FINANÇAS

DOWNSIDE RISK IN COMMODITY AND EQUITY MARKETS

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ABSTRACT

The aim of the present study is to analyse the tail risk of global commodities indices and a set of share indexes of several countries and regions. To measure the downside risk we use two tail risk measures, namely the Value-at-Risk (VaR) and the Conditional Value-at-Risk (CvaR), determined by parametric, semi-parametric and non-parametric approaches.

Using daily prices comprising the period from January of 2002 to December 2016 and considering the pre- and post-global financial crisis sub-periods.

A time-varying correlation between stock and commodity markets returns, comparing returns and downside risk measures was carry out.

Overall, our findings indicate that tail risk of commodity markets is higher than stock market over the period, for almost all commodities, but that over the crisis period analysed the tail risk of stock market indices sharply increases to the same levels of commodities tail risk.

The correlations between commodity and stock returns evolve through time. Considering the tail risk measures, for all analysed pairs, commodity and stock returns, we observe very high contemporaneous correlations during the crisis period.

KEYWORDS: Downside risk, CvaR, dynamic conditional correlation.

1. INTRODUCTION

Over the last two decades has been a big and increasing interest of commodities as an alternative to traditional share market investment. Corresponding to several phases of rising and falling price trends, the commodity prices experienced a high volatility.

The relationship between commodity and stock market prices and returns has been analysed in literature. Most of this literature presents evidence on the impact of energy prices on stock prices, for instance, Park and Ratti (2008), find a significant impact of oil price shocks on real stock returns for developed countries over the period from January 1986 to December 2005.

The analysis of the spillover effects, namely the volatility transmission between commodities and stock markets, has been carried out, among others by Malik and Ewing (2009), Choi and Hammoudeh (2010) or Creti, *et all.* (2013), and the results show that commodity and share markets correlations have increased over time, limiting the hedging in portfolios.

Few studies have examined the relationship between markets, at downside risk level. Powell *et all.* (2017) study the tail risk of commodities and Asian indexes using measures of tail risk and concluded that that the relationship between equities and commodities is inconsistent in both strength and direction over time.

The high and growing importance of commodities in the formation of portfolios justifies the analysis of tail risk and measure.

This paper is structured as follows: Section 2 presents the models used in this study. The data and empirical results are described in 3 Empirical results. Summary conclusions are presented in Section 4.

2. METHODOLOGY

We estimate the downside risk of share and commodities markets and carry out a time-varying analysis of these measures.

2.1 Downside risk measures

The downside risk, i.e., the potential loss of the value of an asset resulting from declining prices.

Value at Risk, often referred to as VaR, is one of the most used risk measures. The Value-at-Risk (VaR) which measures the largest potential loss over a certain period of time for a particular confidence level. Generally, the $(1-\alpha)$ percent VaR of returns is expressed as

$$VaR_x(1-\alpha) = -\sigma_x q_x(\alpha) \tag{1}$$

where $q_x(\alpha)$ is the α percent quantile of the standardized distribution of returns and σ_x is the standard deviation of asset x.

The Conditional Value-at-Risk (CVaR) is introduced by Rockafellar and Uryasev (2000) and it is usually defined as the conditional expectation of losses exceeding VaR for continuous distributions.

$$CVaR_x(1-\alpha) = \frac{1}{\alpha}\int_{1-\alpha}^{1} VaR_x(x)\,dx = -\frac{1}{\alpha}\sigma_x\int_{1-\alpha}^{1} q_x^x dx \tag{2}$$

This paper we use as nonparametric method the historical simulation (HS) approach, where no distributional assumption are needed while the semi-parametric estimation for VaR/CVaR is based on the Cornish–Fisher expansion. The Cornish-Fisher expansion is an approximation of the quantiles of a distribution using polynomials in the quantiles of a normal distribution with coefficients depending on the moments of the distribution under scrutiny (see Maillard (2018)).

The Cornish– Fisher approximations for CVaR are expressed as

$$CVaR_x(1-\alpha) = -\sigma_x\left[M_1 + \frac{1}{6}(M_2-1)s_x + \frac{1}{24}(M_3-3M_1)k_x - \frac{1}{36}(2M_3-5M_1)s_x^2\right] \tag{3}$$

where $M_i = \frac{1}{\alpha}\int_{-\infty}^{c(\alpha)} x^i f(x)\, dx$, i=1, 2, 3, s_x is the skewness of the asset, and k_x is the kurtosis and $f(.)$ is the standard normal probability density function.

2.2 Dynamic Conditional Correlation (DCC) model

Multivariate GARCH processes are a generalization of univariate models. The Dynamic Conditional Correlation (DCC) model of Engle (2002) and Tse and Tsui (2002) is a nonlinear combination of univariate GARCH models.

In the DCC-model, the correlation matrix is time varying and the covariance matrix can be decomposed into:

$$H_t = D_t R_t D_t \tag{4}$$

Where D_t is a diagonal matrix of time varying standard deviations from univariate GARCH processes, whose elements are the conditional standard deviations obtained in a previous univariate model.

$$\mathrm{D}_t = diag\left(\sqrt{h_{11,t}}, \ldots, \sqrt{h_{nn,t}}\right) \tag{5}$$

The R_t is the time-varying conditional correlation matrix,

$$R_t = diag(Q_t)^{-1/2} Q_t\ diag(Q_t)^{-1/2} \tag{6}$$

Where Q_t is the $n \times n$ symmetric positive definite matrix, which has the form,

$$Q_t = (1-\alpha-\beta)\bar{Q} + \alpha\, u_{t-1}u'_{t-1} + \beta\, Q_{t-1} \tag{7}$$

where $\bar{Q}$ is the unconditional variance-covariance matrix $(n \times n)$ of the standardized error, u_t and α and β are scalars.

To ensure a positive definiteness of Q_t, as well as stationarity simple conditions on the parameters, are imposed, namely

$$\alpha \geq 0 \;\; and\; \beta \geq 0$$

and

$$\alpha + \beta < 1$$

The model can be performed by a two-step procedure, where in the first step, the conditional variance is estimated via univariate GARCH model for each series and the second step correspond to estimate the parameters for the conditional correlation.

A review of multivariate GARCH models may be found in Silvennoinen and Terasvirta (2008) as well as in Bauwens, Laurent and Rombouts (2006).

3. EMPIRICAL RESULTS

3.1 Data

We use five S&P GSCI commodity sub-indexes, Energy (EN), Industrial Metals (IM), Precious Metals (PM), Agriculture (AG) and Livestock (LS) and a global index Total Commodities (TC), at daily frequency.
We use the MSCI World Index, MSCI U.S. index, MSCI Europe Index and MSCI Japan Index to capture the stock indexes of main developed markets, covering the Europe, America and Asia geographical areas.

The data for the period Jan – 2002 to Dec-2016 was obtained from Datastream database. We consider the crisis period, a period of overall economic instability covering the global financial crisis in 2007-2009 and the sovereign debt crisis 2010-2012, and the sample is divided into pre- and post-crisis sub-periods.

