTRE achieved high average profitabilities than the market (PSI), and greater profitabilities by unit of risk. They also failed to prove that between them, CAPM and ELECTRE, ELECTRE III assumptions had better performance (greater profitability) than portfolios formed under CAPM assumptions, and greater profitability by unit of risk. Despite these conclusions, certain means could be differentiated based on Kruskal-Wallis results expressed in tables 207 and 221, for instance, average Sharpe's index for 1 and 2 years holding, to a error probability of 10% (answering to *T2* and *T4*). Also with Mann-Withney test some means could be differentiated, in particular, PSI vs. ELECTRE (1 year) for average profitability (see tables 208 and 209, answering to *T1*), ELECTRE vs. CAPM (2, 3 year) for average profitability (see tables 230, 231, 244, and 245, answering to *T3*), PSI vs. ELECTRE (3 years) for average Sharpe's index (see tables 238 and 239, answering to *T2*), ELECTRE vs. CAPM (3 years) for average Sharpe's index (see tables 246 to 247, answering to *T4*).

Finally comparing both sub periods,

- Parametric test (t-student test to compare two groups) results obtained for all holding period allow us to rejected H_0, (except average profitability in a two years holding, to a error probability of 5%) so average profitability - *T1* and *T3* - and average Sharpe's index - *T2* and *T4* - for these two groups are statistically different (see tables nº 152 to 163), confirming the conclusions drawn from charts 27 and 28;

- Nonparametric test (Kruskal-Wallis and Mann-Withney tests) results obtained for all holding period allow us to rejected H_0, meaning average profitability - *T1* and *T3* - and average Sharpe's index - *T2* and *T4* - for these two groups are statistically different (tables nºs 248 to 271), also confirming the conclusions drawn from charts 27 and 28.

CONCLUSIONS

The scope of this thesis is to propose a form to improve estimates on expected returns or portfolio selection, and help investor to decide which are the best assets to invest. So, transforming this problem into a multicriteria problem, and using ELECTRE III methodology, we explore the use of financial theory - financial ratios - in a buy and hold perspective and construct defensive portfolios, where financial ratios and shares are equally weighed.

The multi-attribute representation of alternatives assumes that a decision maker can demarcate a set of criteria that he considers relevant. For the decision maker, a portfolio represents a basket of *w* alternatives according to *k* criteria, and these criteria can fully be characterized by attribute values. In this view, when choosing a portfolio, a decision maker is actually defining to witch attributes he is exposed to. The issue of multi-attribute portfolio selection is to balance the attributes of the individual assets on the portfolio level. This multicriteria approach, based on financial theory, allow investor to support his decision to invest based on his desires (preferences) and in the characteristics of the investment alternatives, adequately understood and related to each other.

As referred by Yap, Yong and Poon (2010), financial theory has been very successful in distinguish the weak companies from the healthy ones. In this sense, supported in the company's financial statements, financial theory provides information about its solvency position and its borrowing power, and whether if it is a suitable investment to consider, throws financial ratios and studies of trends. This idea is enhanced by several studies, of which we emphasize Beaver (1966) and Altman (1968).

To define which financial ratios, among the so many found in the literature are useful to evaluate the financial performance and financial condition of a company, we based on Beaver (1966), Altman (1968, 2000), Yap et al. (2010), and Chen and Shimerda (1981) studies. These authors' search indicates which ratios best predicts business failures, and concluded that there is no need for many ratios. Thereafter, in our study, we use five ratios: Return on Assets (ROA), Return on Equity (ROE), Financial Autonomy (FA), General Liquidity (GL), and Reduced Liquidity (RL).

Literature emphasizes some multi criteria decision models, due to their success in real-life cases, in particular, ELECTRE, AHP, PROMETHEE, MINORA and ADELAIS. In our work, we only use one method: ELECTRE III from ELECTRE family. One of the reason that leads us to choose ELECTRE III, within all versions existent in ELECTRE family, was the possibility for taking into account indifference and preference threshold, the necessity of a quantification of the relative importance of criteria, and to be specific for ranking problems. Another reason is that all the other methods, although being suitable for our decision problem, are more indicated when we want to focus in a group of decision makers, were subjectivity is presented due to qualitative variables and discussion between them. In our work, we focus in a single decision maker rather than in a focus group.

Concerning ELECTRE III, from the "European school", this method is pointed out as being relevant when speaking about multi criteria decision problems, as stated by Roy (1991) and Figueira et al. (2005). So, ELECTRE methods are developed in two main phases. Firstly, the construction of the outranking relations, and secondly the exploitation of those relations to get the final ranking of the alternatives. In the exploitation procedure, recommendations are elaborate from the results obtained in the first phase.

To implement such an investment decision process, our study is developed through a step by step methodology. In order to construct defensive portfolios, in a buy and hold strategy, we first selected shares trading in Portuguese Stock Index (PSI-Geral), from 1999 to 2011, equally weighed. We collected from Euronext weekly closing prices for each share traded, and for PSI-20 TR (this index includes dividends distributed to shareholders).

Secondly, we define two main periods. On one hand an initial/historical period (eight portfolios were established, one for each period); on the other hand, a follow-up period considering one, two and three years holding, allowing us to see how portfolios behave across time. So, we conducted a descriptive analysis to each portfolio individually considering all periods, and a statistical analysis only to follow-up periods. In the specific case of statistical analysis, we conducted parametric (*t*-student and ANOVA one-way tests) and nonparametric tests (Kruskal-Wallis and Mann-Whitney tests) in order to compare profitabilities and Sharpe's index means, subdivided into two analysis: on one hand a unit sample analysis from 2005 to 2011; on the other hand a two sample analysis, one from 2005 to 2007, and another one from 2008 to 2011. This subdivision of follow-up period analysis was necessary because average profitability and Sharpe's index from 2005-2007 are positive to ELECTRE, CAPM and PSI, but, after this period, from 2008-2011, they are negative, except for 2009. Economically this situation can be explained based on subprime crises that erupted in middle of 2007 in American market, with its effect in Europe in early 2008.

Thus, for each period, criteria and thresholds of the model were defined. As *criteria* we used five financial ratios previously calculated from 2000 to 2011, in order to assess profitability, leverage and liquidity. In particular, ROA, ROE, FA, GL, and RL, and all of them have the same weight in the model, 20%. In order to analyze each portfolio, we calculated their average for each period. As *thresholds*, we define a q, the indifference threshold, and a p, the preference threshold, for each criteria, from 2000 to 2011. Threshold q corresponds to the interest treasury bonds, 3 months, annual average, and p threshold corresponds to q threshold plus 30% (historically, in average, market gave us profitabilities greater than assets without risk plus 30%), being applied to ROA and ROE criteria. Relatively to FA, GL and RL, thresholds were defined based on the rules used in government subsidies attribution. In the particular case of ROA and ROE, thresholds were defined based on annual treasury bonds interest rates, three months, and respective data were collected from Institute for the Management of Treasury and Public Credit, IP (IGCP, IP), for each period. For each year, financial ratios were calculated based on financial statements, in particular, consolidated balance sheet, consolidated income statement and respective annual report, and then, for each portfolio, the financial ratio annual average for the considered period was calculated. The financial statements for each company were obtained consulting respective electronic address. As they are companies traded in PSI-Geral, they are required to publish this information.

Thirdly, and having defined models alternatives, criteria and thresholds, ELECTRE III methodology is applied in order to obtain a ranking of the alternatives, using a distillation process, for each period. At the same time, we also form portfolios using CAPM assumptions (shares must have positive profitability, and greater than CAPM results).

Finally, monthly average profitabilities and Sharpe's Index were calculated for each follow-up period – unit sample and two samples – considering one, two and three years holding, for ELECTRE, CAPM and PSI, and then parametric tests (*t*-student and ANOVA one-way tests) and nonparametric tests (Kruskal-Wallis and Mann-Whitney tests) were conducted to those follow-up periods.

Considering the specific case of evaluation of risk measure, as Dowd (2000) pointed out, in portfolio management, the general problem of risk adjustment has two main aspects. On one hand, in which asset invest, in asset *A*, with a high expected return and a relatively high risk, or in asset *B*, with low expected return but relatively safe. On the other hand, the evaluation of actual portfolio performance when decisions have already been made and the results of those decisions are apparent. Given its importance, several statistical measures are used to assess risk: Coefficient Beta, Sharpe's, Treynor's and Jensen's measures are commonly used.

Regarding ELECTRE III results, descriptive analysis tell us that for certain portfolios this method behaved better than market (PSI) besides being more efficient (Sharpe's index). Thus, in order to be able to drawn an overall conclusion, average profitability and Sharpe's Index, by holding periods, was calculated for ELECTRE III and PSI. Results obtained allow us to conclude that, in absolute terms, portfolios based on ELECTRE III assumptions achieved greater average profitabilities and high Sharpe's index than PSI-20 TR, answering to empirical hypothesis *T1* and *T2*. Relatively to CAPM results similar conclusions can be drawn, this is, looking to descriptive analysis portfolios constructed under CAPM assumptions mainly behaved better than market, being less risky. But witch behaved better face to market, ELECTRE III or CAPM? All results obtained, based on descriptive analysis, indicate that, in a three years holding (long term), ELECTRE had greater average profitabilities than market and CAPM, with also greater profitability by unit of risk, in general, answering to empirical hypothesis *T3* and *T4*.

Despite this conclusion based on a descriptive analysis, statistically we found different results, probably due to sample size limitation. So, considering statistical results to a unit sample, parametric test (ANOVA test) indicates that all results obtained, by multiple comparations, lead us to conclude that in every follow-up we cannot infer that one way of calculating portfolio's average profitability (concerning empirical hypothesis *T1* and *T3*) and portfolio's Sharpe's index (empirical hypothesis *T2* and *T4*) is better than the other statistically. With nonparametric test (Kruskal-Wallis and Mann-Withney tests), we can also say that, generally, we cannot reject H_0, meaning that no way of calculating portfolio's average profitability (empirical hypothesis *T1* and *T3*) and portfolio's Sharpe's index (empirical hypothesis *T2* and *T4*) is greater than the other way. But, Mann-Whitney test allow us to differentiated statistically certain means, for instance, to average profitability: PSI vs.

ELECTRE (1 year) and ELECTRE vs. CAPM (2, 3 years); to Sharpe's index: PSI vs. ELECTRE (1 year), PSI vs. CAPM (1, 2 years), ELECTRE vs. CAPM (2, 3 years), PSI vs. ELECTRE (3 years) and finally PSI vs. CAPM (3 years) but only to a error probability of 5%.

For two samples, conclusions are different for 2nd sub period. Results obtained to 1st sub period, with parametric test, *t-student*, indicates that to a error probability of 5% and 10%, all way's of calculating portfolio's average profitability in a 1 year holding are different from 0%, except to PSI, when considering a error of 5%. To portfolio's average Sharpe's index, results to 1 year holding tells that every mean are statistically different from 0%, with an error probability of 10%. In a 2 and 3 years holding, only ELECTRE is statistically different from 0% with an error probability of 10%. ANOVA test show us that all results obtained, by multiple comparations, lead us to conclude that in every follow-up we cannot infer that one way of calculating portfolio's average profitability (empirical hypothesis *T1* and *T3*) and portfolio's average Sharpe's index (empirical hypothesis *T2* and *T4*) is better than the other statistically. Nonparametric test (Kruskal-Wallis and Mann-Withney tests) generally failed to demonstrate that, statistically, portfolio's average profitability and average Sharpe's index can be differentiated to PSI, ELECTRE and CAPM. But, there are particular cases where Mann-Withney test differentiated means, in particular, to average profitability, ELECTRE vs. CAPM (2, 3 years) and PSI vs. CAPM (3 years); to average Sharpe's index, ELECTRE vs. CAPM (1, 2, 3 years) and PSI vs. CAPM (3 years).

Results obtained to 2nd sub period, parametric test (t-student) indicates that to an error probability of 5% and 10%, to average Sharpe's Index, only ELECTRE to a 1 year holding is statistically significant from 0%. All the others are statistically significant from 0%, but only to an error probability of 10%. To a 2 years holding, Sharpe's Index means calculated with ELECTRE are significantly different from 0% to an error probability of 10%. ANOVA test, by pair of group and by follow-up period, lead us to conclude that in every follow-up we cannot infer that one way of calculating portfolio's average profitability (empirical hypothesis *T1* and *T3*) and portfolio's average Sharpe's index (empirical hypothesis *T2* and *T4*) is better than the other, statistically. Nonparametric test (Kruskal-Wallis and Mann-Withney tests) tell us that certain means could be differentiated based on Kruskal-Wallis results for instance, average Sharpe's index for 1 and 2 years holding, to an error probability of 10%. Also with Mann-Withney test some means could be differentiated, in particular, to average profitability, PSI vs. ELECTRE (1 year) and ELECTRE vs. CAPM (2, 3 years); to average Sharpe's index, PSI vs. ELECTRE (3 years) and for average Sharpe's index ELECTRE vs. CAPM (3 years).

Finally looking to results obtained when comparing both sub periods, and as expected, parametric and nonparametric results show us that means can be differentiated statistically. On one hand, parametric test (t-student test to compare two groups) results obtained for all holding period allow us to rejected H_0, (except average profitability in a two years holding, to a error probability of 5%) so average profitability (empirical hypothesis *T1* and *T3*) and average Sharpe's index (empirical hypothesis *T2* and *T4*) for these two groups are statistically different. On the other hand, nonparametric test (Kruskal-Wallis and Mann-Withney tests) results obtained for all holding period allow us to rejected H_0, meaning average profitability (empirical hypothesis *T1* and *T3*) and average Sharpe's index (empirical hypothesis *T2* and *T4*) for these two groups are statistically different.

With all this, we can say that in a certain sense and attending to sample limitations, the ELECTRE III methodology proved to be a good tool to select assets to invest in a buy and hold perspective, and form defensive portfolios, within shares traded in the Portuguese stock index (PSI). However, the findings left by this empirical work, leaves us open other lines for future research, for instance, explore other stock markets, explore other alternatives as investment funds, use another multicriteria method as PROMETHEE, or even enunciate other assumptions (criteria and threshold).