The daily values of five commodity and four market indexes were transformed into series of returns, by applying the expression $ln(P_t/P_{t-1})$, in which P_t and P_{t-1} represent the daily values of a given index, t and $t-1$ day, respectively.

3.2 Results

The main descriptive statistics on Stock and Commodity indices are presented in table 1. The analysis of these statistics allows conclude that, without exception the indexes presents a positive, although small, mean return.

The Jarque-Bera test has applied, in order to verify the adjustment of the normal distribution to the empirical distributions of series, whose statistical probabilities are presented in table 1. The results allow us to conclude that the series of logarithmic variations are statistically significant to 1%, clearly rejecting the hypothesis of their normality.

Table 1: Descriptive statistics

Panel A						
SP GSCI	Total	Energy	Ind. Metals	Prec. Metals	Agriculture	Livestock
Mean	0,012	0,012	0,013	0,030	0,010	0,010
Maximum	7,215	9,809	7,577	8,759	7,154	3,245
Minimum	-8,446	-9,347	-9,109	-10,106	-7,635	-3,954
Std. Dev.	1,498	1,966	1,556	1,272	1,347	0,897
Skewness	-0,157	-0,051	-0,248	-0,462	-0,109	-0,078
Kurtosis	5,748	5,551	5,529	8,051	5,340	3,739
J-B	1079,0	918,8	936,5	3717,6	778,7	80,4
	0,0000	0,0000	0,0000	0,0000	0,0000	0,0000
Panel B						
MSCI			World	Europe	US	Japan
Mean			0,016	0,007	0,021	0,008
Maximum			9,097	10,698	11,042	12,187
Minimum			-7,325	-10,178	-9,514	-9,594
Std. Dev.			1,020	1,405	1,180	1,399
Skewness			-0,4976	-0,1830	-0,3627	-0,1601
Kurtosis			12,818	10,979	15,082	8,701
J-B			13730,6	8995,8	20658,2	4596,6
			0,0000	0,0000	0,0000	0,0000

Notes: Daily data for the period 2 January 2002 to 31 December 2016.
J-B denotes the Jarque-Bera statistic for the null of normality.

In order to estimate the VaR/CVaR we use a non-parametric, a parametric and a semi-parametric approach, at 99% and 95% confidence level and rolling windows of 90, 250 and 520 days.

Tables 2 to 5 present the main VaR/CVaR results for stocks and commodities.

Table 2A
Total Commodities, VaR/CVaR estimated by non-parametric, parametric and semi-parametric models

All period	Non-parametric				Parametric				Semi-parametric			
	VaR_{99}	$CVaR_{99}$	VaR_{95}	$CVaR_{95}$	VaR_{99}	$CVaR_{99}$	VaR_{95}	$CVaR_{95}$	VaR_{99}	$CVaR_{99}$	VaR_{95}	$CVaR_{95}$
Panel A: 90 days rolling window forecasting												
Mean	3,563	3,835	2,312	2,924	3,237	3,711	2,286	2,869	3,451	4,105	2,301	3,013

	VaR99	CVaR99	VaR95	CVaR95	VaR99	CVaR99	VaR95	CVaR95	VaR99	CVaR99	VaR95	CVaR95
Max	8,253	8,446	6,428	7,305	9,025	10,236	6,590	8,083	8,292	9,466	6,407	7,539
Min	1,158	1,244	0,808	0,960	1,194	1,358	0,853	1,066	1,111	1,272	0,792	0,992
Std	1,418	1,550	1,012	1,175	1,326	1,506	0,966	1,187	1,375	1,684	0,954	1,204
Panel B: 250 days rolling window forecasting												
Mean	3,612	4,126	2,353	3,118	3,273	3,752	2,310	2,900	3,712	4,581	2,332	3,193
Max	7,082	7,836	5,367	6,411	6,903	7,868	4,962	6,152	7,279	8,716	4,978	6,382
Min	1,533	1,702	0,984	1,262	1,440	1,646	1,025	1,280	1,431	1,644	1,006	1,272
Std	1,299	1,462	0,940	1,125	1,167	1,331	0,839	1,040	1,299	1,654	0,849	1,117
Panel B: 520 days rolling window forecasting												
Mean	3,856	4,590	2,420	3,306	3,331	3,820	2,348	2,951	3,995	5,092	2,371	3,391
Max	6,482	7,346	4,181	5,519	5,434	6,217	3,860	4,824	6,494	8,570	3,928	5,450
Min	2,080	2,444	1,132	1,649	1,668	1,907	1,189	1,483	1,998	2,478	1,145	1,713
Std	1,256	1,392	0,780	1,030	0,948	1,085	0,673	0,842	1,167	1,567	0,695	0,978

Table 2B

Total Commodities, VaR/CVaR estimated by non-parametric, parametric and semi-parametric models

Crisis Period	Non-parametric				Parametric				Semi-parametric			
	VaR_{99}	$CVaR_{99}$	VaR_{95}	$CVaR_{95}$	VaR_{99}	$CVaR_{99}$	VaR_{95}	$CVaR_{95}$	VaR_{99}	$CVaR_{99}$	VaR_{95}	$CVaR_{95}$
Panel A: 90 days rolling window forecasting												
Mean	4,304	4,602	2,809	3,545	3,826	4,388	2,698	3,390	4,199	5,005	2,774	3,655
Max	8,253	8,446	6,428	7,305	9,025	10,236	6,590	8,083	8,292	9,466	6,407	7,539
Min	1,908	2,036	1,231	1,674	2,031	2,329	1,416	1,793	2,051	2,373	1,369	1,798
Std	1,627	1,721	1,295	1,424	1,660	1,875	1,230	1,494	1,586	1,870	1,184	1,418
Panel B: 250 days rolling window forecasting												
Mean	4,390	5,012	2,899	3,856	3,906	4,480	2,753	3,460	4,511	5,533	2,845	3,882
Max	7,082	7,836	5,367	6,411	6,903	7,868	4,962	6,152	7,279	8,716	4,978	6,382
Min	2,427	2,778	1,695	2,325	2,521	2,909	1,742	2,220	2,590	2,995	1,754	2,271
Std	1,511	1,671	1,163	1,318	1,415	1,605	1,034	1,267	1,501	1,855	1,014	1,309
Panel B: 520 days rolling window forecasting												
Mean	4,761	5,575	3,006	4,091	4,003	4,590	2,823	3,547	4,823	6,091	2,899	4,105
Max	6,482	7,346	4,181	5,519	5,434	6,217	3,860	4,824	6,494	8,570	3,928	5,450
Min	2,950	3,245	1,809	2,592	2,847	3,275	1,984	2,513	2,923	3,329	2,017	2,576
Std	1,445	1,593	0,836	1,151	1,024	1,166	0,740	0,914	1,350	1,837	0,737	1,121

Table 2C

Total Commodities, VaR/CVaR estimated by non-parametric, parametric and semi-parametric models

Pre-Crisis	Non-parametric				Parametric				Semi-parametric			
	VaR_{99}	$CVaR_{99}$	VaR_{95}	$CVaR_{95}$	VaR_{99}	$CVaR_{99}$	VaR_{95}	$CVaR_{95}$	VaR_{99}	$CVaR_{99}$	VaR_{95}	$CVaR_{95}$
Panel A: 90 days rolling window forecasting												
Mean	3,258	3,449	2,227	2,765	3,244	3,727	2,272	2,868	3,152	3,599	2,235	2,797
Max	4,710	4,757	2,933	4,058	4,158	4,757	2,954	3,692	4,577	5,747	3,103	3,982
Min	2,298	2,375	1,392	1,850	2,066	2,388	1,417	1,816	2,108	2,388	1,459	1,855
Std	0,571	0,569	0,308	0,447	0,418	0,477	0,302	0,373	0,508	0,693	0,292	0,419
Panel B: 250 days rolling window forecasting												
Mean	3,440	3,728	2,250	2,895	3,285	3,774	2,303	2,905	3,309	3,878	2,262	2,908
Max	4,190	4,532	2,716	3,354	3,717	4,269	2,607	3,288	3,862	4,738	2,537	3,339

Min	2,829	2,983	1,908	2,531	2,751	3,155	1,938	2,437	2,891	3,200	1,982	2,541
Std	0,526	0,556	0,162	0,249	0,227	0,263	0,156	0,199	0,311	0,469	0,144	0,243
Panel B: 520 days rolling window forecasting												
Mean	3,481	4,021	2,350	2,999	3,313	3,807	2,320	2,929	3,414	4,069	2,276	2,982
Max	4,148	4,448	2,568	3,365	3,516	4,040	2,471	3,110	3,770	4,500	2,502	3,287
Min	2,913	3,265	2,177	2,659	3,103	3,566	2,171	2,743	3,051	3,550	2,106	2,698
Std	0,288	0,355	0,109	0,160	0,116	0,134	0,082	0,103	0,160	0,234	0,077	0,126

Table 2D

Total Commodities, VaR/CVaR estimated by non-parametric, parametric and semi-parametric models

Pos-Crisis	Non-parametric				Parametric				Semi-parametric			
	VaR_{99}	$CVaR_{99}$	VaR_{95}	$CVaR_{95}$	VaR_{99}	$CVaR_{99}$	VaR_{95}	$CVaR_{95}$	VaR_{99}	$CVaR_{99}$	VaR_{95}	$CVaR_{95}$
Panel A: 90 days rolling window forecasting												
Mean	2,868	3,187	1,728	2,247	2,438	2,782	1,744	2,169	2,740	3,396	1,729	2,362
Max	5,589	6,586	2,627	3,730	4,346	4,939	3,153	3,884	5,422	7,765	3,172	4,428
Min	1,158	1,244	0,808	0,960	1,194	1,358	0,853	1,066	1,111	1,272	0,792	0,992
Std	1,234	1,537	0,628	0,831	0,946	1,074	0,689	0,846	1,156	1,564	0,662	0,967
Panel B: 250 days rolling window forecasting												
Mean	2,734	3,329	1,718	2,344	2,406	2,748	1,718	2,140	3,036	3,999	1,709	2,548
Max	4,484	5,369	2,597	3,481	3,870	4,409	2,785	3,450	4,827	7,554	2,729	3,775
Min	1,533	1,702	0,984	1,262	1,440	1,646	1,025	1,280	1,431	1,644	1,006	1,272
Std	0,827	1,110	0,543	0,673	0,720	0,818	0,524	0,644	1,017	1,499	0,509	0,819
Panel B: 520 days rolling window forecasting												
Mean	3,011	3,831	1,698	2,553	2,443	2,795	1,736	2,169	3,459	4,766	1,755	2,838
Max	3,964	4,779	2,112	3,270	3,022	3,443	2,177	2,695	4,243	5,912	2,248	3,471
Min	2,080	2,444	1,132	1,649	1,668	1,907	1,189	1,483	1,998	2,478	1,145	1,713
Std	0,657	0,859	0,308	0,500	0,429	0,491	0,303	0,380	0,726	1,061	0,342	0,578

Table 3A

MSCI World, VaR/CVaR estimated by non-parametric, parametric and semi-parametric models

All sample	Non-parametric				Parametric				Semi-parametric			
	VaR_{99}	$CVaR_{99}$	VaR_{95}	$CVaR_{95}$	VaR_{99}	$CVaR_{99}$	VaR_{95}	$CVaR_{95}$	VaR_{99}	$CVaR_{99}$	VaR_{95}	$CVaR_{95}$
Panel A: 90 days rolling window forecasting												
Mean	2,342	2,513	1,553	1,953	2,058	2,361	1,450	1,823	2,310	2,815	1,477	1,995
Max	7,302	7,325	6,282	6,941	7,903	8,989	5,719	7,059	8,137	11,453	5,637	7,166
Min	0,820	0,868	0,486	0,709	0,814	0,948	0,543	0,709	0,841	0,885	0,548	0,743
Std	1,381	1,448	1,063	1,234	1,280	1,455	0,929	1,144	1,448	1,818	0,926	1,248
Panel B: 250 days rolling window forecasting												
Mean	2,616	2,896	1,590	2,152	2,115	2,426	1,489	1,873	2,638	3,411	1,518	2,223
Max	7,192	7,264	4,399	5,776	5,593	6,384	4,002	4,977	7,288	11,027	4,013	5,936
Min	1,017	1,116	0,678	0,850	1,064	1,230	0,728	0,934	0,968	1,051	0,722	0,872
Std	1,628	1,635	0,912	1,254	1,131	1,287	0,815	1,009	1,569	2,184	0,808	1,289
Panel B: 520 days rolling window forecasting												
Mean	2,859	3,495	1,621	2,359	2,232	2,560	1,572	1,977	3,060	4,167	1,588	2,522
Max	6,134	6,941	3,092	4,737	4,243	4,854	3,030	3,774	7,026	11,347	3,011	5,436
Min	1,053	1,163	0,745	0,950	1,129	1,303	0,779	0,994	1,079	1,219	0,769	0,958
Std	1,600	1,795	0,713	1,173	0,977	1,114	0,702	0,871	1,689	2,569	0,685	1,326

Table 3B
MSCI World, VaR/CVaR estimated by non-parametric, parametric and semi-parametric models