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ANNEXES

ANNEX A:

ELECTRE III / IV - [j:\finbolsa\trabem~1\simula~1\v2_jul~1\p1.elp] - [Concordance Matrix]

	ESTOR	T.DUA	ZON	BPI	FISIP	GRA P	IBERS	INAPA	J_MAR	LISGR	M_ENG	OREY	PORTU	PT	REDIT	SEMAP	S_COS	SONAE	S_IND	SUMOL	TOYOT	BCP	BES	BRISA	C_AMO	CIMPO	COFIN	COMPT	EDP
ESTOR	1	0.8	1	0.52	1	1	0.6	1	1	1	0.8	0.4	0.67	0.8	0.85	0.6	1	0.8	1	1	0.8	0.59	0.74	0.6	0.93	0.6	0.8	1	0.8
T.DUA	1	1	1	0.8	1	1	0.6	1	1	1	1	0.42	0.93	0.81	0.8	0.6	1	0.8	1	0.8	1	0.8	0.8	0.6	1	0.6	0.8	1	1
ZON	0.6	0.6	1	0.58	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.2	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6
BPI	0.96	1	0.8	1	1	1	0.4	1	1	1	1	0.86	0.6	1	0.8	0.8	1	1	1	0.8	0.8	1	1	0.6	0.8	0.64	1	1	0.89
FISIP	0.6	0.6	1	0.6	1	0.8	0.6	0.6	0.8	0.8	0.6	0.23	0.6	0.6	0.55	0.6	0.6	0.6	0.8	0.68	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.8	0.6
GRA P	0.6	0.6	1	0.23	0.8	1	0.6	0.61	1	1	0.6	0.2	0.56	0.6	0.4	0.55	0.7	0.6	1	0.49	0.53	0.26	0.45	0.6	0.46	0.6	0.6	0.8	0.6
IBERS	1	1	1	0.66	1	1	1	1	1	1	1	0.6	0.98	1	1	0.96	1	1	1	1	0.94	0.73	0.92	1	0.87	1	1	1	1
INAPA	1	0.8	1	0.78	1	1	0.6	1	1	1	0.8	0.2	0.6	0.64	0.6	0.6	1	0.8	1	0.8	0.8	0.8	0.8	0.6	0.89	0.6	0.71	1	0.8
J_MAR	0.8	0.76	0.8	0.53	0.8	1	0.44	0.8	1	1	0.76	0.2	0.4	0.6	0.52	0.6	0.8	0.75	1	0.6	0.4	0.6	0.75	0.6	0.63	0.56	0.6	0.8	0.6
LISGR	0.6	0.6	0.9	0.6	0.8	0.88	0.6	0.8	1	1	0.6	0.2	0.49	0.6	0.4	0.6	0.8	0.6	1	0.6	0.58	0.63	0.8	0.6	0.61	0.6	0.6	0.8	0.6
M_ENG	1	1	1	0.8	1	1	0.6	1	1	1	1	0.42	0.94	0.8	0.8	0.6	1	0.8	1	0.97	1	0.8	0.8	0.6	1	0.6	0.8	1	1
OREY	1	1	1	1	1	1	0.6	1	1	1	1	1	1	1	0.93	1	1	1	1	1	1	1	1	0.8	1	1	0.97	1	1
PORTU	1	1	1	0.8	1	1	0.6	1	1	1	1	0.55	1	0.98	1	0.8	1	0.8	1	1	1	0.8	0.8	0.8	1	0.8	0.8	1	1
PT	1	1	1	0.72	1	1	0.6	1	1	1	1	0.54	1	1	0.87	0.9	1	1	1	1	1	0.78	1	0.76	1	0.8	0.8	1	1
REDIT	1	0.81	1	0.79	1	1	0.6	1	1	1	1	0.4	0.78	0.8	1	0.6	1	0.8	1	1	1	0.8	0.8	0.6	1	0.6	0.8	1	1
SEMAP	1	1	1	1	1	1	0.8	1	1	1	1	0.78	1	1	0.8	1	1	1	1	0.87	1	1	1	0.8	1	1	0.84	1	1
S_COS	1	0.8	1	0.79	1	1	0.6	1	1	1	0.8	0.2	0.6	0.6	1	0.6	1	0.8	1	1	0.8	0.8	0.8	0.6	1	0.6	0.62	1	0.77
SONAE	1	1	0.83	0.87	1	1	0.6	1	1	1	1	0.6	0.75	1	0.8	0.8	1	1	1	0.8	0.9	0.92	1	0.6	1	0.8	0.89	1	1
S_IND	0.68	0.61	1	0.77	0.8	1	0.6	0.8	1	1	0.61	0.2	0.59	0.6	0.4	0.6	0.8	0.61	1	0.6	0.6	0.8	0.8	0.6	0.74	0.6	0.6	0.8	0.6
SUMOL	1	0.8	1	0.8	1	1	0.6	1	1	1	0.8	0.24	0.6	0.6	0.8	0.6	0.85	0.8	1	1	0.8	0.8	0.8	0.6	0.8	0.6	0.63	1	0.78
TOYOT	1	1	1	0.8	1	1	0.6	1	1	1	1	0.58	0.86	0.8	1	0.6	1	0.8	1	1	1	0.8	0.8	0.6	1	0.6	0.8	1	1
BCP	1	1	0.8	1	1	1	0.4	1	1	1	1	0.9	0.6	1	0.8	0.8	1	1	1	0.8	0.8	1	1	0.6	0.83	0.7	1	1	0.95
BES	0.97	1	0.8	0.83	1	1	0.4	1	1	1	1	0.75	0.6	0.98	0.8	0.8	1	1	1	0.8	0.8	1	1	0.6	0.8	0.54	0.8	1	0.91
BRISA	1	1	1	0.77	1	1	1	1	1	1	1	0.6	1	1	1	1	1	1	1	1	0.99	0.84	1	1	0.92	1	1	1	1
C_AMO	1	0.8	1	0.77	1	1	0.6	1	1	1	0.8	0.57	0.6	0.8	1	0.6	1	0.8	1	1	0.98	0.8	0.8	0.6	1	0.6	0.8	1	0.9
CIMPO	1	1	1	0.96	1	1	0.8	1	1	1	1	0.6	1	1	1	1	1	1	1	1	1	1	1	0.8	1	1	1	1	1
COFIN	1	1	1	1	1	1	0.6	1	1	1	1	0.66	1	1	0.8	0.83	1	1	1	0.92	1	1	1	1	0.92	1	1	1	1
COMPT	0.61	0.6	0.99	0.58	1	1	0.6	0.8	0.94	1	0.6	0.2	0.57	0.6	0.4	0.6	0.6	0.6	1	0.6	0.6	0.6	0.8	0.6	1	0.6	0.8	1	0.9
EDP	1	1	1	0.52	1	1	0.6	1	1	1	1	0.4	0.93	0.8	1	0.59	1	0.8	1	1	0.97	0.59	1	0.8	1	1	1	1	1

Figure 2 – Concordance Matrix (Portfolio 1)

Source: ELECTRE III software output estimation, May 2012

ANNEX B:

ELECTRE III / IV - [j:\finbolsa\trabem~1\simula~1\v2_jul~1\p1.elp] - [Ranking Matrix]

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	ESTOR	T.DUA	ZON	BPI	FISIP	GRA P	IBERS	INAPA	J_MAR	LISGR	M_ENG	OREY	PORTU	PT	REDIT	SEMAP	S_COS	SONAE	S_IND	SUMOL	TOYOT	BCP	BES	BRISA	C_AMO	CIMPO	COFIN	COMPT	EDP
ESTOR	I	P-	P	P-	P	P	P-	P	P	P	P-	P-	P-	P-	P-	P-	P-	P-	P	P-	P-	P-	P-	P-	P-	P-	P-	P	P-
T.DUA	P	I	P	P-	P	P	P-	P	P	P	R	P-	P-	P-	P-	P-	P	P-	P	P	P-	P-	P-	P-	P	P-	P-	P	P-
ZON	P-	P-	I	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-
BPI	P	P	P	I	P	P	P-	P	P	P	P	P-	P	P	P	P-	P	P	P	P	P	P	P	P-	P	P-	R	P	P
FISIP	P-	P-	P	P-	I	I	P-	P-	R	I	P-	P-	P-	P-	P-	P-	P-	P-	I	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-
GRA P	P-	P-	P	P-	I	I	P-	P-	R	I	P-	P-	P-	P-	P-	P-	P-	P-	I	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-
IBERS	P	P	P	P	P	P	I	P	P	P	P	P-	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
INAPA	P-	P-	P	P-	P	P	P-	I	P	P	P-	P-	P-	P-	P-	P-	P-	P-	P	P-	P-	P-	P-	P-	P-	P-	P-	P	P-
J_MAR	P-	P-	P	P-	R	R	P-	P-	I	R	P-	P-	P-	P-	P-	P-	P-	P-	R	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-
LISGR	P-	P-	P	P-	I	I	P-	P-	R	I	P-	P-	P-	P-	P-	P-	P-	P-	I	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-
M_ENG	P	R	P	P-	P	P	P-	P	P	P	I	P-	P-	P-	P-	P-	P	P-	P	P	P-	P-	P-	P-	P	P-	P-	P	P-
OREY	P	P	P	P	P	P	P	P	P	P	P	I	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
PORTU	P	P	P	P-	P	P	P-	P	P	P	P	P-	I	R	P	P-	P	P-	P	P	P	R	P	P-	P	P-	P-	P	P
PT	P	P	P	P-	P	P	P-	P	P	P	P	P-	R	I	R	P-	P	R	P	P	P	R	P	P-	P	P-	P-	P	P
REDIT	P	P	P	P-	P	P	P-	P	P	P	P	P-	P-	R	I	P-	P	P-	P	P	P	P-	P	P-	P	P-	P-	P	P
SEMAP	P	P	P	P	P	P	P-	P	P	P	P	P-	P	P	P	I	P	P	P	P	P	P	P	P-	P	P-	P	P	P
S_COS	P	P-	P	P-	P	P	P-	P	P	P	P-	P-	P-	P-	P-	P-	I	P-	P	R	P-	P-	P-	P-	P-	P-	P-	P	P-
SONAE	P	P	P	P-	P	P	P-	P	P	P	P	P-	P	R	P	P-	P	I	P	P	P	R	P	P-	P	P-	P-	P	P
S_IND	P-	P-	P	P-	I	I	P-	P-	R	I	P-	P-	P-	P-	P-	P-	P-	P-	I	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-
SUMOL	P	P-	P	P-	P	P	P-	P	P	P	P-	P-	P-	P-	P-	P-	R	P-	P	I	P-	P-	P-	P-	P-	P-	P-	P	P-
TOYOT	P	P	P	P-	P	P	P-	P	P	P	P	P-	P-	P-	P-	P-	P	P-	P	P	I	P-	I	P-	P	P-	P-	P	P
BCP	P	P	P	P-	P	P	P-	P	P	P	P	P-	R	R	P	P-	P	R	P	P	P	I	P	P-	P	P-	R	P	P
BES	P	P	P	P-	P	P	P-	P	P	P	P	P-	P-	P-	P-	P-	P	P-	P	P	I	P-	I	P-	P	P-	P-	P	P
BRISA	P	P	P	P	P	P	P-	P	P	P	P	P-	P	P	P	P	P	P	P	P	P	P	P	I	P	P	P	P	P
C_AMO	P	P-	P	P-	P	P	P-	P	P	P	P-	P-	P-	P-	P-	P-	P	P-	P	P	P-	P-	P-	P-	I	P-	P-	P	P-
CIMPO	P	P	P	P	P	P	P-	P	P	P	P	P-	P	P	P	P	P	P	P	P	P	P	P	P-	P	I	P	P	P
COFIN	P	P	P	R	P	P	P-	P	P	P	P	P-	P	P	P	P-	P	P	P	P	P	R	P	P-	P	P-	I	P	P
COMPT	P-	P-	P	P-	P	P	P-	P-	P	P	P-	P-	P-	P-	P-	P-	P-	P-	P	P-	P-	P-	P-	P-	P-	P-	P-	I	P-
EDP	P	P	P	P-	P	P	P-	P	P	P	P	P-	P-	P-	P-	P-	P	P-	P	P	P-	P-	P-	P-	P	P-	P-	P	I

Legend (symbol at the intersection of the row for *a* and the column for *b*):

The couple (*a*, *b*) verifies one of the 4 following relations :

- if *a* is better than *b*, the symbol is **P**;

- if *a* is equivalent to *b*, the symbol is **I**;

- if *a* is as good as *b*, the symbol is **P-**;

- if *a* is incomparable to *b*, the symbol is **R**.

Figure 3 – Ranking Matrix (Portfolio 1)

Source: ELECTRE III software output estimation, May 2012

ANNEX C:

ELECTRE III / IV - [j:\finbolsa\trabem~1\simula~1\v2_jul~1\p2.elp] - [Concordance Matrix]