Crisis	Non-parametric				Parametric				Semi-parametric			
	VaR_{99}	$CVaR_{99}$	VaR_{95}	$CVaR_{95}$	VaR_{99}	$CVaR_{99}$	VaR_{95}	$CVaR_{95}$	VaR_{99}	$CVaR_{99}$	VaR_{95}	$CVaR_{95}$
Panel A: Rolling window (90 days)												
Mean	3,345	3,523	2,334	2,865	3,009	3,444	2,132	2,670	3,352	4,085	2,156	2,901
Max	7,302	7,325	6,282	6,941	7,903	8,989	5,719	7,059	8,137	11,453	5,637	7,166
Min	1,674	1,768	0,942	1,434	1,386	1,594	0,969	1,225	1,566	1,905	0,974	1,347
Std	1,590	1,638	1,273	1,449	1,532	1,737	1,118	1,372	1,702	2,139	1,105	1,472
Panel B: 250 days rolling window forecasting												
Mean	3,934	4,219	2,334	3,187	3,051	3,493	2,162	2,707	3,902	5,119	2,195	3,272
Max	7,192	7,264	4,399	5,776	5,593	6,384	4,002	4,977	7,288	11,027	4,013	5,936
Min	1,540	2,096	0,973	1,411	1,253	1,444	0,869	1,105	1,777	2,300	0,980	1,479
Std	1,779	1,752	1,005	1,357	1,216	1,382	0,881	1,086	1,721	2,449	0,859	1,403
Panel B: 520 days rolling window forecasting												
Mean	4,124	4,865	2,218	3,314	3,014	3,452	2,134	2,674	4,387	6,139	2,144	3,573
Max	6,134	6,941	3,092	4,737	4,243	4,854	3,030	3,774	7,026	11,347	3,011	5,436
Min	1,591	2,063	1,000	1,402	1,362	1,568	0,949	1,202	1,688	2,139	0,996	1,429
Std	1,715	1,874	0,688	1,198	0,982	1,118	0,708	0,876	1,812	2,848	0,674	1,399

Table 3C
MSCI World, VaR/CVaR estimated by non-parametric, parametric and semi-parametric models

Pre-Crisis	Non-parametric				Parametric				Semi-parametric			
	VaR_{99}	$CVaR_{99}$	VaR_{95}	$CVaR_{95}$	VaR_{99}	$CVaR_{99}$	VaR_{95}	$CVaR_{95}$	VaR_{99}	$CVaR_{99}$	VaR_{95}	$CVaR_{95}$
Panel A: 90 days rolling window forecasting												
Mean	1,412	1,534	0,884	1,144	1,271	1,464	0,883	1,121	1,361	1,611	0,903	1,186
Max	2,116	2,515	1,491	1,730	1,981	2,263	1,413	1,761	2,014	2,898	1,392	1,744
Min	0,820	0,868	0,486	0,709	0,814	0,948	0,543	0,709	0,841	0,885	0,548	0,743
Std	0,433	0,532	0,252	0,326	0,276	0,311	0,206	0,248	0,407	0,548	0,231	0,338
Panel B: 250 days rolling window forecasting												
Mean	1,481	1,685	0,950	1,239	1,321	1,522	0,917	1,165	1,474	1,794	0,937	1,270
Max	2,178	2,396	1,302	1,746	1,838	2,122	1,269	1,618	2,105	2,642	1,284	1,796
Min	1,017	1,116	0,678	0,850	1,064	1,230	0,728	0,934	0,968	1,051	0,722	0,872
Std	0,300	0,367	0,159	0,226	0,159	0,183	0,109	0,140	0,295	0,437	0,124	0,232
Panel B: 520 days rolling window forecasting												
Mean	1,723	2,103	1,091	1,474	1,551	1,784	1,082	1,370	1,755	2,197	1,075	1,501
Max	2,928	3,372	1,812	2,373	2,541	2,912	1,795	2,252	2,883	3,780	1,711	2,430
Min	1,053	1,163	0,745	0,950	1,129	1,303	0,779	0,994	1,079	1,219	0,769	0,958
Std	0,574	0,604	0,340	0,434	0,452	0,517	0,323	0,402	0,527	0,719	0,287	0,440

Table 3D
MSCI World, VaR/CVaR estimated by non-parametric, parametric and semi-parametric models

Pos Crisis	Non-parametric				Parametric				Semi-parametric			
	VaR_{99}	$CVaR_{99}$	VaR_{95}	$CVaR_{95}$	VaR_{99}	$CVaR_{99}$	VaR_{95}	$CVaR_{95}$	VaR_{99}	$CVaR_{99}$	VaR_{95}	$CVaR_{95}$
Panel A: 90 days rolling window forecasting												

Mean	1,921	2,130	1,168	1,529	1,563	1,795	1,097	1,383	1,852	2,306	1,134	1,582
Max	3,372	3,794	2,175	2,742	2,512	2,868	1,802	2,237	3,257	4,305	1,954	2,752
Min	1,071	1,154	0,663	0,880	0,897	1,034	0,621	0,791	1,059	1,224	0,657	0,929
Std	0,628	0,800	0,335	0,458	0,410	0,464	0,300	0,367	0,612	0,848	0,325	0,500
Panel B: 250 days rolling window forecasting												
Mean	1,968	2,320	1,224	1,667	1,643	1,887	1,151	1,453	2,092	2,720	1,185	1,756
Max	4,395	4,708	2,290	3,353	3,221	3,684	2,292	2,862	4,074	5,256	2,395	3,443
Min	1,338	1,659	0,832	1,172	1,143	1,315	0,795	1,009	1,417	1,767	0,840	1,202
Std	0,679	0,718	0,333	0,474	0,476	0,543	0,340	0,423	0,599	0,823	0,333	0,496
Panel B: 520 days rolling window forecasting												
Mean	2,284	3,033	1,343	1,952	1,855	2,130	1,301	1,641	2,571	3,471	1,348	2,122
Max	3,530	4,395	1,918	2,834	2,648	3,035	1,869	2,347	3,679	5,123	1,953	2,995
Min	1,527	1,783	0,949	1,301	1,291	1,488	0,896	1,139	1,507	1,870	0,929	1,290
Std	0,713	1,006	0,335	0,554	0,516	0,589	0,368	0,459	0,738	1,023	0,374	0,605

Table 4
MSCI World, MSCI Europe, MSCI US and MSCI Japan, VaR/CVaR estimated by non-parametric, parametric and semi-parametric models

all sample	Non-parametric				Parametric				Semi-parametric			
	VaR_{99}	$CVaR_{99}$	VaR_{95}	$CVaR_{95}$	VaR_{99}	$CVaR_{99}$	VaR_{95}	$CVaR_{95}$	VaR_{99}	$CVaR_{99}$	VaR_{95}	$CVaR_{95}$
Panel A: 520 days rolling window forecasting - MSCI WORLD												
Mean	2,859	3,495	1,621	2,359	2,232	2,560	1,572	1,977	3,060	4,167	1,588	2,522
Max	6,134	6,941	3,092	4,737	4,243	4,854	3,030	3,774	7,026	11,347	3,011	5,436
Min	1,053	1,163	0,745	0,950	1,129	1,303	0,779	0,994	1,079	1,219	0,769	0,958
Std	1,600	1,795	0,713	1,173	0,977	1,114	0,702	0,871	1,689	2,569	0,685	1,326
Panel B: 520 days rolling window forecasting - MSCI EUROPE												
Mean	3,779	4,578	2,189	3,168	3,080	3,532	2,173	2,729	4,048	5,484	2,141	3,352
Max	6,978	8,300	3,982	5,753	5,560	6,358	3,956	4,939	8,616	13,521	3,708	6,646
Min	1,566	1,720	1,053	1,346	1,537	1,774	1,062	1,353	1,507	1,732	1,050	1,330
Std	1,684	1,935	0,895	1,335	1,242	1,416	0,892	1,106	1,876	2,859	0,801	1,483
Panel C: 520 days rolling window forecasting - MSCI US												
Mean	3,219	4,035	1,851	2,696	2,586	2,966	1,822	2,290	3,576	4,929	1,812	2,934
Max	6,575	8,586	3,508	5,505	5,178	5,918	3,691	4,602	8,581	14,226	3,552	6,600
Min	1,486	1,573	0,989	1,261	1,403	1,613	0,978	1,239	1,459	1,707	0,959	1,281
Std	1,658	2,297	0,791	1,359	1,191	1,360	0,853	1,060	2,107	3,267	0,801	1,637
Panel D: 520 days rolling window forecasting - MSCI JAPAN												
Mean	3,683	4,978	2,194	3,187	3,195	3,662	2,255	2,831	4,469	6,157	2,263	3,665
Max	6,245	7,637	3,571	5,190	5,144	5,881	3,662	4,571	7,621	11,855	3,524	5,927
Min	2,417	2,848	1,547	2,084	2,250	2,582	1,584	1,993	2,389	2,815	1,519	2,092
Std	1,073	1,212	0,549	0,839	0,793	0,903	0,572	0,707	1,328	2,225	0,489	1,004

Table 5
SP Total Commodities, Energy, Industrial Metals, Precious Metals, Agriculture and Livestock, VaR/CVaR estimated by non-parametric, parametric and semi-parametric models

all sample	Non-parametric				Parametric				Semi-parametric			
	VaR_{99}	$CVaR_{99}$	VaR_{95}	$CVaR_{95}$	VaR_{99}	$CVaR_{99}$	VaR_{95}	$CVaR_{95}$	VaR_{99}	$CVaR_{99}$	VaR_{95}	$CVaR_{95}$
Panel A: 520 days rolling window forecasting - Total Commodities												
Mean	3,856	4,590	2,420	3,306	3,331	3,820	2,348	2,951	3,995	5,092	2,371	3,391
Max	6,482	7,346	4,181	5,519	5,434	6,217	3,860	4,824	6,494	8,570	3,928	5,450
Min	2,080	2,444	1,132	1,649	1,668	1,907	1,189	1,483	1,998	2,478	1,145	1,713
Std	1,256	1,392	0,780	1,030	0,948	1,085	0,673	0,842	1,167	1,567	0,695	0,978
Panel B: 520 days rolling window forecasting - Energy												
Mean	4,931	5,868	3,071	4,188	4,258	4,883	3,003	3,773	5,106	6,542	3,009	4,328
Max	7,979	8,976	5,039	6,723	6,682	7,648	4,753	5,931	7,836	10,314	4,777	6,643
Min	2,645	3,134	1,509	2,152	2,147	2,455	1,527	1,907	2,577	3,213	1,456	2,205
Std	1,465	1,665	0,930	1,207	1,144	1,309	0,812	1,015	1,374	1,913	0,822	1,144
Panel C: 520 days rolling window forecasting - Industrial Metals												
Mean	4,225	5,241	2,456	3,484	3,523	4,040	2,484	3,121	4,417	5,729	2,529	3,718
Max	5,814	6,779	3,830	5,206	5,418	6,198	3,850	4,812	6,087	9,016	3,957	5,129
Min	1,975	2,198	1,355	1,726	2,005	2,308	1,394	1,769	1,978	2,274	1,321	1,748
Std	1,217	1,394	0,705	0,945	0,953	1,090	0,678	0,846	1,179	1,681	0,707	0,967
Panel D: 520 days rolling window forecasting - Precious Metals												
Mean	3,666	4,863	2,089	3,089	2,914	3,344	2,048	2,579	4,393	6,112	2,139	3,571
Max	4,900	7,206	2,803	3,970	4,003	4,589	2,824	3,547	6,568	10,343	2,760	5,048
Min	2,373	2,712	1,451	2,076	2,091	2,406	1,456	1,845	2,345	2,925	1,478	2,024
Std	0,557	0,972	0,359	0,489	0,515	0,590	0,365	0,457	1,153	1,975	0,326	0,838
Panel E: 520 days rolling window forecasting - Agriculture												
Mean	3,454	4,196	2,155	2,949	3,078	3,529	2,171	2,728	3,473	4,343	2,146	2,977
Max	5,377	6,446	3,479	4,797	4,661	5,332	3,313	4,139	5,545	7,093	3,420	4,721
Min	1,983	2,455	1,465	1,854	2,166	2,489	1,516	1,915	2,160	2,537	1,381	1,909
Std	1,039	1,262	0,569	0,896	0,739	0,847	0,521	0,655	1,045	1,349	0,595	0,875
Panel F: 520 days rolling window forecasting - Livestock												
Mean	2,182	2,571	1,450	1,912	2,011	2,307	1,416	1,781	2,194	2,651	1,427	1,903
Max	2,652	3,289	1,899	2,422	2,448	2,807	1,733	2,172	2,842	3,539	1,788	2,444
Min	1,299	1,609	0,927	1,211	1,361	1,568	0,944	1,200	1,403	1,672	0,930	1,223
Std	0,284	0,396	0,238	0,301	0,278	0,318	0,198	0,247	0,355	0,441	0,219	0,303

Nonparametric and semi-parametric methods have performed well during all period. However, the behaviour of the models is not constant over time. During the crisis period all CVaR models included, using nonparametric and semi-parametric methods was accepted at both confidence levels, while almost included VaR models were rejected.

Figures 1a to 1d CVaR show the daily CVaR based on a rolling window of 520 days and a confidence level of 95%, for the pairs formed by each stock market index MSCI World, Europe, US and Japan, and each commodity index.