	ESTOR	T.DUA	ZON	BPI	FISIP	GRA P	IBERS	INAPA	J_MAR	LISGR	M_ENG	OREY	PORTU	PT	REDIT	SEMAP	S_COS	SONAE	S_IND	SUMOL	TOYOT	BCP	BES	BRISA	C_AMO	CIMPO	COFIN	COMPT	EDP
ESTOR	1	0.8	0.91	0.49	1	1	0.6	1	1	1	0.8	0.2	0.68	0.71	0.8	0.6	0.8	0.8	1	1	0.8	0.71	0.78	0.6	0.93	0.6	0.74	1	0.8
T.DUA	1	1	0.94	0.98	1	1	0.6	1	1	1	1	0.76	0.98	1	0.96	0.8	1	1	1	0.8	1	1	1	0.6	1	0.6	1	1	1
ZON	1	0.8	1	0.77	1	1	0.6	1	1	1	0.8	0.44	0.8	0.8	0.87	0.6	0.94	0.8	1	1	1	0.8	0.8	0.6	1	0.6	0.8	1	0.8
BPI	1	1	0.8	1	1	1	0.4	1	1	1	1	0.8	0.73	0.96	0.8	0.8	1	1	1	0.8	0.8	1	1	0.53	0.8	0.43	0.99	1	1
FISIP	0.6	0.6	0.6	0.59	1	0.8	0.6	0.6	0.62	0.8	0.6	0.38	0.6	0.6	0.6	0.6	0.6	0.6	0.8	0.8	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.8	0.6
GRA P	0.6	0.6	0.6	0.2	0.8	1	0.6	0.6	0.6	1	0.6	0.2	0.51	0.6	0.6	0.51	0.6	0.6	0.6	0.6	0.54	0.37	0.52	0.6	0.44	0.6	0.6	0.8	0.6
IBERS	1	1	1	0.63	1	1	1	1	1	1	1	0.6	0.94	1	1	0.94	1	1	1	1	0.97	0.86	0.99	1	0.87	1	1	1	1
INAPA	1	0.8	0.72	0.74	1	1	0.6	1	1	1	0.8	0.31	0.55	0.6	0.67	0.6	0.8	0.72	1	0.8	0.8	0.8	0.8	0.6	1	0.6	0.6	1	0.8
J_MAR	1	0.8	0.6	0.52	1	1	0.54	1	1	1	0.8	0.2	0.41	0.64	0.6	0.6	0.8	0.8	1	0.8	0.67	0.73	0.8	0.6	0.91	0.6	0.67	1	0.8
LISGR	0.6	0.6	0.4	0.37	0.8	1	0.54	0.67	0.6	1	0.6	0.2	0.4	0.6	0.4	0.6	0.8	0.6	0.8	0.6	0.46	0.59	0.6	0.6	0.53	0.6	0.6	0.8	0.6
M_ENG	1	0.8	1	0.8	1	1	0.6	1	1	1	1	0.62	1	0.8	1	0.8	1	1	1	0.95	1	0.8	0.88	0.6	1	0.6	0.8	1	1
OREY	1	1	1	1	1	1	0.6	1	1	1	1	1	1	1	1	0.8	1	1	1	1	1	1	1	0.8	1	0.8	1	1	1
PORTU	1	0.8	1	0.8	1	1	0.6	1	1	1	1	0.71	1	0.8	1	0.6	1	0.88	1	1	1	0.8	0.8	0.66	1	0.6	0.8	1	1
PT	1	1	1	0.69	1	1	0.6	1	1	1	1	0.6	1	1	1	0.8	1	1	1	1	1	1	1	0.63	0.99	0.6	1	1	1
REDIT	1	0.8	1	0.77	1	1	0.6	1	1	1	0.83	0.45	1	0.8	1	0.6	1	0.8	1	1	1	0.8	0.8	0.6	1	0.6	0.8	1	1
SEMAP	1	0.8	1	0.8	1	1	0.8	1	1	1	1	0.78	1	1	1	1	1	1	1	1	1	0.99	1	0.8	1	0.8	0.8	1	1
S_COS	1	0.68	0.97	0.78	1	1	0.6	1	1	1	0.8	0.32	0.8	0.6	0.81	0.6	1	0.6	1	1	0.93	0.8	0.8	0.6	1	0.6	0.6	1	0.92
SONAE	1	0.8	0.8	0.7	1	1	0.56	1	1	1	1	0.43	0.82	0.8	0.8	0.8	1	1	1	0.8	0.89	0.8	1	0.6	1	0.6	0.8	1	1
S_IND	0.8	0.6	0.6	0.78	0.8	1	0.6	0.8	0.8	1	0.6	0.34	0.6	0.6	0.6	0.6	0.8	0.6	1	0.67	0.6	0.8	0.8	0.6	0.8	0.6	0.6	0.8	0.6
SUMOL	0.8	0.6	0.6	0.78	1	1	0.6	0.83	0.9	1	0.6	0.34	0.6	0.6	0.6	0.6	0.8	0.6	1	1	0.6	0.8	0.8	0.6	0.8	0.6	0.6	1	0.6
TOYOT	1	0.8	1	0.8	1	1	0.6	1	1	1	0.8	0.65	0.98	0.8	1	0.6	1	0.8	1	1	1	0.8	0.8	0.6	1	0.6	0.8	1	1
BCP	1	1	0.8	0.86	1	1	0.4	1	1	1	1	0.8	0.75	0.99	0.8	0.8	1	1	1	0.8	0.8	1	1	0.59	0.81	0.5	1	1	1
BES	1	0.8	0.8	0.8	1	1	0.4	1	1	1	1	0.6	0.68	0.91	0.8	0.8	1	1	1	0.8	0.8	1	1	0.55	0.8	0.45	0.74	1	1
BRISA	1	1	1	0.74	1	1	1	1	1	1	1	0.6	0.99	1	1	0.99	1	1	1	1	1	0.97	1	1	0.92	1	1	1	1
C_AMO	1	0.8	1	0.75	1	1	0.6	1	1	1	0.8	0.4	0.8	0.8	0.8	0.6	0.8	0.8	1	1	0.87	0.8	0.8	0.6	1	0.6	0.8	1	0.8
CIMPO	1	1	1	1	1	1	1	1	1	1	1	0.9	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
COFIN	1	1	1	1	1	1	0.6	1	1	1	1	0.82	1	1	1	0.8	1	1	1	1	1	1	1	0.6	1	0.6	1	1	1
COMPT	0.6	0.6	0.4	0.48	1	0.87	0.4	0.63	0.7	0.8	0.6	0.2	0.4	0.6	0.4	0.6	0.6	0.6	0.8	0.6	0.4	0.6	0.6	0.53	0.39	0.43	0.6	1	0.6
EDP	1	0.8	1	0.5	1	1	0.6	1	1	1	1	0.4	0.97	0.8	1	0.57	1	0.8	1	1	1	0.72	0.8	0.6	0.9	0.6	0.8	1	1

Figure 4 – Concordance Matrix (Portfolio 2)

Source: ELECTRE III software output estimation, May 2012

ANNEX D:

ELECTRE III / IV - [j:\finbolsa\trabem~1\simula~1\v2_jul~1\p2.elp] - [Ranking Matrix]

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	ESTOR	T.DUA	ZON	BPI	FISIP	GRA P	IBERS	INAPA	J_MAR	LISGR	M_ENG	OREY	PORTU	PT	REDIT
ESTOR	I	P-	P-	P-	P	P	P-	P	P	P	P-	P-	P-	P-	P-
T.DUA	P	I	P	R	P	P	P-	P	P	P	P	P-	P	P	P
ZON	P	P-	I	P-	P	P	P-	P	P	P	P-	P-	P-	P-	P-
BPI	P	R	P	I	P	P	P-	P	P	P	P	P-	P	R	P
FISIP	P-	P-	P-	P-	I	I	P-	P-	P-	I	P-	P-	P-	P-	P-
GRA P	P-	P-	P-	P-	I	I	P-	P-	P-	I	P-	P-	P-	P-	P-
IBERS	P	P	P	P	P	P	I	P	P	P	P	P-	P	P	P
INAPA	P-	P-	P-	P-	P	P	P-	I	I	P	P-	P-	P-	P-	P-
J_MAR	P-	P-	P-	P-	P	P	P-	I	I	P	P-	P-	P-	P-	P-
LISGR	P-	P-	P-	P-	I	I	P-	P-	P-	I	P-	P-	P-	P-	P-
M_ENG	P	P-	P	P-	P	P	P-	P	P	P	I	P-	R	P-	P
OREY	P	P	P	P	P	P	P	P	P	P	P	I	P	P	P
PORTU	P	P-	P	P-	P	P	P-	P	P	P	R	P-	I	P-	P
PT	P	P-	P	R	P	P	P-	P	P	P	P	P-	P	I	P
REDIT	P	P-	P	P-	P	P	P-	P	P	P	P-	P-	P-	P-	I
SEMAP	P	P	P	P	P	P	P-	P	P	P	P	P-	P	P	P
S_COS	P	P-	R	P-	P	P	P-	P	P	P	P-	P-	P-	P-	P-
SONAE	P	P-	R	P-	P	P	P-	P	P	P	P-	P-	P-	P-	P-
S_IND	P-	P-	P-	P-	P	P	P-	P-	P-	P	P-	P-	P-	P-	P-
SUMOL	R	P-	P-	P-	P	P	P-	P	P	P	P-	P-	P-	P-	P-
TOYOT	P	P-	P	P-	P	P	P-	P	P	P	P-	P-	P-	P-	P-
BCP	P	P-	P	P-	P	P	P-	P	P	P	R	P-	R	R	P
BES	P	P-	P	P-	P	P	P-	P	P	P	R	P-	P-	P-	R
BRISA	P	P	P	P	P	P	R	P	P	P	P	P-	P	P	P
C_AMO	P	P-	P-	P-	P	P	P-	P	P	P	P-	P-	P-	P-	P-
CIMPO	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
COFIN	P	P	P	R	P	P	P-	P	P	P	P	P-	P	P	P
COMPT	P-	P-	P-	P-	P	P	P-	P-	P-	P	P-	P-	P-	P-	P-
EDP	P	P-	R	P-	P	P	P-	P	P	P	P-	P-	P-	P-	P-

	SEMAP	S_COS	SONAE	S_IND	SUMOL	TOYOT	BCP	BES	BRISA	C_AMO	CIMPO	COFIN	COMPT	EDP
ESTOR	P-	P-	P-	P	R	P-	P-	P-	P-	P-	P-	P-	P	P-
T.DUA	P-	P	P	P	P	P	P	P	P-	P	P-	P-	P	P
ZON	P-	R	R	P	P	P-	P-	P-	P-	P	P-	P-	P	R
BPI	P-	P	P	P	P	P	P	P	P-	P	P-	R	P	P
FISIP	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-
GRA P	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-
IBERS	P	P	P	P	P	P	P	P	R	P	P-	P	P	P
INAPA	P-	P-	P-	P	P-	P-	P-	P-	P-	P-	P-	P-	P	P-
J_MAR	P-	P-	P-	P	P-	P-	P-	P-	P-	P-	P-	P-	P	P-
LISGR	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-
M_ENG	P-	P	P	P	P	P	R	R	P-	P	P-	P-	P	P
OREY	P	P	P	P	P	P	P	P	P	P	P-	P	P	P
PORTU	P-	P	P	P	P	P	R	P	P-	P	P-	P-	P	P
PT	P-	P	P	P	P	P	R	P	P-	P	P-	P-	P	P
REDIT	P-	P	P	P	P	P	P-	R	P-	P	P-	P-	P	P
SEMAP	I	P	P	P	P	P	P	P	P-	P	P-	P	P	P
S_COS	P-	I	P-	P	P	R	P-	P-	P-	R	P-	P-	P	P-
SONAE	P-	P	I	P	P	R	P-	R	P-	P	P-	P-	P	P
S_IND	P-	P-	P-	I	P-	P-	P-	P-	P-	P-	P-	P-	P	P-
SUMOL	P-	P-	P-	P	I	P-	P-	P-	P-	P-	P-	P-	P	P-
TOYOT	P-	R	R	P	P	I	P-	P-	P-	P	P-	P-	P	R
BCP	P-	P	P	P	P	P	I	P	P-	P	P-	P-	P	P
BES	P-	P	R	P	P	P	P-	I	P-	P	P-	P-	P	P
BRISA	P	P	P	P	P	P	P	P	I	P	P-	P	P	P
C_AMO	P-	R	P-	P	P	P-	P-	P-	P-	I	P-	P-	P	P-
CIMPO	P	P	P	P	P	P	P	P	P	P	I	P	P	P
COFIN	P-	P	P	P	P	P	P	P	P-	P	P-	I	P	P
COMPT	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	I	P-
EDP	P-	P	P-	P	P	R	P-	P-	P-	P	P-	P-	P	I

Legend (symbol at the intersection of the row for *a* and the column for *b*):

The couple (*a*, *b*) verifies one of the 4 following relations :

- if *a* is better than *b*, the symbol is **P**;

- if *a* is equivalent to *b*, the symbol is **I**;

- if *a* is as good as *b*, the symbol is **P-**;

- if *a* is incomparable to *b*, the symbol is **R**.

Figure 5– Ranking Matrix (Portfolio 2)

Source: ELECTRE III software output estimation, May 2012

ANNEX E:

ELECTRE III / IV - [j:\finbolsa\trabem~1\simula~1\v2_jul~1\p3.elp] - [Concordance Matrix]

File Edit Calculate Results Options Window Help

	ESTOR	T.DUA	ZON	BPI	FISIP	GRA P	IBERS	INAPA	J_MAR	LISGR	M_ENG	OREY	PORTU	PT	REDIT	SEMAP	S_COS	SONAE	S_IND	SUMOL	TOYOT	BCP	BES	BRISA	C_AMO	CIMPO	COFIN	COMPT	EDP
ESTOR	1	0.78	0.6	0.49	1	1	0.6	1	0.8	1	0.8	0.54	0.51	0.6	0.73	0.6	0.8	0.6	1	0.97	0.97	0.76	0.8	0.6	0.94	0.56	0.6	1	0.8
T.DUA	1	1	0.91	0.95	1	1	0.6	1	1	1	1	1	0.98	1	1	0.8	1	1	1	0.84	1	1	1	0.99	1	0.7	1	1	1
ZON	1	0.8	1	0.75	1	1	0.67	1	1	1	1	0.8	0.96	0.8	1	0.93	1	1	1	1	1	1	1	0.8	1	0.8	0.8	1	1
BPI	1	1	0.6	1	1	1	0.4	1	1	1	1	0.8	0.72	0.8	0.93	0.8	1	0.85	1	0.8	0.8	1	1	0.64	0.8	0.4	0.83	1	1
FISIP	0.66	0.6	0.6	0.56	1	0.8	0.6	0.8	0.6	0.8	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.8	0.79	0.6	0.6	0.6	0.6	0.6	0.6	0.6	1	0.6
GRA P	0.6	0.6	0.6	0.2	0.8	1	0.6	1	0.6	1	0.6	0.39	0.33	0.6	0.6	0.54	0.7	0.6	0.67	0.66	0.57	0.45	0.56	0.6	0.44	0.42	0.6	0.8	0.6
IBERS	1	1	1	0.61	1	1	1	1	1	1	1	0.86	0.78	1	1	0.95	1	1	1	1	0.98	0.92	1	1	0.86	0.88	1	1	1
INAPA	0.8	0.6	0.4	0.74	0.8	1	0.6	1	0.6	1	0.6	0.6	0.4	0.6	0.6	0.6	0.8	0.6	0.8	0.6	0.52	0.8	0.8	0.6	0.59	0.6	0.6	0.8	0.6
J_MAR	1	0.8	0.71	0.51	1	1	0.6	1	1	1	1	0.75	0.74	0.8	1	0.8	1	0.8	1	0.8	1	0.94	1	0.6	0.94	0.57	0.8	1	1
LISGR	0.7	0.6	0.4	0.37	0.8	1	0.47	0.8	0.6	1	0.6	0.6	0.25	0.6	0.6	0.6	0.8	0.6	0.8	0.6	0.4	0.62	0.69	0.59	0.37	0.5	0.6	0.8	0.6
M_ENG	1	0.8	0.66	0.8	1	1	0.6	1	1	1	1	0.8	0.99	0.8	1	0.8	1	0.8	1	0.85	1	0.8	0.91	0.6	1	0.6	0.8	1	1
OREY	1	1	1	1	1	1	0.6	1	1	1	1	1	1	0.8	1	0.8	1	1	1	1	1	1	1	1	1	0.6	1	1	1
PORTU	1	0.8	0.8	0.8	1	1	0.6	1	0.8	1	1	0.8	1	0.8	1	0.8	0.99	0.8	1	1	1	0.8	0.8	0.8	1	0.6	0.8	1	1
PT	1	1	1	0.83	1	1	0.8	1	1	1	1	1	0.9	1	1	0.8	1	1	1	0.92	1	1	1	1	0.97	0.8	1	1	1
REDIT	1	0.8	0.75	0.69	1	1	0.6	1	0.85	1	1	0.8	0.94	0.8	1	0.8	1	0.8	1	1	1	0.8	0.8	0.8	1	0.6	0.8	1	1
SEMAP	1	0.8	0.8	0.8	1	1	0.8	1	1	1	1	0.8	1	0.8	1	1	1	0.8	1	1	1	0.8	0.83	0.8	1	0.8	0.8	1	1
S_COS	1	0.6	0.79	0.76	1	1	0.6	1	0.94	1	1	0.6	0.8	0.6	0.8	0.8	1	0.6	1	1	0.8	0.8	1	0.6	0.94	0.6	0.6	1	0.89
SONAE	1	0.8	0.88	0.67	1	1	0.6	1	1	1	1	1	0.8	0.8	1	0.8	1	1	1	0.8	1	1	1	0.95	1	0.6	0.92	1	1
S_IND	0.8	0.6	0.6	0.79	0.8	1	0.6	1	0.77	1	0.8	0.6	0.6	0.6	0.6	0.6	0.8	0.6	1	0.7	0.6	0.8	0.8	0.6	0.76	0.6	0.6	1	0.71
SUMOL	0.8	0.6	0.6	0.76	1	1	0.6	1	0.78	1	0.8	0.6	0.6	0.6	0.6	0.6	0.8	0.6	1	1	0.6	0.8	0.8	0.6	0.78	0.6	0.6	1	0.73
TOYOT	1	0.8	0.7	0.78	1	1	0.6	1	0.8	1	0.91	0.8	1	0.8	1	0.79	0.8	0.8	1	1	1	0.8	0.8	0.79	1	0.6	0.8	1	1
BCP	1	0.8	0.6	0.8	1	1	0.4	1	1	1	1	0.6	0.73	0.6	0.99	0.8	1	0.86	1	0.8	0.8	1	1	0.5	0.8	0.46	0.64	1	1
BES	1	0.79	0.6	0.8	1	1	0.4	1	1	1	1	0.6	0.66	0.6	0.91	0.8	1	0.79	1	0.8	0.79	1	1	0.49	0.8	0.44	0.6	1	1
BRISA	1	1	1	0.73	1	1	0.6	1	1	1	1	1	0.84	0.82	1	0.84	1	1	1	1	1	1	1	1	0.92	0.74	1	1	1
C_AMO	1	0.8	0.6	0.74	1	1	0.6	1	0.8	1	0.8	0.8	1	0.8	0.91	0.6	0.8	0.8	1	1	1	0.8	0.8	0.6	1	0.6	0.8	1	0.84
CIMPO	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
COFIN	1	1	1	1	1	1	0.6	1	1	1	1	1	1	0.84	1	0.8	1	1	1	1	1	1	1	1	1	0.6	1	1	1
COMPT	0.4	0.4	0.4	0.38	0.6	0.6	0.4	0.8	0.48	0.8	0.4	0.4	0.26	0.4	0.4	0.4	0.4	0.51	0.6	0.55	0.4	0.6	0.6	0.4	0.34	0.37	0.4	1	0.4
EDP	1	0.8	0.6	0.49	1	1	0.6	1	0.92	1	1	0.76	0.82	0.8	1	0.8	1	0.8	1	1	1	0.8	0.8	0.65	0.9	0.53	0.8	1	1