Fig. 1a. CVaR 95% MSCI World vs Commodities

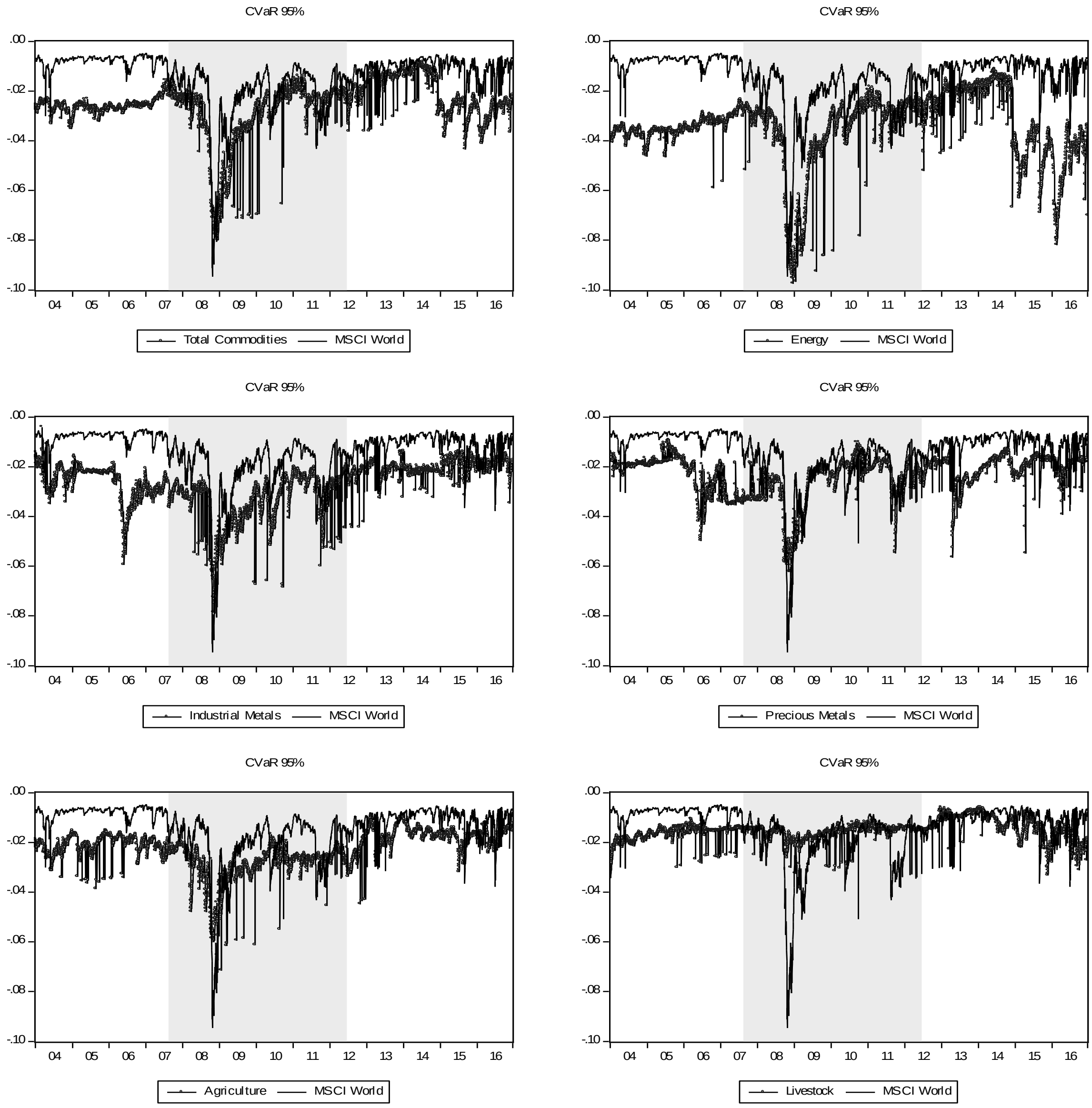


Fig. 1b. CVaR 95%. MSCI Europe vs Commodities

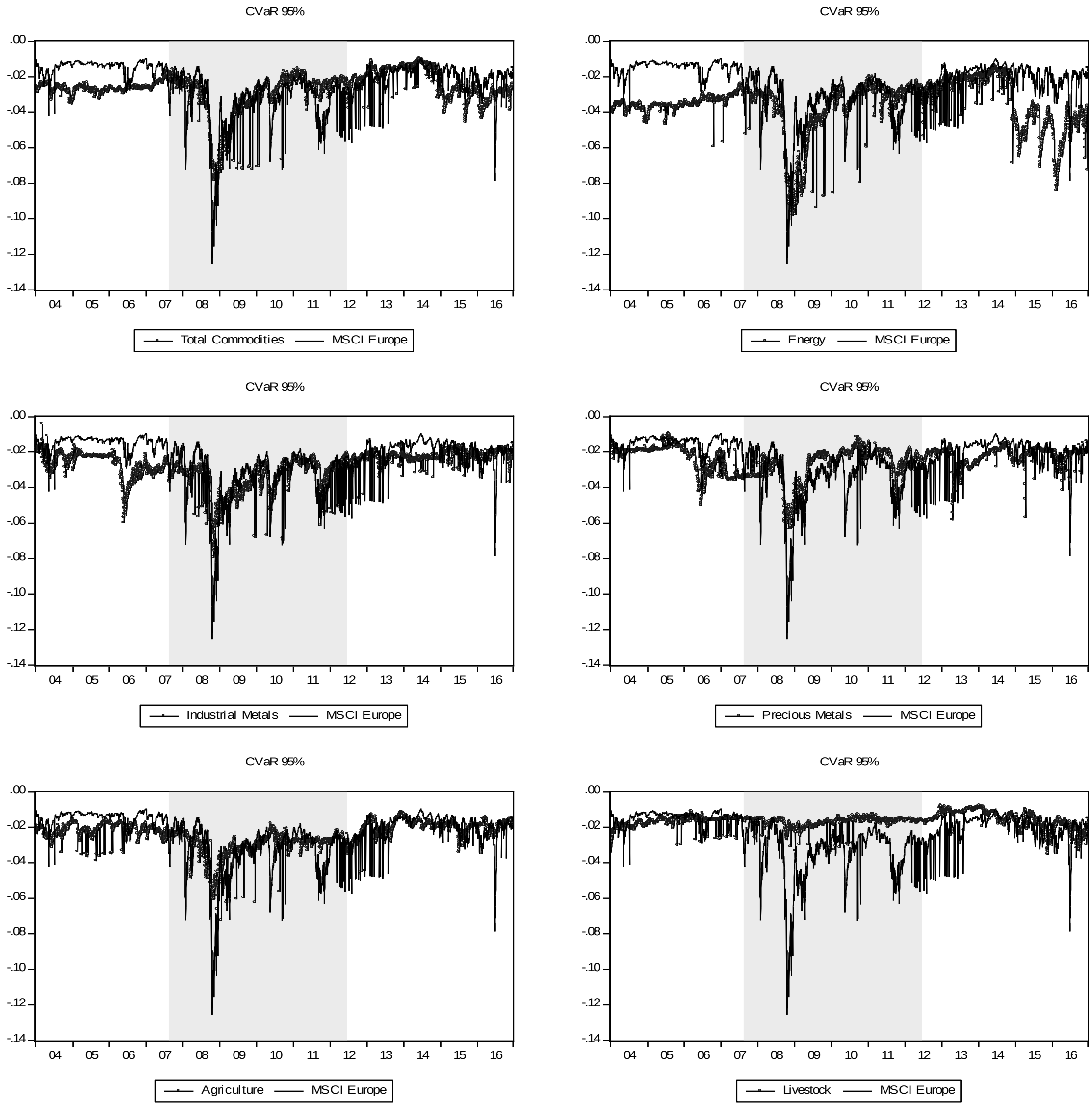


Fig. 1c. CVaR 95% MSCI US vs Commodities

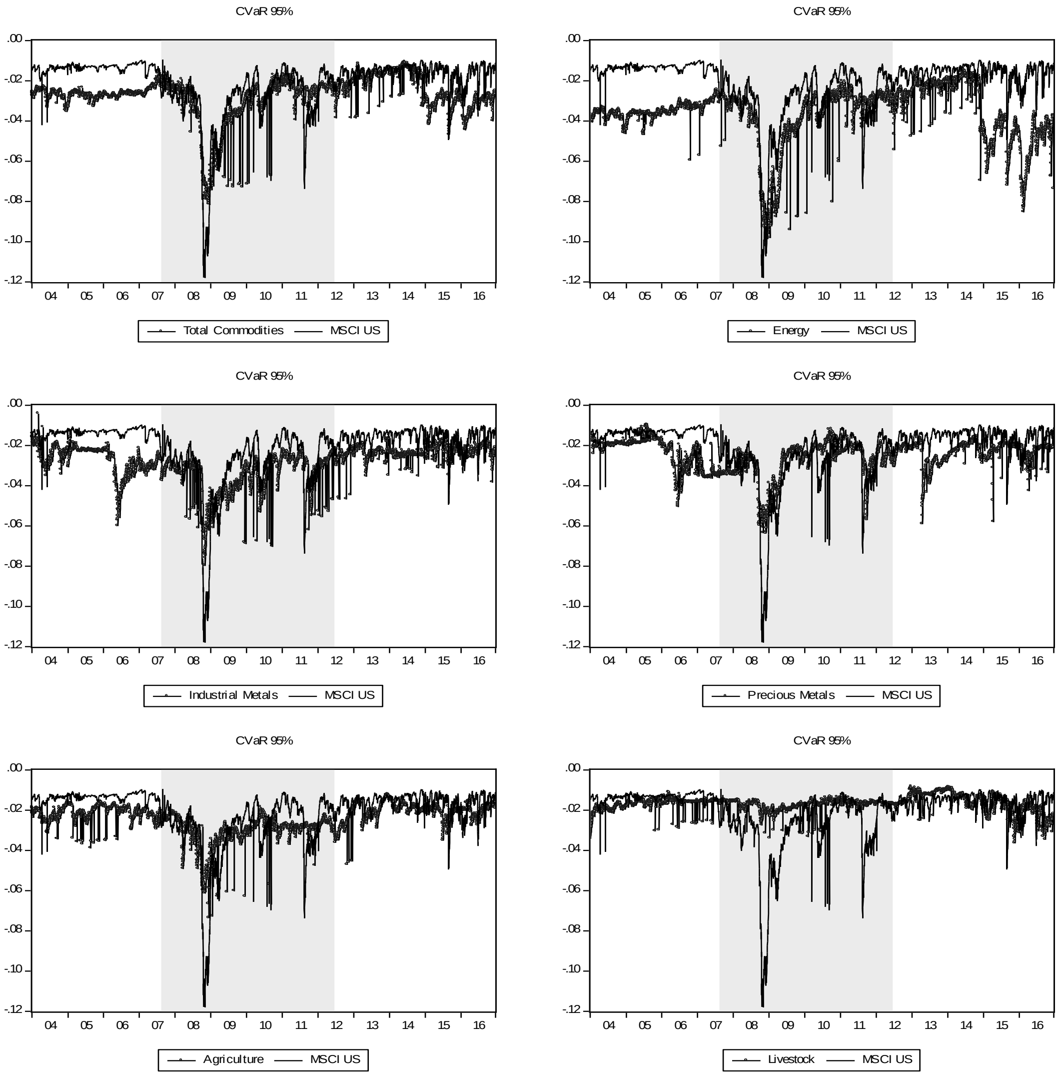
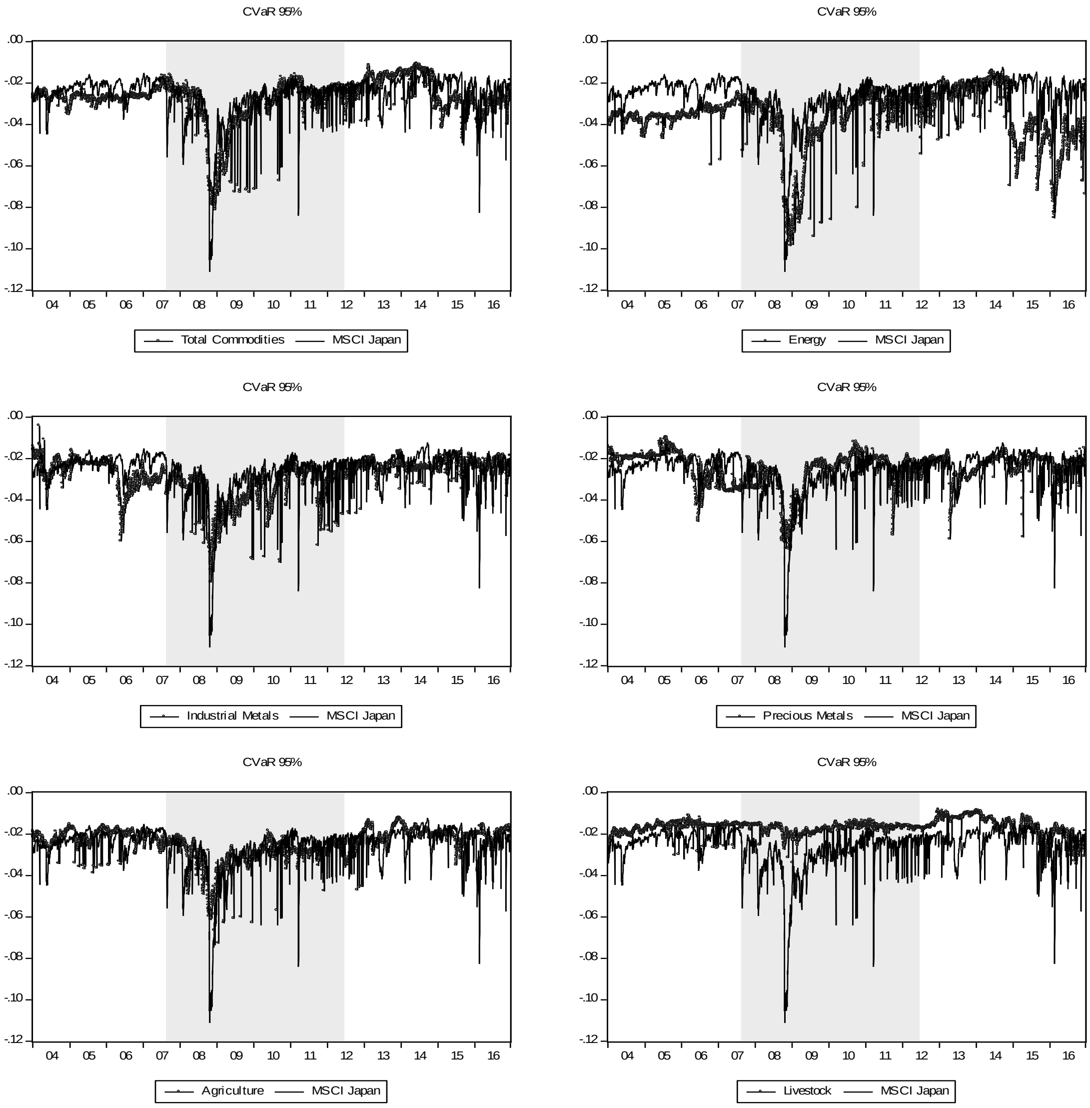


Fig. 1d. CvaR 95%. MSCI Japan vs Commodities



The analysis of figures allows us to observe the significant difference of magnitude of tail metrics during the crisis period and calm periods.