Figure 6 – Concordance Matrix (Portfolio 3)

Source: ELECTRE III software output estimation, May 2012

ANNEX F:

ELECTRE III / IV - [j:\finbolsa\trabem~1\simula~1\v2_jul~1\p3.elp] - [Ranking Matrix]

	ESTOR	T.DUA	ZON	BPI	FISIP	GRA P	IBERS	INAPA	J_MAR	LISGR	M_ENG	OREY	PORTU	PT	REDIT	SEMAP	S_COS	SONAE	S_IND	SUMOL	TOYOT	BCP	BES	BRISA	C_AMO	CIMPO	COFIN	COMPT	EDP
ESTOR	I	P^-	P^-	P^-	P	P	P^-	P	P^-	P	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P	R	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P	P^-
T.DUA	P	I	P	P	P	P	P^-	P	P	P	P	R	P	P^-	P	R	P	P	P	P	P	P	P	P	P	P^-	R	P	P
ZON	P	P^-	I	R	P	P	P^-	P	P	P	P	P^-	P	P^-	P	R	P	R	P	P	P	P	P	P^-	P	P^-	P^-	P	P
BPI	P	P^-	R	I	P	P	P^-	P	P	P	P	P^-	P	P^-	P	P^-	P	R	P	P	P	P	P	P^-	P	P^-	P^-	P	P
FISIP	P^-	P^-	P^-	P^-	I	P	P^-	P	P^-	P	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P	P^-
GRA P	P^-	P^-	P^-	P^-	P^-	I	P^-	P^-	P^-	P	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P	P^-
IBERS	P	P	P	P	P	P	I	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P^-	P	P	P
INAPA	P^-	P^-	P^-	P^-	P^-	P	P^-	I	P^-	P	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P	P^-
J_MAR	P	P^-	P^-	P^-	P	P	P^-	P	I	P	P	P^-	P^-	P^-	P	P^-	P	P^-	P	P	P	P^-	R	P^-	P	P^-	P^-	P	P
LISGR	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	I	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	I	P^-
M_ENG	P	P^-	P^-	P^-	P	P	P^-	P	P^-	P	I	P^-	P^-	P^-	R	P^-	P	P^-	P	P	R	P^-	P^-	P^-	P	P^-	P^-	P	P
OREY	P	R	P	P	P	P	P^-	P	P	P	P	I	P	P^-	P	R	P	P	P	P	P	P	P	P	P	P^-	I	P	P
PORTU	P	P^-	P^-	P^-	P	P	P^-	P	P	P	P	P^-	I	P^-	P	P^-	P	P^-	P	P	P	R	P	P^-	P	P^-	P^-	P	P
PT	P	P	P	P	P	P	P^-	P	P	P	P	P	P	I	P	R	P	P	P	P	P	P	P	P	P	P^-	P	P	P
REDIT	P	P^-	P^-	P^-	P	P	P^-	P	P^-	P	R	P^-	P^-	P^-	I	P^-	P	P^-	P	P	R	P^-	R	P^-	R	P^-	P^-	P	P
SEMAP	P	R	R	P	P	P	P^-	P	P	P	P	R	P	R	P	I	P	P	P	P	P	P	P	R	P	P^-	R	P	P
S_COS	P	P^-	P^-	P^-	P	P	P^-	P	P^-	P	P^-	P^-	P^-	P^-	P^-	P^-	I	P^-	P	P	R	P^-	P^-	P^-	R	P^-	P^-	P	P
SONAE	P	P^-	R	R	P	P	P^-	P	P	P	P	P^-	P	P^-	P	P^-	P	I	P	P	P	P	P	P^-	P	P^-	P^-	P	P
S_IND	P^-	P^-	P^-	P^-	P	P	P^-	P	P^-	P	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	I	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P	P^-
SUMOL	R	P^-	P^-	P^-	P	P	P^-	P	P^-	P	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P	I	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P	P^-
TOYOT	P	P^-	P^-	P^-	P	P	P^-	P	P^-	P	R	P^-	P^-	P^-	R	P^-	R	P^-	P	P	I	P^-	P^-	P^-	P	P^-	P^-	P	R
BCP	P	P^-	P^-	P^-	P	P	P^-	P	P	P	P	P^-	R	P^-	P	P^-	P	P^-	P	P	P	I	P	P^-	P	P^-	P^-	P	P
BES	P	P^-	P^-	P^-	P	P	P^-	P	R	P	P	P^-	P^-	P^-	R	P^-	P	P^-	P	P	P	P^-	I	P^-	P	P^-	P^-	P	P
BRISA	P	P^-	P	P	P	P	P^-	P	P	P	P	P^-	P	P^-	P	R	P	P	P	P	P	P	P	I	P	P^-	P^-	P	P
C_AMO	P	P^-	P^-	P^-	P	P	P^-	P	P^-	P	P^-	P^-	P^-	P^-	R	P^-	R	P^-	P	P	P^-	P^-	P^-	P^-	I	P^-	P^-	P	R
CIMPO	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	I	P	P	P
COFIN	P	R	P	P	P	P	P^-	P	P	P	P	I	P	P^-	P	R	P	P	P	P	P	P	P	P	P	P^-	I	P	P
COMPT	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	I	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	I	P^-
EDP	P	P^-	P^-	P^-	P	P	P^-	P	P^-	P	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P	P	R	P^-	P^-	P^-	R	P^-	P^-	P	I

Legend (symbol at the intersection of the row for *a* and the column for *b*):

The couple (*a*, *b*) verifies one of the 4 following relations :

- if *a* is better than *b*, the symbol is **P**;

- if *a* is equivalent to *b*, the symbol is **I**;

- if *a* is as good as *b*, the symbol is **P^-**;

- if *a* is incomparable to *b*, the symbol is **R**.

Figure 7 – Ranking Matrix (Portfolio 3)

Source: ELECTRE III software output estimation, May 2012

ANNEX G:

ELECTRE III / IV - [j:\finbolsa\trabem~1\simula~1\v2_jul~1\p4.elp] - [Concordance Matrix]

File Edit Calculate Results Options Window Help

	ESTOR	T.DUA	ZON	BPI	FISIP	GRA P	IBERS	INAPA	J_MAR	LISGR	M_ENG	OREY	PORTU	PT	REDIT	SEMAP	S_COS	SONAE	S_IND	SUMOL	TOYOT	BCP	BES	BRISA	C_AMO	CIMPO	COFIN	COMPT	EDP
ESTOR	1	0.65	0.6	0.45	1	1	0.6	1	0.6	1	0.8	0.69	0.56	0.6	0.64	0.75	0.8	0.6	1	0.89	1	0.8	0.8	0.6	0.89	0.47	0.6	1	0.8
T.DUA	1	1	0.8	0.91	1	1	0.8	1	1	1	1	1	1	0.8	1	0.8	1	1	1	0.89	1	1	1	1	1	0.8	1	1	1
ZON	1	1	1	0.87	1	1	1	1	1	1	1	1	0.92	0.8	1	1	1	1	1	1	1	1	1	1	0.99	0.8	1	1	1
BPI	1	0.87	0.6	1	1	1	0.6	1	0.8	1	1	0.91	0.6	0.6	0.86	0.8	1	0.8	1	0.8	0.8	1	1	0.62	0.8	0.54	0.8	1	1
FISIP	0.6	0.6	0.6	0.5	1	0.8	0.6	0.8	0.6	0.8	0.6	0.6	0.56	0.6	0.6	0.6	0.6	0.6	0.6	0.65	0.6	0.6	0.6	0.6	0.6	0.6	0.6	1	0.6
GRA P	0.6	0.6	0.6	0.2	0.8	1	0.6	0.98	0.6	0.8	0.6	0.6	0.26	0.6	0.6	0.42	0.6	0.6	0.6	0.6	0.6	0.52	0.58	0.6	0.4	0.31	0.6	0.8	0.6
IBERS	1	1	1	0.6	1	1	1	1	1	1	1	1	0.72	0.8	1	0.88	1	1	1	1	1	0.98	1	1	0.82	0.79	1	1	1
INAPA	0.74	0.6	0.42	0.64	0.8	1	0.6	1	0.6	0.8	0.6	0.6	0.4	0.6	0.6	0.6	0.8	0.6	0.6	0.6	0.52	0.72	0.78	0.6	0.55	0.6	0.6	0.8	0.6
J_MAR	1	0.8	0.6	0.48	1	1	0.7	1	1	1	1	1	0.81	0.8	0.8	0.78	1	0.8	1	0.89	1	1	1	1	0.92	0.68	1	1	1
LISGR	0.6	0.6	0.4	0.29	0.8	1	0.4	1	0.6	1	0.6	0.6	0.22	0.6	0.6	0.55	0.6	0.6	0.6	0.4	0.4	0.6	0.6	0.46	0.3	0.3	0.6	0.8	0.6
M_ENG	1	0.8	0.56	0.78	1	1	0.6	1	0.98	1	1	1	0.83	0.8	0.8	0.8	1	0.8	1	0.8	1	1	1	0.95	1	0.6	0.85	1	1
OREY	1	0.8	0.6	0.8	1	1	0.6	1	1	1	1	1	1	0.8	0.8	0.8	1	0.8	1	0.99	1	1	1	1	1	0.6	0.95	1	1
PORTU	1	0.8	0.6	0.8	1	1	0.6	1	0.8	1	0.8	0.8	1	0.8	0.8	0.8	0.8	0.8	1	1	1	0.8	0.8	0.8	1	0.6	0.8	1	1
PT	1	1	0.8	0.77	1	1	0.8	1	1	1	1	1	0.8	1	1	0.79	1	1	1	0.82	1	1	1	1	0.93	0.79	1	1	1
REDIT	1	1	0.8	0.83	1	1	0.8	1	1	1	1	1	0.9	0.8	1	0.8	1	1	1	0.99	1	1	1	1	0.97	0.66	1	1	1
SEMAP	1	0.8	0.8	0.8	1	1	0.8	1	0.8	1	0.8	0.8	1	0.8	0.8	1	0.8	0.8	1	1	1	0.8	0.85	0.8	1	0.8	0.8	1	1
S_COS	1	0.6	0.6	0.73	1	1	0.6	1	0.8	1	0.92	0.8	0.8	0.6	0.6	0.8	1	0.6	1	1	0.81	1	1	0.8	0.82	0.6	0.6	1	0.89
SONAE	1	1	0.8	0.8	1	1	0.87	1	1	1	1	1	0.82	0.8	1	0.8	1	1	1	0.8	1	1	1	1	0.99	0.92	1	1	1
S_IND	0.89	0.8	0.6	0.76	1	1	0.6	1	0.6	1	0.8	0.8	0.68	0.6	0.8	0.6	0.8	0.6	1	1	0.8	0.8	0.8	0.6	0.8	0.6	0.66	1	0.8
SUMOL	0.8	0.6	0.6	0.7	1	1	0.6	1	0.6	1	0.69	0.6	0.56	0.6	0.6	0.6	0.8	0.6	0.8	1	0.6	0.8	0.8	0.6	0.6	0.6	0.6	1	0.65
TOYOT	1	0.8	0.6	0.72	1	1	0.6	1	0.8	1	0.8	0.8	1	0.8	0.8	0.8	0.8	0.8	1	1	1	0.8	0.8	0.8	1	0.6	0.8	1	0.8
BCP	1	0.67	0.4	0.8	1	1	0.4	1	0.8	1	1	0.92	0.6	0.6	0.67	0.8	1	0.6	1	0.8	0.8	1	1	0.68	0.8	0.4	0.6	1	1
BES	1	0.6	0.4	0.8	1	1	0.4	1	0.6	1	0.8	0.65	0.6	0.6	0.6	0.8	1	0.6	1	0.8	0.76	0.81	1	0.49	0.76	0.4	0.6	1	1
BRISA	1	0.8	0.6	0.56	1	1	0.73	1	1	1	1	1	0.85	0.8	0.8	0.78	1	0.8	1	1	1	1	1	1	0.92	0.71	0.85	1	1
C_AMO	1	0.8	0.6	0.72	1	1	0.6	1	0.8	1	0.8	0.8	1	0.8	0.8	0.8	0.8	0.8	1	1	1	0.8	0.8	0.8	1	0.6	0.8	1	0.8
CIMPO	1	1	1	1	1	1	1	1	1	1	1	1	1	0.99	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
COFIN	1	0.96	0.8	1	1	1	0.65	1	1	1	1	1	1	0.8	1	0.8	1	0.92	1	0.99	1	1	1	1	1	0.6	1	1	1
COMPT	0.4	0.4	0.4	0.04	0.8	0.6	0.4	0.6	0.4	0.6	0.4	0.4	0.18	0.4	0.4	0.32	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.4	0.26	0.27	0.4	1	0.4
EDP	1	0.8	0.6	0.46	1	1	0.6	1	0.8	1	0.8	0.8	0.8	0.8	0.8	0.74	0.81	0.8	1	1	1	0.8	1	0.79	0.88	0.49	0.8	1	1

Figure 8 – Concordance Matrix (Portfolio 4)

Source: ELECTRE III software output estimation, May 2012

ANNEX H:

ELECTRE III / IV - [j:\finbolsa\trabem~1\simula~1\v2_jul~1\p4.elp] - [Ranking Matrix]

	ESTOR	T.DUA	ZON	BPI	FISIP	GRA P	IBERS	INAPA	J_MAR	LISGR	M_ENG	OREY	PORTU	PT	REDIT	SEMAP	S_COS	SONAE	S_IND	SUMOL	TOYOT	BCP	BES	BRISA	C_AMO	CIMPO	COFIN	COMPT	EDP
ESTOR	I	P-	P-	P-	P	P	P-	P	P-	P	P-	P-	P-	P-	P-	P-	P-	P-	R	P-	P-	P-	P-	P-	P-	P-	P-	P	P-
T.DUA	P	I	P-	R	P	P	P-	P	P	P	P	P	P	P-	P	R	P	R	P	P	P	P	P	P	P	P-	R	P	P
ZON	P	P	I	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P-	P	P	P
BPI	P	R	P-	I	P	P	P-	P	R	P	P	R	P	P-	R	P-	P	P-	P	P	P	P	P	R	P	P-	P-	P	P
FISIP	P-	P-	P-	P-	I	P	P-	P	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P	P-
GRA P	P-	P-	P-	P-	P-	I	P-	I	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P	P-
IBERS	P	P	P-	P	P	P	I	P	P	P	P	P	P	I	P	P	P	P	P	P	P	P	P	P	P	P-	P	P	P
INAPA	P-	P-	P-	P-	P-	I	P-	I	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P	P-
J_MAR	P	P-	P-	R	P	P	P-	P	I	P	P	P-	R	P-	P-	R	P	P-	P	P	P	P	P	P	P	P-	P-	P	P
LISGR	P-	P-	P-	P-	P	P	P-	P	P-	I	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P	P-
M_ENG	P	P-	P-	P-	P	P	P-	P	P-	P	I	P-	R	P-	P-	P-	P	P-	P	P	P	P	P	P-	P	P-	P-	P	P
OREY	P	P-	P-	R	P	P	P-	P	P	P	P	I	P	P-	P-	R	P	P-	P	P	P	P	P	P	P	P-	P-	P	P
PORTU	P	P-	P-	P-	P	P	P-	P	R	P	R	P-	I	P-	P-	P-	P	P-	P	P	P	P	P	R	P	P-	P-	P	P
PT	P	P	P-	P	P	P	I	P	P	P	P	P	P	I	P	P	P	P	P	P	P	P	P	P	P	P-	P	P	P
REDIT	P	P-	P-	R	P	P	P-	P	P	P	P	P	P	P-	I	R	P	R	P	P	P	P	P	P	P	P-	R	P	P
SEMAP	P	R	P-	P	P	P	P-	P	R	P	P	R	P	P-	R	I	P	P-	P	P	P	P	P	R	P	P-	R	P	P
S_COS	P	P-	P-	P-	P	P	P-	P	P-	P	P-	P-	P-	P-	P-	P-	I	P-	P	P	P	P-	P	P-	R	P-	P-	P	R
SONAE	P	R	P-	P	P	P	P-	P	P	P	P	P	P	P-	R	P	P	I	P	P	P	P	P	P	P	P-	P	P	P
S_IND	R	P-	P-	P-	P	P	P-	P	P-	P	P-	P-	P-	P-	P-	P-	P-	P-	I	R	P-	P-	P-	P-	P-	P-	P-	P	P-
SUMOL	P	P-	P-	P-	P	P	P-	P	P-	P	P-	P-	P-	P-	P-	P-	P-	P-	R	I	P-	P-	P-	P-	P-	P-	P-	P	P-
TOYOT	P	P-	P-	P-	P	P	P-	P	P-	P	P-	P-	P-	P-	P-	P-	P-	P-	P	P	I	P-	P	P-	P-	P-	P-	P	R
BCP	P	P-	P-	P-	P	P	P-	P	P-	P	P-	P-	P-	P-	P-	P-	P	P-	P	P	P	I	P	P-	R	P-	P-	P	P
BES	P	P-	P-	P-	P	P	P-	P	P-	P	P-	P-	P-	P-	P-	P-	P-	P-	P	P	P-	P-	I	P-	P-	P-	P-	P	R
BRISA	P	P-	P-	R	P	P	P-	P	P-	P	P	P-	R	P-	P-	R	P	P-	P	P	P	P	P	I	P	P-	P-	P	P
C_AMO	P	P-	P-	P-	P	P	P-	P	P-	P	P-	P-	P-	P-	P-	P-	R	P-	P	P	P	R	P	P-	I	P-	P-	P	R
CIMPO	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	I	P	P	P
COFIN	P	R	P-	P	P	P	P-	P	P	P	P	P	P	P-	R	R	P	P-	P	P	P	P	P	P	P	P-	I	P	P
COMPT	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	I	P-
EDP	P	P-	P-	P-	P	P	P-	P	P-	P	P-	P-	P-	P-	P-	P-	R	P-	P	P	R	P-	R	P-	R	P-	P-	P	I

Legend (symbol at the intersection of the row for *a* and the column for *b*):

The couple (*a*, *b*) verifies one of the 4 following relations :

- if *a* is better than *b*, the symbol is **P**;

- if *a* is equivalent to *b*, the symbol is **I**;

- if *a* is as good as *b*, the symbol is **P-**;

- if *a* is incomparable to *b*, the symbol is **R**.

Figure 9 – Ranking Matrix (Portfolio 4)

Source: ELECTRE III software output estimation, May 2012

ANNEX I:

ELECTRE III / IV - [j:\finbolsa\trabem~1\simula~1\v2_jul~1\p5.elp] - [Concordance Matrix]

	ESTOR	T.DUA	ZON	BPI	FISIP	GRA P	IBERS	INAPA	J_MAR	LISGR	M_ENG	OREY	PORTU	PT	REDIT	SEMAP	S_COS	SONAE	S_IND	SUMOL	TOYOT	BCP	BES	BRISA	C_AMO	CIMPO	COFIN	COMPT	EDP
ESTOR	1	1	0.6	0.4	1	1	0.6	1	0.6	1	0.8	0.8	0.42	0.6	0.6	0.68	0.8	0.61	1	1	1	0.8	1	0.6	0.85	0.37	0.8	1	0.8
T.DUA	1	1	0.6	0.59	1	1	0.6	1	0.6	1	0.8	0.8	0.65	0.6	0.6	0.8	0.8	0.6	1	1	1	0.96	1	0.6	1	0.6	0.8	1	1
ZON	1	1	1	0.69	1	1	1	1	1	1	1	1	0.85	0.8	0.8	0.98	1	1	1	1	1	1	1	1	0.95	0.93	1	1	1
BPI	1	1	0.6	1	1	0.98	0.4	1	0.8	1	1	1	0.6	0.6	0.6	0.8	1	0.8	1	0.8	0.8	1	1	0.66	0.8	0.6	1	1	1
FISIP	0.6	0.6	0.6	0.43	1	1	0.6	0.8	0.6	1	0.6	0.6	0.57	0.6	0.6	0.6	0.6	0.6	0.6	0.8	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.97	0.6
GRA P	0.6	0.6	0.6	0.2	0.8	1	0.6	0.6	0.6	0.8	0.6	0.6	0.23	0.6	0.6	0.4	0.6	0.6	0.6	0.61	0.6	0.56	0.6	0.6	0.41	0.28	0.6	0.8	0.6
IBERS	1	1	1	0.6	1	1	1	1	1	1	1	1	0.68	0.8	0.8	0.83	1	1	1	1	1	1	1	1	0.8	0.73	1	1	1
INAPA	0.65	0.72	0.6	0.59	0.8	1	0.6	1	0.6	0.8	0.6	0.6	0.34	0.6	0.6	0.6	0.8	0.6	0.75	0.63	0.58	0.8	0.8	0.6	0.53	0.6	0.8	0.8	0.6
J_MAR	1	1	0.95	0.61	1	1	1	1	1	1	1	1	0.72	0.8	0.8	0.88	1	1	1	1	1	1	1	1	0.9	0.85	1	1	1
LISGR	0.6	0.6	0.4	0.2	0.8	0.75	0.4	0.8	0.6	1	0.6	0.6	0.14	0.6	0.6	0.48	0.6	0.6	0.6	0.6	0.4	0.6	0.6	0.44	0.26	0.24	0.6	1	0.6
M_ENG	1	1	0.6	0.86	1	1	0.6	1	0.67	1	1	1	0.76	0.73	0.8	0.8	1	1	1	0.86	1	1	1	1	0.96	0.6	1	1	1
OREY	1	1	0.6	0.91	1	1	0.6	1	0.8	1	1	1	0.98	0.8	0.8	0.8	1	1	1	1	1	1	1	1	1	0.6	1	1	1
PORTU	1	1	0.6	0.8	1	1	0.65	1	0.8	1	0.8	0.8	1	0.8	0.8	0.8	0.8	0.8	1	1	1	0.8	1	0.8	1	0.8	0.8	1	0.85
PT	1	1	0.89	0.62	1	1	1	1	1	1	1	1	0.6	1	1	0.83	1	1	1	0.94	1	1	1	1	0.91	0.89	1	1	1
REDIT	1	1	0.8	0.71	1	1	0.92	1	1	1	1	1	0.67	0.97	1	0.79	1	1	1	1	1	1	1	1	0.96	0.94	1	1	1
SEMAP	1	1	0.8	0.8	1	1	0.8	1	0.8	1	0.8	0.8	1	0.8	0.8	1	0.8	0.8	1	1	1	0.8	0.97	0.8	1	0.8	0.8	1	0.8
S_COS	1	1	0.65	0.82	1	1	0.6	1	0.6	1	1	0.91	0.55	0.6	0.6	0.8	1	0.8	1	0.91	1	1	1	0.8	1	0.6	1	1	0.96
SONAE	1	1	0.62	0.67	1	1	0.66	1	0.8	1	1	1	0.74	0.8	0.8	0.79	1	1	1	1	1	1	1	1	0.96	0.74	1	1	1
S_IND	1	0.99	0.6	0.7	1	1	0.6	1	0.6	1	0.8	0.8	0.8	0.6	0.6	0.8	0.8	0.6	1	1	1	0.8	0.94	0.6	1	0.6	0.8	1	0.8
SUMOL	0.6	0.67	0.6	0.49	1	1	0.6	1	0.6	1	0.6	0.6	0.49	0.6	0.6	0.6	0.8	0.6	0.69	1	0.6	0.77	0.8	0.6	0.59	0.57	0.8	1	0.6
TOYOT	1	0.8	0.6	0.63	1	1	0.6	1	0.71	1	0.8	0.8	0.9	0.77	0.8	0.8	0.8	0.8	1	1	1	0.8	0.8	0.8	1	0.6	0.8	1	0.8
BCP	1	1	0.4	0.8	1	1	0.4	1	0.6	1	1	0.99	0.6	0.6	0.6	0.8	0.81	0.64	1	0.8	0.8	1	1	0.72	0.8	0.4	1	1	1
BES	1	1	0.41	0.8	1	1	0.4	1	0.6	1	0.8	0.75	0.6	0.6	0.6	0.8	0.8	0.6	1	0.8	0.8	1	1	0.54	0.8	0.4	0.8	1	1
BRISA	1	1	0.6	0.44	1	1	0.74	1	0.8	1	1	1	0.81	0.8	0.8	0.74	1	1	1	1	1	1	1	1	0.91	0.69	1	1	1
C_AMO	1	0.91	0.6	0.66	1	1	0.6	1	0.76	1	0.8	0.8	1	0.8	0.8	0.8	0.8	0.8	1	1	1	0.8	0.85	0.8	1	0.6	0.8	1	0.8
CIMPO	1	1	1	1	1	1	1	1	1	1	1	1	1	0.8	0.8	1	1	1	1	1	1	1	1	1	1	1	1	1	1
COFIN	1	1	0.6	0.74	1	1	0.6	1	0.6	1	0.82	0.8	0.55	0.6	0.6	0.8	1	0.8	1	1	0.8	1	1	0.8	0.8	0.6	1	1	0.8
COMPT	0.4	0.4	0.4	0	0.6	0.6	0.4	0.6	0.4	0.8	0.4	0.4	0.11	0.4	0.4	0.25	0.4	0.4	0.4	0.6	0.4	0.4	0.4	0.4	0.22	0.16	0.4	1	0.4
EDP	1	1	0.6	0.4	1	1	0.6	1	0.8	1	1	1	0.75	0.8	0.8	0.7	0.8	0.8	1	1	1	1	1	1	0.86	0.44	1	1	1

Figure 10 – Concordance Matrix (Portfolio 5)

Source: ELECTRE III software output estimation, May 2012

ANNEX J:

ELECTRE III / IV - [j:\finbolsa\trabem~1\simula~1\v2_jul~1\p5.elp] - [Ranking Matrix]