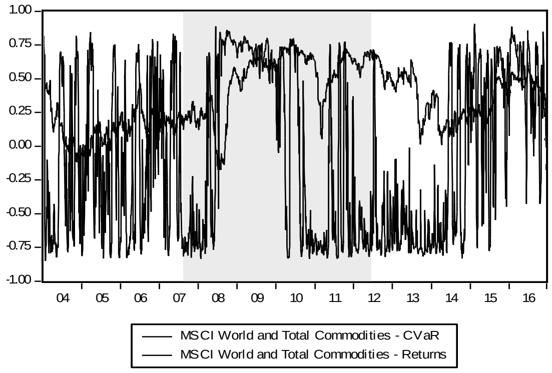
The CVaR of MSCI World and Total Commodities presented at the top left in Figure 1a shows that tail risk of commodity market is much higher than stock market prior to the crisis period. The crisis period changed the pattern, as the tail risk of stock markets increase significantly, to the same levels of commodity indices. Similar pattern can also be observed for energy, industrial metals, precious metals and agricultural indices.

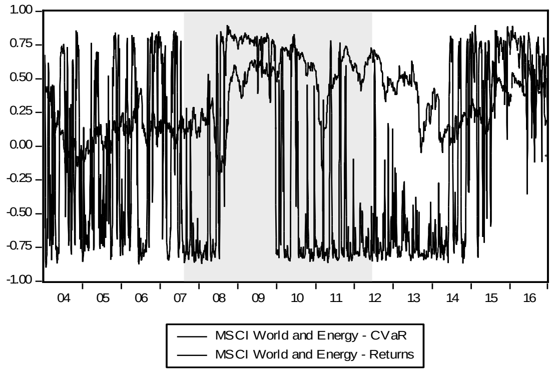
Not surprisingly, the US case shows as well as the global case that the post-crises display almost the pattern than pre-crises, while the European stock market continuous to present relatively high tail risk values in the post-crises period.

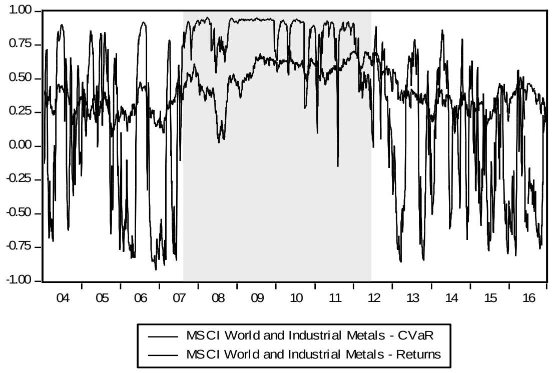
To infer the relations established between the performances of these tail risk measures we carry out the analysis of the dynamic conditional correlations, to stablish a set of co-movements between the two variables.

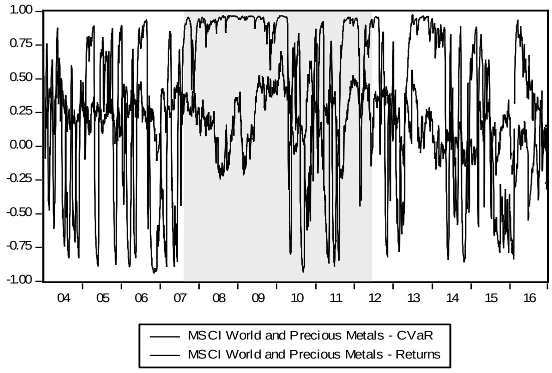
With the aim of analyse the time-varying correlations of stock markets returns and commodity indices we adopt the DCC model.

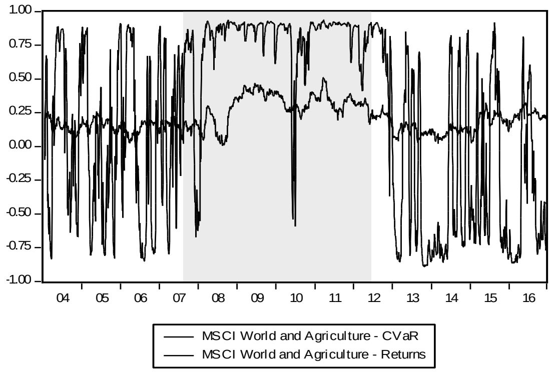
Fig. 2. Dynamic Conditional Correlations.

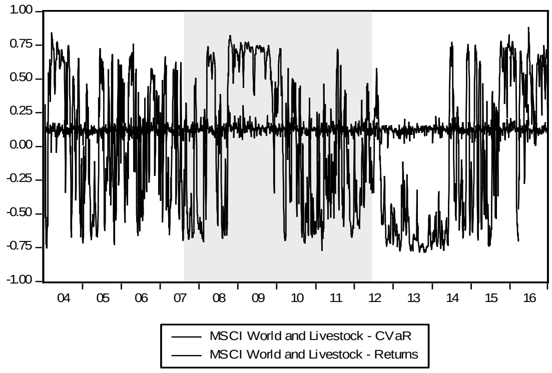












In general, the contemporary conditional correlations between stock markets, measured by the MSCI World and commodities were positive for returns, with the exception of Agriculture and Livestock with a near zero correlation. The global financial crises and the sovereign debt crises clearly increased correlations between MSCI World index and commodities at global level, for energy, industrial metals and agriculture, although livestock remain unaltered, we can see a correlation decrease in precious metals.

At tail level less intense contemporaneous correlations is presented with frequent and strong oscillations suggesting a temporal mismatch. During crisis periods, the tail risk between Market return and commodity indices present higher contemporaneous correlations for all pairwise analysed. We can clearly notice that during the crisis period, especially during the global financial crisis, the highest values of these correlations occurred and settled

We can clearly see that during the crisis period, especially during the global financial crisis, when the highest values of these correlations were found, the correlations at the tails level were clearly higher than the correlations at the mean level.

6. CONCLUSION

This paper analyses the links between commodity and stock markets, considering the total and the tail risk.

To this end, we examining two tail risk measures - VaR and CVaR, with several approaches each commodity and stock market index over a period of fourteen years.

We study the consistence of measures over time, namely considering the periods of pre-crises, global financial and sovereign debt crisis and post-crisis. Using the dynamic conditional correlation (DCC) GARCH methodology to establish whether the correlations between commodity and stock market evolve over time and depend on the economy phase situation.

Our main findings can be summarized as follows.
First, in order to measure the tail risk, nonparametric and semi-parametric methods have performed well during all period. However, this behaviour of the considered methods is not constant over time.

Second, the tail risk of commodity markets is higher than stock market over the period, for energy, industrial metals, precious metals and agricultural indices as well as total commodities, but over the crisis period analysed the tail risk of stock market indices sharply increases to the same levels of commodities tail risk.

Third, the correlations between commodity and stock returns evolve through time. Considering the total returns, we can observe an increase of correlation over high volatility periods, particularly between 2007–2012.
At tail risk level, for all analysed pairs, commodity and stock returns, present a very high contemporaneous correlations during the crisis period.

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