	ESTOR	T.DUA	ZON	BPI	FISIP	GRA P	IBERS	INAPA	J_MAR	LISGR	M_ENG	OREY	PORTU	PT	REDIT	SEMAP	S_COS	SONAE	S_IND	SUMOL	TOYOT	BCP	BES	BRISA	C_AMO	CIMPO	COFIN	COMPT	EDP
ESTOR	I	P^-	P^-	P^-	P	P	P^-	P	P^-	P	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	I	P	P^-	P^-	R	P^-	P^-	P^-	P^-	P	P^-
T.DUA	P	I	P^-	P^-	P	P	P^-	P	P^-	P	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P	P	P	P^-	P	P^-	R	P^-	P^-	P	P^-
ZON	P	P	I	P	P	P	P	P	P	P	P	P	P	I	I	P	P	P	P	P	P	P	P	P	P	P^-	P	P	P
BPI	P	P	P^-	I	P	P	R	P	R	P	P	R	P	P^-	P^-	P	P	R	P	P	P	P	P	R	P	P^-	P	P	P
FISIP	P^-	P^-	P^-	P^-	I	P	P^-	P	P^-	P	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P	P^-
GRA P	P^-	P^-	P^-	P^-	P^-	I	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P	P^-
IBERS	P	P	P^-	R	P	P	I	P	I	P	P	P	P	P^-	P^-	R	P	P	P	P	P	P	P	P	P	P^-	P	P	P
INAPA	P^-	P^-	P^-	P^-	P^-	P	P^-	I	P^-	P	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P	P^-
J_MAR	P	P	P^-	R	P	P	I	P	I	P	P	P	P	P^-	P^-	R	P	P	P	P	P	P	P	P	P	P^-	P	P	P
LISGR	P^-	P^-	P^-	P^-	P^-	P	P^-	P^-	P^-	I	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P	P^-
M_ENG	P	P	P^-	P^-	P	P	P^-	P	P^-	P	I	P^-	R	P^-	P^-	P^-	P	P^-	P	P	P	P	P	P^-	P	P^-	P	P	P
OREY	P	P	P^-	R	P	P	P^-	P	P^-	P	P	I	R	P^-	P^-	R	P	R	P	P	P	P	P	P	P	P^-	P	P	P
PORTU	P	P	P^-	P^-	P	P	P^-	P	P^-	P	R	R	I	P^-	P^-	P^-	R	R	P	P	P	P	P	R	P	P^-	P	P	P
PT	P	P	I	P	P	P	P	P	P	P	P	P	P	I	I	P	P	P	P	P	P	P	P	P	P	P^-	P	P	P
REDIT	P	P	I	P	P	P	P	P	P	P	P	P	P	I	I	P	P	P	P	P	P	P	P	P	P	P^-	P	P	P
SEMAP	P	P	P^-	P^-	P	P	R	P	R	P	P	R	P	P^-	P^-	I	P	R	P	P	P	P	P	R	P	P^-	P	P	P
S_COS	P	P	P^-	P^-	P	P	P^-	P	P^-	P	P^-	P^-	R	P^-	P^-	P^-	I	P^-	P	P	P	P	P	P^-	P	P^-	P	P	P
SONAE	P	P	P^-	R	P	P	P^-	P	P^-	P	P	R	R	P^-	P^-	R	P	I	P	P	P	P	P	P	P	P^-	P	P	P
S_IND	I	P^-	P^-	P^-	P	P	P^-	P	P^-	P	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	I	P	P^-	P^-	R	P^-	P^-	P^-	P^-	P	P^-
SUMOL	P^-	P^-	P^-	P^-	P	P	P^-	P	P^-	P	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	I	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P	P^-
TOYOT	P	P^-	P^-	P^-	P	P	P^-	P	P^-	P	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P	P	I	P^-	P	P^-	P^-	P^-	P^-	P	P^-
BCP	P	P	P^-	P^-	P	P	P^-	P	P^-	P	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P	P	P	I	P	P^-	R	P^-	R	P	P^-
BES	R	P^-	P^-	P^-	P	P	P^-	P	P^-	P	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	R	P	P^-	P^-	I	P^-	P^-	P^-	P^-	P	P^-
BRISA	P	P	P^-	R	P	P	P^-	P	P^-	P	P	P^-	R	P^-	P^-	R	P	P^-	P	P	P	P	P	I	P	P^-	P	P	P
C_AMO	P	R	P^-	P^-	P	P	P^-	P	P^-	P	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P	P	P	R	P	P^-	I	P^-	R	P	R
CIMPO	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	I	P	P	P
COFIN	P	P	P^-	P^-	P	P	P^-	P	P^-	P	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P	P	P	R	P	P^-	R	P^-	I	P	P^-
COMPT	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	I	P^-
EDP	P	P	P^-	P^-	P	P	P^-	P	P^-	P	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P^-	P	P	P	P	P	P^-	R	P^-	P	P	I

Legend (symbol at the intersection of the row for *a* and the column for *b*):

The couple (*a*, *b*) verifies one of the 4 following relations :

- if *a* is better than *b*, the symbol is **P**;

- if *a* is equivalent to *b*, the symbol is **I**;

- if *a* is as good as *b*, the symbol is **P^-**;

- if *a* is incomparable to *b*, the symbol is **R**.

Figure 11 – Ranking Matrix (Portfolio 5)

Source: ELECTRE III software output estimation, May 2012

ANNEX K:

ELECTRE III / IV - [j:\finbolsa\trabem~1\simula~1\v2_jul~1\p6.elp] - [Concordance Matrix]

	ESTOR	T.DUA	ZON	BPI	FISIP	GRA P	IBERS	INAPA	J_MAR	LISGR	M_ENG	OREY	PORTU	PT	REDIT	SEMAP	S_COS	SONAE	S_IND	SUMOL	TOYOT	BCP	BES	BRISA	C_AMO	CIMPO	COFIN	COMPT	EDP
ESTOR	1	0.8	0.6	0.4	1	1	0.6	1	0.6	1	0.8	1	0.55	0.6	0.8	0.7	0.84	1	1	1	1	1	1	0.99	0.8	0.33	1	1	0.8
T.DUA	1	1	0.6	0.76	1	1	0.6	1	0.6	1	0.97	1	0.66	0.6	0.8	0.8	1	0.96	1	1	1	1	1	0.88	1	0.6	1	1	1
ZON	1	1	1	0.7	1	1	1	1	0.8	1	1	1	0.91	0.8	1	1	1	1	1	1	1	1	1	1	0.93	0.94	1	1	1
BPI	1	1	0.76	1	1	1	0.4	1	0.6	1	1	1	0.6	0.6	0.8	0.62	1	0.8	1	0.8	0.8	1	1	0.74	0.8	0.6	1	1	0.92
FISIP	0.6	0.6	0.6	0.61	1	1	0.6	1	0.6	1	0.6	0.6	0.6	0.6	0.6	0.6	0.69	0.6	1	1	0.6	0.74	0.73	0.6	0.6	0.6	0.8	1	0.6
GRA P	0.6	0.6	0.6	0.2	0.61	1	0.6	0.8	0.6	0.8	0.6	0.6	0.28	0.6	0.6	0.49	0.6	0.6	0.6	0.6	0.6	0.56	0.58	0.6	0.4	0.3	0.6	0.8	0.6
IBERS	1	1	1	0.6	1	1	1	1	0.8	1	1	1	0.73	0.8	1	0.89	1	1	1	1	1	1	1	1	0.8	0.74	1	1	1
INAPA	0.6	0.62	0.6	0.61	0.8	1	0.6	1	0.6	0.8	0.6	0.6	0.4	0.6	0.6	0.6	0.8	0.6	0.8	0.8	0.6	0.8	0.8	0.6	0.52	0.6	0.8	0.8	0.6
J_MAR	1	1	1	0.62	1	1	1	1	1	1	1	1	0.83	0.8	1	0.99	1	1	1	1	1	1	1	1	0.88	0.86	1	1	1
LISGR	0.6	0.6	0.4	0.2	0.4	0.6	0.4	0.8	0.45	1	0.6	0.41	0.17	0.6	0.6	0.33	0.6	0.6	0.54	0.41	0.4	0.6	0.6	0.4	0.22	0.21	0.6	0.6	0.57
M_ENG	1	1	0.8	0.82	1	1	0.68	1	0.62	1	1	1	0.8	0.65	1	0.83	1	1	1	1	0.93	1	1	1	0.86	0.78	1	1	1
OREY	1	0.8	0.6	0.72	1	1	0.6	1	0.6	1	0.8	1	1	0.63	0.8	0.81	0.8	1	1	1	1	1	1	1	1	0.6	1	1	0.8
PORTU	1	0.8	0.8	0.8	1	1	0.79	1	0.8	1	0.8	1	1	0.8	0.8	1	0.8	1	1	1	1	1	1	0.84	1	0.8	1	1	0.8
PT	1	1	1	0.69	1	1	1	1	1	1	1	1	0.7	1	1	1	1	1	1	1	1	1	1	1	0.84	0.93	1	1	1
REDIT	1	1	0.8	0.69	1	1	0.69	1	0.6	1	1	1	0.54	0.6	1	0.8	1	1	1	0.94	0.8	1	1	0.95	0.72	0.56	1	1	1
SEMAP	1	0.8	0.8	0.8	1	1	0.8	1	0.8	1	0.8	1	1	0.8	0.8	1	0.8	1	1	1	1	1	1	0.8	1	0.8	1	1	0.8
S_COS	1	1	0.6	0.82	1	1	0.6	1	0.6	1	0.92	1	0.6	0.6	0.8	0.8	1	0.8	1	1	0.95	1	1	0.8	0.87	0.59	1	1	0.96
SONAE	1	0.8	0.8	0.46	1	1	0.6	1	0.8	1	0.8	1	0.88	0.8	0.8	1	0.8	1	1	1	1	1	1	0.8	0.92	0.73	1	1	0.8
S_IND	0.8	0.8	0.6	0.68	1	1	0.6	1	0.6	1	0.79	0.8	0.6	0.6	0.8	0.6	0.8	0.6	1	1	0.7	0.8	0.8	0.6	0.8	0.6	0.8	1	0.6
SUMOL	0.6	0.6	0.6	0.49	1	1	0.6	1	0.6	1	0.6	0.6	0.53	0.6	0.6	0.6	0.76	0.6	0.8	1	0.6	0.8	0.8	0.6	0.54	0.55	0.8	0.97	0.6
TOYOT	0.8	0.8	0.6	0.63	1	1	0.6	1	0.71	1	0.8	0.8	0.89	0.74	0.8	0.92	0.8	0.97	1	1	1	0.8	0.8	0.8	1	0.6	1	1	0.8
BCP	1	0.8	0.6	0.8	1	1	0.4	1	0.6	1	0.8	1	0.6	0.6	0.8	0.68	0.8	0.8	1	0.8	0.8	1	1	0.8	0.8	0.4	1	1	0.71
BES	1	0.8	0.6	0.8	1	1	0.4	1	0.6	1	0.8	1	0.6	0.6	0.8	0.72	0.8	0.8	1	0.82	0.8	1	1	0.8	0.8	0.4	1	1	0.72
BRISA	1	1	0.8	0.61	1	1	0.67	1	0.8	1	0.8	1	0.86	0.8	0.8	0.98	1	1	1	1	1	1	1	1	0.87	0.68	1	1	1
C_AMO	0.8	0.8	0.6	0.67	1	1	0.6	1	0.61	1	0.8	0.8	0.8	0.64	0.8	0.62	0.8	0.8	1	1	1	0.8	0.8	0.8	1	0.6	0.87	1	0.8
CIMPO	1	1	1	1	1	1	1	1	0.8	1	1	1	1	0.8	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
COFIN	1	0.8	0.6	0.64	1	1	0.58	1	0.6	1	0.74	0.96	0.57	0.6	0.8	0.8	0.8	0.8	1	1	0.73	1	1	0.6	0.74	0.52	1	1	0.6
COMPT	0.4	0.4	0.4	0	0.6	0.8	0.4	0.6	0.4	0.98	0.4	0.4	0.16	0.4	0.4	0.31	0.4	0.4	0.6	0.6	0.4	0.4	0.4	0.4	0.2	0.18	0.4	1	0.4
EDP	1	1	0.75	0.6	1	1	0.6	1	0.8	1	1	1	0.84	0.8	0.8	0.96	1	1	1	1	1	1	1	1	0.85	0.48	1	1	1

Figure 12 – Concordance Matrix (Portfolio 6)

Source: ELECTRE III software output estimation, May 2012

ANNEX L:

ELECTRE III / IV - [j:\finbolsa\trabem~1\simula~1\v2_jul~1\p6.elp] - [Ranking Matrix]

File Edit Calculate Results Options Window Help

	ESTOR	T.DUA	ZON	BPI	FISIP	GRA P	IBERS	INAPA	J_MAR	LISGR	M_ENG	OREY	PORTU	PT	REDIT	SEMAP	S_COS	SONAE	S_IND	SUMOL	TOYOT	BCP	BES	BRISA	C_AMO	CIMPO	COFIN	COMPT	EDP
ESTOR	I	P-	P-	P-	P	P	P-	P	P-	P	P-	P-	P-	P-	P-	P-	P-	P-	P	P	R	P	P	P-	R	P-	P	P	P-
T.DUA	P	I	P-	P-	P	P	P-	P	P-	P	R	P	R	P-	P-	R	P	P	P	P	P	P	P	R	P	P-	P	P	P-
ZON	P	P	I	R	P	P	P	P	P-	P	P	P	R	P-	P	P	P	P	P	P	P	P	P	P	P	P-	P	P	P
BPI	P	P	R	I	P	P	R	P	P-	P	P	P	R	P-	R	P	P	P	P	P	P	P	P	P	P	P-	P	P	P
FISIP	P-	P-	P-	P-	I	P	P-	P	P-	P	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P	P-
GRA P	P-	P-	P-	P-	P-	I	P-	P-	P-	P	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-
IBERS	P	P	P-	R	P	P	I	P	P-	P	P	P	R	P-	P	P	P	P	P	P	P	P	P	P	P	P-	P	P	P
INAPA	P-	P-	P-	P-	P-	P	P-	I	P-	P	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P	P-
J_MAR	P	P	P	P	P	P	P	P	I	P	P	P	P	P-	P	P	P	P	P	P	P	P	P	P	P	P-	P	P	P
LISGR	P-	P-	P-	P-	P-	P-	P-	P-	P-	I	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-
M_ENG	P	R	P-	P-	P	P	P-	P	P-	P	I	P	R	P-	P-	P	P	P	P	P	P	P	P	P	P	P-	P	P	R
OREY	P	P-	P-	P-	P	P	P-	P	P-	P	P-	I	P-	P-	P-	P-	P-	P-	P	P	R	P	P	P-	R	P-	P	P	P-
PORTU	P	R	R	R	P	P	R	P	P-	P	R	P	I	P-	R	R	P	P	P	P	P	P	P	R	P	P-	P	P	R
PT	P	P	P	P	P	P	P	P	P	P	P	P	P	I	P	P	P	P	P	P	P	P	P	P	P	P-	P	P	P
REDIT	P	P	P-	R	P	P	P-	P	P-	P	P	P	R	P-	I	P	P	P	P	P	P	P	P	P	P	P-	P	P	P
SEMAP	P	R	P-	P-	P	P	P-	P	P-	P	P-	P	R	P-	P-	I	P	P	P	P	P	P	P	P-	P	P-	P	P	P-
S_COS	P	P-	P-	P-	P	P	P-	P	P-	P	P-	P	P-	P-	P-	P-	I	P	P	P	P	P	P	P-	R	P-	P	P	P-
SONAE	P	P-	P-	P-	P	P	P-	P	P-	P	P-	P	P-	P-	P-	P-	P-	I	P	P	R	P	P	P-	R	P-	P	P	P-
S_IND	P-	P-	P-	P-	P	P	P-	P	P-	P	P-	P-	P-	P-	P-	P-	P-	P-	I	R	P-	P-	P-	P-	P-	P-	P-	P	P-
SUMOL	P-	P-	P-	P-	P	P	P-	P	P-	P	P-	P-	P-	P-	P-	P-	P-	P-	R	I	P-	P-	P-	P-	P-	P-	P-	P	P-
TOYOT	R	P-	P-	P-	P	P	P-	P	P-	P	P-	R	P-	P-	P-	P-	P-	R	P	P	I	P	P	P-	R	P-	P	P	P-
BCP	P-	P-	P-	P-	P	P	P-	P	P-	P	P-	P-	P-	P-	P-	P-	P-	P-	P	P	P-	I	I	P-	P-	P-	R	P	P-
BES	P-	P-	P-	P-	P	P	P-	P	P-	P	P-	P-	P-	P-	P-	P-	P-	P-	P	P	P-	I	I	P-	P-	P-	R	P	P-
BRISA	P	R	P-	P-	P	P	P-	P	P-	P	P-	P	R	P-	P-	P	P	P	P	P	P	P	P	I	P	P-	P	P	R
C_AMO	R	P-	P-	P-	P	P	P-	P	P-	P	P-	R	P-	P-	P-	P-	R	R	P	P	R	P	P	P-	I	P-	P	P	P-
CIMPO	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P	I	P	P	P
COFIN	P-	P-	P-	P-	P	P	P-	P	P-	P	P-	P-	P-	P-	P-	P-	P-	P-	P	P	P-	R	R	P-	P-	P-	I	P	P-
COMPT	P-	P-	P-	P-	P-	P	P-	P-	P-	P	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	I	P-
EDP	P	P	P-	P-	P	P	P-	P	P-	P	R	P	R	P-	P-	P	P	P	P	P	P	P	P	R	P	P-	P	P	I

Legend (symbol at the intersection of the row for *a* and the column for *b*):

The couple (*a*, *b*) verifies one of the 4 following relations :

- if *a* is better than *b*, the symbol is **P**;

- if *a* is equivalent to *b*, the symbol is **I**;

- if *a* is as good as *b*, the symbol is **P-**;

- if *a* is incomparable to *b*, the symbol is **R**.

Figure 13 – Ranking Matrix (Portfolio 6)

Source: ELECTRE III software output estimation, May 2012

ANNEX M:

ELECTRE III / IV - [j:\finbolsa\trabem~1\simula~1\v2_jul~1\p7.elp] - [Concordance Matrix]

	ESTOR	T.DUA	ZON	BPI	FISIP	GRA P	IBERS	INAPA	J_MAR	LISGR	M_ENG	OREY	PORTU	PT	REDIT	SEMAP	S_COS	SONAE	S_IND	SUMOL	TOYOT	BCP	BES	BRISA	C_AMO	CIMPO	COFIN	COMPT	EDP
ESTOR	1	1	0.71	0.6	1	1	0.6	1	0.6	1	0.8	1	0.54	0.6	0.8	0.88	1	1	1	1	1	1	1	0.6	0.8	0.49	1	1	0.8
T.DUA	0.8	1	0.6	0.58	1	1	0.6	1	0.6	1	0.8	1	0.43	0.6	0.8	0.6	0.8	0.8	1	1	0.95	1	0.86	0.6	1	0.6	1	1	0.62
ZON	1	1	1	0.79	1	1	0.97	1	0.8	1	1	1	0.98	0.6	1	1	1	1	1	1	1	1	1	0.8	1	1	1	1	1
BPI	1	1	0.6	1	0.93	0.99	0.4	1	0.6	1	0.81	0.94	0.6	0.6	0.8	0.61	1	0.99	1	0.89	0.74	1	1	0.59	0.8	0.6	1	1	0.81
FISIP	0.6	0.8	0.6	0.7	1	1	0.6	1	0.6	1	0.6	0.6	0.6	0.6	0.79	0.6	0.76	0.6	1	1	0.6	0.8	0.8	0.6	0.6	0.6	0.8	1	0.6
GRA P	0.6	0.6	0.6	0.31	0.6	1	0.6	0.74	0.6	0.8	0.6	0.6	0.48	0.6	0.6	0.58	0.6	0.6	0.6	0.6	0.6	0.56	0.57	0.6	0.6	0.59	0.6	0.8	0.6
IBERS	1	1	1	0.6	1	1	1	1	0.8	1	1	1	0.78	0.6	1	0.92	1	1	1	1	1	1	1	1	0.83	0.92	1	1	1
INAPA	0.6	0.8	0.6	0.66	0.8	1	0.6	1	0.6	0.8	0.6	0.6	0.4	0.6	0.74	0.6	0.71	0.6	0.8	0.8	0.59	0.8	0.8	0.6	0.52	0.6	0.8	0.8	0.6
J_MAR	1	1	1	0.6	1	1	1	1	1	1	1	1	0.84	0.6	1	0.97	1	1	1	1	1	1	1	1	0.88	0.98	1	1	1
LISGR	0.4	0.4	0.4	0.2	0.6	0.6	0.4	0.72	0.4	1	0.6	0.4	0.22	0.46	0.6	0.34	0.6	0.4	0.6	0.45	0.4	0.6	0.6	0.4	0.25	0.34	0.6	0.8	0.4
M_ENG	1	1	0.98	0.8	1	1	0.65	1	0.6	1	1	1	0.6	0.6	1	1	1	1	1	1	0.83	1	1	0.6	0.8	0.74	1	1	1
OREY	1	1	0.8	0.71	1	1	0.6	1	0.6	1	0.8	1	0.73	0.6	0.8	0.85	0.87	0.93	1	1	1	1	1	0.6	1	0.6	1	1	0.8
PORTU	1	1	0.8	1	1	1	0.8	1	0.8	1	0.8	1	1	0.6	0.8	1	1	1	1	1	1	1	1	0.8	1	0.89	1	1	0.93
PT	1	1	1	0.98	1	1	1	1	1	1	1	1	0.8	1	1	1	1	1	1	1	1	1	1	1	0.98	1	1	1	1
REDIT	1	1	0.8	0.68	1	1	0.69	1	0.6	1	1	1	0.52	0.6	1	0.8	1	1	1	1	0.8	1	1	0.76	0.74	0.66	1	1	0.92
SEMAP	1	1	0.8	1	1	1	0.65	1	0.8	1	0.8	1	1	0.6	0.8	1	1	1	1	1	1	1	1	0.8	1	0.97	1	1	1
S_COS	1	1	0.6	0.82	1	1	0.55	1	0.6	1	0.8	1	0.6	0.6	0.8	0.8	1	1	1	1	0.81	1	1	0.6	0.8	0.68	1	1	0.93
SONAE	1	1	0.8	0.64	1	1	0.6	1	0.6	1	0.8	1	0.68	0.6	0.8	1	1	1	1	1	1	1	1	0.62	0.92	0.72	1	1	0.93
S_IND	0.6	0.8	0.6	0.65	0.98	1	0.6	1	0.6	1	0.6	0.6	0.57	0.6	0.8	0.6	0.78	0.6	1	0.8	0.6	0.8	0.8	0.6	0.6	0.6	0.8	1	0.6
SUMOL	0.6	0.8	0.6	0.51	1	1	0.6	1	0.6	1	0.6	0.6	0.56	0.6	0.8	0.6	0.8	0.6	1	1	0.6	0.8	0.8	0.6	0.58	0.6	0.8	1	0.6
TOYOT	0.8	1	0.8	0.63	1	1	0.6	1	0.6	1	0.8	1	0.67	0.6	0.8	0.8	0.8	0.8	1	1	1	1	0.94	0.67	1	0.67	1	1	0.8
BCP	1	1	0.6	0.8	1	1	0.4	1	0.6	1	0.8	1	0.4	0.6	0.8	0.48	0.8	0.77	1	0.96	0.71	1	1	0.6	0.8	0.4	1	1	0.6
BES	1	1	0.6	0.91	1	1	0.4	1	0.6	1	0.8	1	0.6	0.6	0.8	0.74	1	1	1	1	0.77	1	1	0.6	0.8	0.4	1	1	0.64
BRISA	1	1	1	0.95	1	1	1	1	0.95	1	1	1	1	0.6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C_AMO	0.8	1	0.8	0.68	1	1	0.6	1	0.6	1	0.8	0.8	0.6	0.6	0.8	0.8	0.8	0.8	1	1	1	0.95	0.8	0.6	1	0.6	1	1	0.8
CIMPO	1	1	1	1	1	1	0.8	1	0.8	1	1	1	1	0.6	0.88	1	1	1	1	1	1	1	1	0.8	1	1	1	1	1
COFIN	0.78	1	0.6	0.48	1	1	0.41	1	0.6	1	0.72	0.81	0.31	0.6	0.8	0.59	0.8	0.6	1	1	0.6	1	0.8	0.6	0.54	0.4	1	1	0.6
COMPT	0.4	0.4	0.4	0.17	0.4	0.8	0.4	0.6	0.4	1	0.4	0.4	0.24	0.4	0.4	0.35	0.4	0.4	0.6	0.4	0.4	0.5	0.45	0.4	0.26	0.36	0.4	1	0.4
EDP	1	1	1	0.6	1	1	0.6	1	0.72	1	1	1	0.87	0.6	0.85	0.98	1	1	1	1	1	1	1	0.8	0.89	0.99	1	1	1

Figure 14 – Concordance Matrix (Portfolio 7)

Source: ELECTRE III software output estimation, May 2012

ANNEX N:

ELECTRE III / IV - [j:\finbolsa\trabem~1\simula~1\v2_jul~1\p7.elp] - [Ranking Matrix]

File Edit Calculate Results Options Window Help

	ESTOR	T.DUA	ZON	BPI	FISIP	GRA P	IBERS	INAPA	J_MAR	LISGR	M_ENG	OREY	PORTU	PT	REDIT	SEMAP	S_COS	SONAE	S_IND	SUMOL	TOYOT	BCP	BES	BRISA	C_AMO	CIMPO	COFIN	COMPT	EDP
ESTOR	I	P	P-	P-	P	P	P-	P	P-	P	P-	P	P-	P-	P-	P-	P	P-	P	P	P	P	P	P-	R	P-	P	P	P-
T.DUA	P-	I	P-	P-	P	P	P-	P	P-	P	P-	P-	P-	P-	P-	P-	P-	P-	P	P	P-	I	P-	P-	P-	P-	P	P	P-
ZON	P	P	I	P	P	P	P-	P	P-	P	P	P	P	P-	P	P	P	P	P	P	P	P	P	P-	P	P	P	P	P
BPI	P	P	P-	I	P	P	P-	P	P-	P	P-	P	P-	P-	R	P-	P	P	P	P	P	P	P	P-	P	P-	P	P	R
FISIP	P-	P-	P-	P-	I	P	P-	P	P-	P	P-	P-	P-	P-	P-	P-	P-	P-	P	P-	P-	P-	P-	P-	P-	P-	P-	P	P-
GRA P	P-	P-	P-	P-	P-	I	P-	P-	P-	P	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-
IBERS	P	P	P	P	P	P	I	P	P-	P	P	P	P	P-	P	P	P	P	P	P	P	P	P	P-	P	P	P	P	P
INAPA	P-	P-	P-	P-	P-	P	P-	I	P-	P	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P	P-
J_MAR	P	P	P	P	P	P	P	P	I	P	P	P	P	P-	P	P	P	P	P	P	P	P	P	P-	P	P	P	P	P
LISGR	P-	P-	P-	P-	P-	P-	P-	P-	P-	I	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-
M_ENG	P	P	P-	P	P	P	P-	P	P-	P	I	P	P-	P-	P	P	P	P	P	P	P	P	P	P-	P	P-	P	P	R
OREY	P-	P	P-	P-	P	P	P-	P	P-	P	P-	I	P-	P-	P-	P-	P-	P-	P	P	P-	P	P	P-	R	P-	P	P	P-
PORTU	P	P	P-	P	P	P	P-	P	P-	P	P	P	I	P-	P	P	P	P	P	P	P	P	P	P-	P	R	P	P	P
PT	P	P	P	P	P	P	P	P	P	P	P	P	P	I	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
REDIT	P	P	P-	R	P	P	P-	P	P-	P	P-	P	P-	P-	I	P-	P	P	P	P	P	P	P	P-	P	P-	P	P	R
SEMAP	P	P	P-	P	P	P	P-	P	P-	P	P-	P	P-	P-	P	I	P	P	P	P	P	P	P	P-	P	P-	P	P	R
S_COS	P-	P	P-	P-	P	P	P-	P	P-	P	P-	P	P-	P-	P-	P-	I	P-	P	P	P	P	P	P-	R	P-	P	P	P-
SONAE	P	P	P-	P-	P	P	P-	P	P-	P	P-	P	P-	P-	P-	P-	P	I	P	P	P	P	P	P-	R	P-	P	P	P-
S_IND	P-	P-	P-	P-	P-	P	P-	P	P-	P	P-	P-	P-	P-	P-	P-	P-	P-	I	P-	P-	P-	P-	P-	P-	P-	P-	P	P-
SUMOL	P-	P-	P-	P-	P	P	P-	P	P-	P	P-	P-	P-	P-	P-	P-	P-	P-	P	I	P-	P-	P-	P-	P-	P-	P-	P	P-
TOYOT	P-	P	P-	P-	P	P	P-	P	P-	P	P-	P	P-	P-	P-	P-	P-	P-	P	P	I	P	P	P-	R	P-	P	P	P-
BCP	P-	I	P-	P-	P	P	P-	P	P-	P	P-	P-	P-	P-	P-	P-	P-	P-	P	P	P-	I	P-	P-	P-	P-	P	P	P-
BES	P-	P	P-	P-	P	P	P-	P	P-	P	P-	P-	P-	P-	P-	P-	P-	P-	P	P	P-	P	I	P-	P-	P-	P	P	P-
BRISA	P	P	P	P	P	P	P	P	P	P	P	P	P	P-	P	P	P	P	P	P	P	P	P	I	P	P	P	P	P
C_AMO	R	P	P-	P-	P	P	P-	P	P-	P	P-	R	P-	P-	P-	P-	R	R	P	P	R	P	P	P-	I	P-	P	P	P-
CIMPO	P	P	P-	P	P	P	P-	P	P-	P	P	P	R	P-	P	P	P	P	P	P	P	P	P	P-	P	I	P	P	P
COFIN	P-	P-	P-	P-	P	P	P-	P	P-	P	P-	P-	P-	P-	P-	P-	P-	P-	P	P	P-	P-	P-	P-	P-	P-	I	P	P-
COMPT	P-	P-	P-	P-	P-	P	P-	P-	P-	P	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	I	P-
EDP	P	P	P-	R	P	P	P-	P	P-	P	R	P	P-	P-	R	R	P	P	P	P	P	P	P	P-	P	P-	P	P	I

Legend (symbol at the intersection of the row for *a* and the column for *b*):

The couple (*a*, *b*) verifies one of the 4 following relations :

- if *a* is better than *b*, the symbol is **P**;

- if *a* is equivalent to *b*, the symbol is **I**;

- if *a* is as good as *b*, the symbol is **P-**;

- if *a* is incomparable to *b*, the symbol is **R**.

Figure 15 – Ranking Matrix (Portfolio 7)

Source: ELECTRE III software output estimation, May 2012

ANNEX O:

ELECTRE III / IV - [j:\finbolsa\trabem~1\simula~1\v2_jul~1\p8.elp] - [Concordance Matrix]

	ESTOR	T.DUA	ZON	BPI	FISIP	GRA P	IBERS	INAPA	J_MAR	LISGR	M_ENG	OREY	PORTU	PT	REDIT	SEMAP	S_COS	SONAE	S_IND	SUMOL	TOYOT	BCP	BES	BRISA	C_AMO	CIMPO	COFIN	COMPT	EDP
ESTOR	1	1	0.6	0.51	1	1	0.6	1	0.6	1	0.8	0.64	0.45	0.6	0.81	0.5	0.8	0.8	1	1	1	0.98	0.78	0.6	0.58	0.56	0.8	1	0.6
T.DUA	0.79	1	0.6	0.52	1	1	0.57	1	0.6	0.8	0.6	0.6	0.4	0.6	0.8	0.6	0.65	0.6	1	1	0.46	0.8	0.78	0.6	0.46	0.6	0.8	1	0.6
ZON	1	1	1	0.78	1	1	1	1	0.6	1	1	1	0.67	0.6	1	1	1	1	1	1	0.95	1	1	1	0.95	1	1	1	1
BPI	1	1	0.6	1	0.92	1	0.4	1	0.6	1	0.8	0.53	0.4	0.6	1	0.4	0.8	0.8	1	1	0.8	1	1	0.6	0.7	0.4	0.8	1	0.6
FISIP	0.8	1	0.6	0.68	1	1	0.6	1	0.6	0.8	0.6	0.6	0.6	0.6	0.8	0.6	0.72	0.6	1	0.86	0.6	0.8	0.8	0.6	0.6	0.6	0.8	1	0.6
GRA P	0.6	0.6	0.6	0.4	0.6	1	0.6	0.6	0.6	0.8	0.6	0.6	0.59	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.6	0.56	0.57	0.6	0.6	0.6	0.6	0.6	0.6
IBERS	1	1	1	0.6	1	1	1	1	0.8	1	1	1	0.95	0.6	1	1	1	1	1	1	1	1	1	1	0.89	1	1	1	1
INAPA	0.8	1	0.6	0.58	1	1	0.6	1	0.6	0.8	0.74	0.6	0.48	0.6	0.8	0.6	0.8	0.69	1	0.8	0.8	0.8	0.8	0.6	0.6	0.6	0.8	1	0.6
J_MAR	1	1	1	0.6	1	1	1	1	1	1	1	1	0.96	0.76	1	1	1	1	1	1	1	1	1	1	0.9	1	1	1	1
LISGR	0.4	0.6	0.4	0	0.6	0.6	0.4	0.6	0.4	1	0.4	0.4	0.32	0.4	0.4	0.37	0.4	0.4	0.6	0.6	0.4	0.48	0.4	0.4	0.26	0.4	0.4	0.8	0.4
M_ENG	1	1	1	0.72	1	1	0.6	1	0.6	1	1	1	0.6	0.6	1	1	1	1	1	1	0.8	1	1	0.82	0.8	0.72	1	1	1
OREY	1	1	0.8	0.78	1	1	0.69	1	0.6	1	0.8	1	0.8	0.6	1	1	1	1	1	1	1	1	1	0.8	1	1	0.8	1	0.8
PORTU	1	1	0.8	1	1	1	0.8	1	0.8	1	0.87	1	1	0.6	1	1	1	1	1	1	1	1	1	0.8	1	1	0.8	1	1
PT	1	1	1	0.99	1	1	1	1	1	1	1	1	0.8	1	1	1	1	1	1	1	0.89	1	1	1	0.89	1	1	1	1
REDIT	1	1	0.6	0.64	1	1	0.47	1	0.6	1	0.63	0.6	0.4	0.6	1	0.6	0.8	0.6	1	1	0.69	1	1	0.6	0.57	0.6	0.8	1	0.6
SEMAP	1	1	0.8	0.99	1	1	0.8	1	0.75	1	0.8	1	1	0.6	1	1	1	1	1	1	1	1	1	0.8	1	1	0.8	1	1
S_COS	1	1	0.65	0.76	1	1	0.4	1	0.6	1	0.8	0.98	0.4	0.6	1	0.67	1	1	1	1	0.8	1	1	0.6	0.8	0.45	0.8	1	0.71
SONAE	1	1	0.8	0.6	1	1	0.6	1	0.6	1	0.8	1	0.54	0.6	1	0.8	1	1	1	1	1	1	1	0.67	0.94	0.6	0.8	1	0.8
S_IND	0.6	0.8	0.6	0.36	0.8	1	0.6	0.8	0.6	0.8	0.6	0.6	0.47	0.6	0.8	0.6	0.6	0.6	1	0.8	0.6	0.67	0.6	0.6	0.6	0.6	0.8	0.8	0.6
SUMOL	0.8	1	0.6	0.45	1	1	0.6	1	0.6	1	0.6	0.6	0.6	0.6	0.8	0.6	0.8	0.6	1	1	0.6	0.8	0.8	0.6	0.59	0.6	0.8	1	0.6
TOYOT	1	1	0.8	0.82	1	1	0.6	1	0.6	1	0.8	0.8	0.6	0.6	1	0.74	0.8	0.8	1	1	1	1	0.91	0.6	1	0.6	0.8	1	0.8
BCP	1	1	0.6	0.8	1	1	0.4	1	0.6	1	0.8	0.6	0.4	0.6	0.8	0.47	0.8	0.8	1	1	0.78	1	0.8	0.6	0.46	0.4	0.8	1	0.6
BES	1	1	0.6	1	1	1	0.4	1	0.6	1	0.8	0.66	0.4	0.6	1	0.53	0.88	1	1	1	0.8	1	1	0.6	0.77	0.4	0.8	1	0.6
BRISA	1	1	1	1	1	1	1	1	0.8	1	1	1	1	0.6	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
C_AMO	1	1	0.8	0.84	1	1	0.6	1	0.6	1	0.8	0.8	0.6	0.6	1	0.8	0.8	0.8	1	1	1	1	1	0.8	1	0.77	0.8	1	0.8
CIMPO	1	1	0.8	0.94	1	1	0.87	1	0.8	1	0.94	1	1	0.6	1	1	1	1	1	1	1	1	1	0.8	1	1	0.87	1	1
COFIN	1	1	0.8	0.6	0.95	1	0.6	1	0.6	1	0.81	0.77	0.56	0.6	1	0.61	1	0.8	1	1	0.67	1	1	0.8	0.5	0.6	1	1	0.8
COMPT	0.6	0.71	0.42	0.4	0.72	0.8	0.4	0.78	0.4	0.8	0.6	0.4	0.38	0.48	0.8	0.4	0.8	0.4	0.8	0.6	0.4	0.8	0.8	0.4	0.32	0.4	0.8	1	0.4
EDP	1	1	0.94	0.6	1	1	0.96	1	0.6	1	1	1	0.79	0.6	1	1	1	1	1	1	1	1	1	0.8	0.93	1	1	1	1

Figure 16 – Concordance Matrix (Portfolio 8)

Source: ELECTRE III software output estimation, May 2012

ELECTRE III / IV - [j:\finbolsa\trabem~1\simula~1\v2_jul~1\p8.elp] - [Ranking Matrix]

File Edit Calculate Results Options Window Help

	ESTOR	T.DUA	ZON	BPI	FISIP	GRA P	IBERS	INAPA	J_MAR	LISGR	M_ENG	OREY	PORTU	PT	REDIT	SEMAP	S_COS	SONAE	S_IND	SUMOL	TOYOT	BCP	BES	BRISA	C_AMO	CIMPO	COFIN	COMPT	EDP
ESTOR	I	P	P-	P-	P	P	P-	P	P-	P	P-	P-	P-	P-	P-	P-	P-	P-	P	P	P-	P	P-	P-	P-	P-	P-	P	P-
T.DUA	P-	I	P-	P-	P-	P	P-	P	P-	P	P-	P-	P-	P-	P-	P-	P-	P-	P	P-	P-	P-	P-	P-	P-	P-	P-	P	P-
ZON	P	P	I	P	P	P	P-	P	P-	P	P	P	P-	P-	P	P	P	P	P	P	P	P	P	P-	P	P	P	P	P
BPI	P	P	P-	I	P	P	P-	P	P-	P	P-	R	P-	P-	P	P-	P	R	P	P	P	P	P	P-	R	P-	R	P	P-
FISIP	P-	P	P-	P-	I	P	P-	P	P-	P	P-	P-	P-	P-	P-	P-	P-	P-	P	R	P-	P-	P-	P-	P-	P-	P-	P	P-
GRA P	P-	P-	P-	P-	P-	I	P-	P-	P-	R	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-
IBERS	P	P	P	P	P	P	I	P	P-	P	P	P	I	P-	P	P	P	P	P	P	P	P	P	P-	P	P	P	P	P
INAPA	P-	P-	P-	P-	P-	P	P-	I	P-	P	P-	P-	P-	P-	P-	P-	P-	P-	P	P-	P-	P-	P-	P-	P-	P-	P-	P	P-
J_MAR	P	P	P	P	P	P	P	P	I	P	P	P	P	P-	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
LISGR	P-	P-	P-	P-	P-	R	P-	P-	P-	I	P-	P-	P-	P-	P-	P-	P-	P-	R	P-	P-	P-	P-	P-	P-	P-	P-	R	P-
M_ENG	P	P	P-	P	P	P	P-	P	P-	P	I	P	P-	P-	P	P	P	P	P	P	P	P	P	P-	P	P-	P	P	P-
OREY	P	P	P-	R	P	P	P-	P	P-	P	P-	I	P-	P-	P	P-	P	P	P	P	P	P	P	P-	P	P-	P	P	P-
PORTU	P	P	P	P	P	P	I	P	P-	P	P	P	I	P-	P	P	P	P	P	P	P	P	P	P-	P	P	P	P	P
PT	P	P	P	P	P	P	P	P	P	P	P	P	P	I	P	P	P	P	P	P	P	P	P	P	P	P	P	P	P
REDIT	P	P	P-	P-	P	P	P-	P	P-	P	P-	P-	P-	P-	I	P-	P-	P-	P	P	P-	P	P-	P-	P-	P-	P-	P	P-
SEMAP	P	P	P-	P	P	P	P-	P	P-	P	P-	P	P-	P-	P	I	P	P	P	P	P	P	P	P-	P	P-	P	P	P-
S_COS	P	P	P-	P-	P	P	P-	P	P-	P	P-	P-	P-	P-	P	P-	I	P-	P	P	R	P	P	P-	P-	P-	P-	P	P-
SONAE	P	P	P-	R	P	P	P-	P	P-	P	P-	P-	P-	P-	P	P-	P	I	P	P	P	P	P	P-	P-	P-	P-	P	P-
S_IND	P-	P-	P-	P-	P-	P	P-	P-	P-	R	P-	P-	P-	P-	P-	P-	P-	P-	I	P-	P-	P-	P-	P-	P-	P-	P-	P	P-
SUMOL	P-	P	P-	P-	R	P	P-	P	P-	P	P-	P-	P-	P-	P-	P-	P-	P-	P	I	P-	P-	P-	P-	P-	P-	P-	P	P-
TOYOT	P	P	P-	P-	P	P	P-	P	P-	P	P-	P-	P-	P-	P	P-	R	P-	P	P	I	P	P	P-	P-	P-	P-	P	P-
BCP	P-	P	P-	P-	P	P	P-	P	P-	P	P-	P-	P-	P-	P-	P-	P-	P-	P	P	P-	I	P-	P-	P-	P-	P-	P	P-
BES	P	P	P-	P-	P	P	P-	P	P-	P	P-	P-	P-	P-	P	P-	P-	P-	P	P	P-	P	I	P-	P-	P-	P-	P	P-
BRISA	P	P	P	P	P	P	P	P	P-	P	P	P	P	P-	P	P	P	P	P	P	P	P	P	I	P	P	P	P	P
C_AMO	P	P	P-	R	P	P	P-	P	P-	P	P-	P-	P-	P-	P	P-	P	P	P	P	P	P	P	P-	I	P-	P	P	P-
CIMPO	P	P	P-	P	P	P	P-	P	P-	P	P	P	P-	P-	P	P	P	P	P	P	P	P	P	P-	P	I	P	P	P
COFIN	P	P	P-	R	P	P	P-	P	P-	P	P-	P-	P-	P-	P	P-	P	P	P	P	P	P	P	P-	P-	P-	I	P	P-
COMPT	P-	P-	P-	P-	P-	P	P-	P-	P-	R	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	P-	I	P-
EDP	P	P	P-	P	P	P	P-	P	P-	P	P	P	P-	P-	P	P	P	P	P	P	P	P	P	P-	P	P-	P	P	I

Legend (symbol at the intersection of the row for *a* and the column for *b*):

The couple (*a*, *b*) verifies one of the 4 following relations :

- if *a* is better than *b*, the symbol is **P**;

- if *a* is equivalent to *b*, the symbol is **I**;

- if *a* is as good as *b*, the symbol is **P-**;

- if *a* is incomparable to *b*, the symbol is **R**.

Figure 17 – Ranking Matrix (Portfolio 8)

Source: ELECTRE III software output estimation, May 2012